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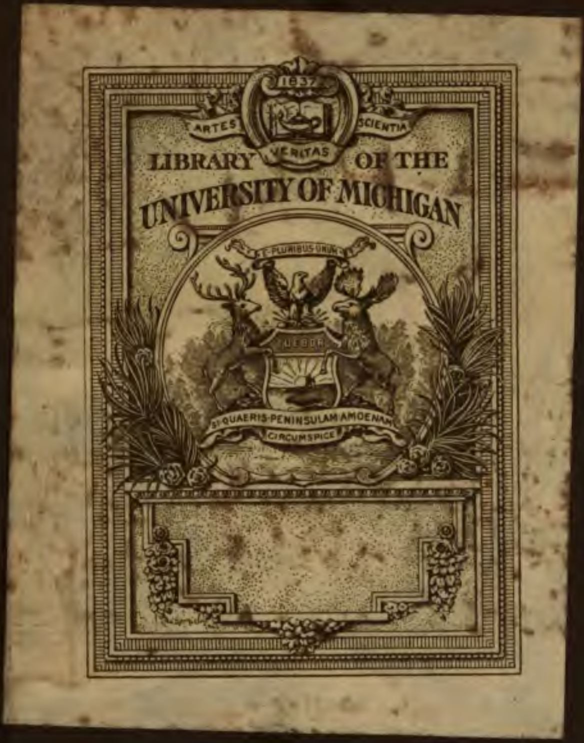

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THE DOCTRINE
OF
INTEREST AND ANNUITIES

Analytically Investigated and Explained;

TOGETHER WITH SEVERAL USEFUL TABLES.

BY FRANCIS BAILY.

ENLARGED WITH AN EXTENSIVE COLLECTION OF FORMULÆ IN COMPOUND INTEREST AND LIFE INSURANCE

SELECTED FROM THE WORKS OF

MESSRS. DODSON, SIMPSON, MORGAN, BAILY, MILNE, SCRATCHLEY,
DR. FARR, PROFESSOR DE MORGAN, THE EDITOR, AND OTHERS.

ACCOMPANIED BY NUMEROUS VALUABLE TABLES.

BY H. FILIPOWSKI.

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THE DOCTRINE OF INTEREST, ANNUITIES, AND LIFE ASSURANCE.

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KEY TO THE NOTATION

THROUGHOUT THE WHOLE OF THE FIRST VOLUME, AND PART SECOND OF THE PRESENT VOLUME, FROM PAGE 140 TO PAGE 174.

a_x	Annuity on a life aged x payable at the end of each year.
$\hat{a}_x$	Annuity on a life aged x payable at the beginning of each year.
$\hat{a}_{x\infty}$	$\frac{r+1}{r}$.
a_{x1}	Annuity on a life aged $x+1$.
a_{1x}	Annuity on a life aged $x-1$.
$a_{x\cdot y}$ or a_{xy}	Annuity to be entered on at the expiration of one year, and to last until the failure of one of the lives x, y.
$a_{\overline{xy}}$	Annuity as above, to last until the failure of the second of two lives x, y.
$a_{xy\|m}$	Annuity on two joint lives aged respectively $x+m$ and $y+m$.
$a_{x)m}$	Annuity on a life aged x for a *temporary* term of m years.
$a_{x(m}$	Annuity on a life aged x to be *deferred* m years hence.
c	Chance of living, or probability.
$c_{x)1}$ or c_x	Chance of a life aged x living one year beyond his present age.
c_{xy}	Chance of two lives, x and y, living one year longer together.
$c_{x)m}$	Chance of a life x living m years longer.
$c_{x(m}$	Chance of a life x surviving above m years.
d_x	Dying, or decrements: number dying at age x.
d	Discount, $=1-v$.
e	Expectation of life.
$f_{x)z}$	Chance of a life x *failing* before another life z.
l_x	The number living at age x.

$l_x l_y$ . .	The number living at age x multiplied by the number living at age y.
$l_x l_{y\mid m}$ . .	The same as above, but the ages respectively increased with m years.
p	Premium (office premium).
q . . .	The highest age attainable according to the tables adopted.
r . . .	Rate of interest.
v^x . . .	Value: present value of £1 due x years hence.
x, y, z . .	Represent three different lives aged respectively x, y, and z.
$x'y$ . . .	Provided x die first of two lives x, y.
xy' . . .	Provided y die first of two lives x, y.
$x''y$ . . .	Provided x die second.
xy'' . . .	Provided y die second.
C_x . . .	The number dying at age x multiplied by v^{x+1}.
D_x . . .	The number living at age x multiplied by v^x.
M_x . . .	The summation of the C column from age q to age x inclusive.
N_x . . .	The summation of the D column from age q to age x inclusive.
R_x . . .	The summation of the M column from age q to age x inclusive.
S_x . . .	The summation of the N columns from age q to age x inclusive.
λ . . .	Logarithm.
$N\lambda$ . . .	Naperian logarithm.
Π_x . . .	Single premium for an insurance of £1 on the life aged x.
π_x . . .	Annual premium for an insurance of £1 on the life aged x.

AUTHOR'S PREFACE.

The subject of the following work is certainly of great and extensive use in the present state of society. The utility of it has in all ages been manifest; and the leading principles of the science have long formed part of the elementary treatises which are common in every place of public education. At this time, however, a more than ordinary degree of attention has been given to it, arising from the great variety and extent of property which is affected by circumstances involving the consideration of this subject. For, when we consider the numerous cases of daily occurrence in which the question of *Interest* is unavoidably concerned, both Simple and Compound; when we consider the great and extensive business which is constantly transacting in the purchase and sale of *Annuities* of various kinds, immediate and reversionary, temporary and perpetual, increased no doubt by the numerous offices which have sprung up within these few years for that very purpose; when we consider also the immense quantity of lands (belonging to corporate bodies and to individuals) which are held on *Leases* for different terms of years, and which leases are continually required to be exchanged, sold, or renewed; but above all, when we consider its application in regard to our *Finances* and the state of the *National Debt*, with the help it affords us in pointing out the easiest and most effectual method of alleviating our present incumbrance; the subject assuredly acquires a degree of importance which it never before aspired to, and demands the attention of every person engaged in the public business of life.

Under this impression, I have often lamented, perhaps in common with many others, that the whole subject has never been formed into a regular system, where an easy and expeditious reference might be made for the solution of those intricate and abstruse questions which oftentimes present themselves. Yet, independent of some detached memoirs on the subject by a few celebrated mathematicians, we shall find that the elementary treatises above alluded to contain, within the limits of a few pages, all that has yet been published on the science.

The first express treatise on the subject of Interest and Annuities that I have been able to find, and that pretends to any degree of merit, is Ward's *Clauvis Usuræ*, published in the year 1709. It had, however, been handled by several authors previously thereto; and even Ward himself had inserted something thereon, in a *Compendium of Algebra* (1695), and afterwards in the *Young Mathematician's Guide* (1707). Indeed, his *Clauvis Usuræ* contains little more than a tedious explanation of some of the most common rules or formulæ, such as are inserted in pages 9, 22, 38, and 48 of the present work.

About this time Mr. Sherwin had published his *Mathematical Tables* (1706), in the preface to which Dr. Halley (a name ever to be revered in the history of mathematics) gave the investigation of the principal theorems relative to compound interest and annnities, and, at the same time, inserted some formulæ for finding the value of ρ, or the rate of interest in annuities* —a subject which appears to have been beyond Ward's capacity, as he still persisted in the old method of finding the value by Trial and Error. These theorems have been transcribed and formed into rules by various authors, but without any improvement or extension, down to the present time, and are the same which are still to be met with in all the elementary treatises on Algebra.†

In the year 1727, Mr. Smart published his very valuable *Tables of Interest and Annuities*. They are the same as the tables at the end of the present work, and will be found not only of great and extensive use, but essentially necessary in the solution of the various cases connected with this subject. I have neither time nor inclination to calculate them anew, and therefore I give them to the world "with all their imperfections on their head." I am happy, however, to observe, that after many years' experience I have not met with any errors but such as might be discovered on inspection, and that I have no reason to be dissatisfied with their accuracy.*

---

* These formulæ were given by that illustrious mathematician in his *Discourse on Compound Interest* without a demonstration. But this was afterwards supplied by M. de Moivre in a *Letter to Dr. Halley*, inserted in *The Mathematical Magazine or Philosophical Repository* for 1761, p. 41, a work which was soon dropped for want of encouragement. The demonstration here alluded to has, however, been preserved by the laudable exertions of Baron Maseres, in the *Scriptores Logarithimici*, vol. v. p. 519. It may also be seen in the *Phil. Trans.*, vol. lx. p. 508, in the preface to Callet's *Logarithms*, p. 72, and in the *Appendix* to the present work.

† See Dodson's *Antilogarithmic Canon*, 1742, p. 52. Dodson's *Mathematical Repository*, 1748, vol. i. p. 300. Simpson's *Algebra*, first published in 1745, sect. 16. Callet's *Logarithmic Tables*, p. 71. Hutton's *Course of Mathematics*, vol. i. p. 255. And other treatises of this kind.

In the *Philosophical Transactions* for the year 1770, Mr. John Robertson, the librarian to the Royal Society, has given *The Investigations of Twenty Cases of Compound Interest.*† He observes that "The late William Jones, Esq.,‡ among the variety of mathematical matters to which he gave attention, considered the business of compound interest fully, and did, many years ago, cause to be engraven on a copper-plate more cases in interest than had been exhibited before that time; several copies of impressions from that plate were distributed among his friends, to whom it appeared that he had treated this subject in a more extensive manner than had been done by other mathematicians. The theorems or rules for the cases of compound interest, without their investigations, were inserted by Mr. Jones in the quarto edition of logarithms published by Gardiner; and the rules were also communicated to Mr. Dodson, who published them, *by Mr. Jones' leave,*§ with examples to illustrate the use of his anti-logarithmic tables; but the investigations of these theorems not having yet been made public, it is apprehended that gentlemen curious in these speculations would be pleased to see them." Mr. Robertson then proceeds to investigate analytically the theorems here alluded to, and has executed his task in a very concise and luminous manner.

About five years after this, Dr. Price (whose labours on this subject are entitled to the highest respect and commendation) sent to the Royal Society a paper containing *Short and easy theorems for finding, in all cases, the differences between the values of annuities payable Yearly, and of the same annuities payable Half-yearly, Quarterly, or Momently.*‖ In this paper, the learned author gives a new method of determining the value of annuities payable at less intervals than a whole year; and the results are *different* from those obtained by the method then in use. For, it should be observed, that, hitherto, the method of finding the values of half-yearly or quarterly annuities was always on the supposition of a given *annual* rate of interest; and this is the only correct mode where the periods of the payments of the *annuity only* are supposed to vary. It is on this principle that all the preceding writers, from Mr. Ward down to the period here alluded to, have founded their rules for determining the values of such annuities; but Dr. Price has assumed as an hypothesis, that the periods for the payment of the *interest also* vary with the annuity in every such case, a circumstance by no means connected with the preceding one, since the two hypotheses are totally independent of each other.

---

* They were re-published by Mr. Brand in 1780; but, though this is called an *improved* edition in the title-page, it contains so many errors that it cannot be depended upon.

† In No. 43, vol. lx.

‡ Father of that accomplished scholar, the late Sir William Jones, and author of the *Synopsis Palmariorum Matheseos*, published in 1706.

§ This can allude to the *form* only in which these theorems are published, they being arranged in a sort of synoptic table; for all the theorems there given had been published near forty years before by Dr. Halley.

‖ *Phil. Trans.* for 1775, vol. lxvi. part 1.

M. D'Alembert, in his memoir *Sur les Annuités*, inserted in the eighth volume of his *Opuscules Mathématiques* (1780), has entered into an investigation of this very subject, and in which he supports the principles laid down by all former authors, in opposition to this new hypothesis. He has also given a very ingenious explanation of his theory by means of the Logarithmic Curve; a method, however, which had been adopted above sixty years before by Dr. Keill, in his elegant little tract *On Logarithms*, inserted at the end of his edition of Euclid.

Lastly, Mr. Baron Maseres,* whose name stands so high as the great patron of mathematical science, has given, in his *Principles of the Doctrine of Life Annuities* (1783), page 261, an investigation of the value and increase of money put out to interest at less periods than a year, and has pointed out, in a clear and perspicuous manner, the limits of the increase which takes place in such cases. To this gentleman, whose laudable exertions in the cause of literature and science entitle him to the warmest thanks of his country, we are also indebted for a valuable collection of almost everything that has been written on the subject of interest and annuities. The fifth volume of the *Scriptores Logarithmici* † contains, among other tracts, Dr. Halley's *Discourse on Compound Interest;* M. de Moivre's *Letter to Dr. Halley;* Mr. John Robertson's *Investigations, &c.;* Mr. William Jones' *Theorems;* Mr. Dodson's explanation and use of the same as inserted in his *Antilogarithmic Canon*, and in his *Mathematical Repository;* all those parts of Mr. Ward's *Young Mathematician's Guide* which relate to interest and annuities, and the whole of his *Clavis*

---

* Francis Maseres, Esq., Cursitor Baron of His Majesty's Court of Exchequer.

† Or a "Collection of several curious tracts on the nature and construction of Logarithms," published at a great labour and expense by Baron Maseres, with the laudable motive of disseminating useful and scientific knowledge, and of rescuing from oblivion many scarce and valuable tracts on the science.

Usuræ, to which the learned editor has added several valuable notes of his own, in order to clear up in a satisfactory manner such passages as may appear rather dark or intricate in the original.

Notwithstanding the labours of these authors, much still remained to be done. Something was wanting in each; and the whole collectively would not form a complete treatise on the science. This I had often experienced in my researches on the subject, and in the investigation of those cases which were laid before me for solution; I therefore set about to compile one for my own use, to which I might always refer with expedition and confidence, and where the unity of design would not be destroyed by a confusion of characters or symbols.* The consequence has been (what, indeed, is common in such cases), that the work increased under my hands to an extent which I hardly expected. In the progress of my inquiries, a variety of new matter presented itself, and I found that the subject not only wanted new-modelling, but was capable of being treated much more extensively than it had hitherto been done.

Such was the origin of the present treatise, which (as I have just observed) was at first intended solely for my own private use, and which has been appropriated by me to that purpose for some years past. But, as others, who are pursuing the same inquiries, may have met with and will continue to meet with similar difficulties and inconveniences, I have conceived that I should be rendering an acceptable service to the public in thus making known the fruits of my labour, and thereby rendering the path more open and easy to those who, in future, may be led, either by inclination or duty, to turn their attention to subjects of this kind.

Should any experienced mathematician honour these pages with a perusal, I would inform him, beforehand, that he will meet with nothing here that is curious or profound: the subject indeed will not admit of it. My object has been to accommodate the work, as much as possible, to those who are acquainted with the first principles only of Algebra: and I apprehend that such as can readily solve a Simple Equation, and have a thorough knowledge of the nature and use of Logarithms and the Method of Series, will find little or no difficulty in their progress through the work.* The more expert analyst, however, may occasionally consult it with advantage to himself; as he will not only find some points that are new, but also, that the various theorems here given will afford him an easy reference for the solution of most cases which may engage his attention. I have chiefly aimed at clearness and perspicuity; being well persuaded that the true ends of science are only retarded by an affectation of profundity and brevity.

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* That which might be deficient in one of these treatises would be but ill supplied by another, owing to the very circumstance here mentioned. For one author denotes the *amount* of £1 by r; another denotes the *rate of interest* by r, and the *amount* by $(1+r)$; some express the *annuity* by a, and the *amount* by m; whilst others again invert this method, and express the *annuity* by m, and the *amount* by a. These things, though perhaps trifling in themselves, do not much *assist* a reader when he is turning to different authors for an *explanation*.

In the first and four following chapters (the superstructure of all the rest) I have entered into a full investigation of the doctrine of INTEREST, both Simple and Compound: and have shown the various results which arise according to the periods at which such interest is payable. The next six chapters contain the principles of the doctrine of ANNUITIES, with their several affections not only according to the times of the payment of interest, but also according to the periods at which the annuity itself becomes due and is payable.† The twelfth and thirteenth chapters contain a full exposition of the doctrine of REVERSIONS and of the RENEWAL OF LEASES; together with several useful tables for calculating the value of the Fines which ought to be paid for the renewal of leases held under corporations and colleges. The four subsequent chapters contain an investigation of several useful and curious points which could not properly be classed under the preceding heads: and which are indeed of sufficient importance to form distinct sections of themselves. The last chapter is devoted principally to the application of this doctrine to various subjects in FINANCE; and in this part I have inserted several new formulæ, which, I think, may be very convenient and useful to such persons as have directed their attention to these studies.

The process for finding the value of ρ, or the rate of interest, in Annuities, being somewhat long and tedious, I thought it best not to interrupt the order of the work by inserting it in each place where the formula is given (see pp. 38, 48, &c.), but have referred the reader to the several notes in the APPENDIX, where he will find the subject treated in a full and comprehensive manner. The last Note is confined to the application of the properties of the Logarithmic Curve to the doctrine of interest and annuities: those who are curious in this branch of mathematics will readily excuse the digression; and those who consult the work with other views are at liberty to overlook it altogether. The former may improve on the ideas there suggested; and the latter will lose nothing by the omission.

---

* Some few of the notes, both in the body of the work and in the Appendix, may perhaps be considered as exceptions to this general remark.

† Many persons may perhaps think that I have been too minute in my inquiries on this subject; and that the real differences which arise in such cases are not worth our attention. But I would observe here, once for all, that most of the cases stated in the body of the work are such as frequently may, and *do* arise in practice; and where information is consequently desirable. The reader, however, must judge for himself *how far* it may be necessary to attend to the distinctions which I have made: but even on this subject he would be incapable of deciding, had I not pointed out the *limits* which arise in these cases.

In order that the TABLES may be more easily consulted, they are all arranged together at the end. The method of forming these tables has been fully explained in the body of the work; so that nothing more need be said on the subject here: but it may be necessary to make some observations on the SUPPLEMENTARY TABLES, which are annexed thereto. They are added for the purpose of facilitating the solution of questions connected with this subject, and for the prevention of those mistakes which would otherwise often arise. The FIRST of these additional tables may be considered as a supplement to Table III., since it shows the amount of £1 at compound interest, at the *annual* rates there given, for periods less than one whole year: by means of which we may determine the amount when any fractional parts of a year are concerned. The method of using it will be evident from an inspection of the formula in § 19, where ν will, in this case, denote a proper fractional quantity. It would have been easy to have inserted a similar table for determining the present value of £1 for periods less than a year; but, since these values would be only the reciprocals of the values in the table just mentioned, and as the values in both tables are so readily and expeditiously found by means of logarithms, it will be unnecessary to enlarge more on the subject. The SECOND and THIRD of these additional tables show respectively the decimal parts of a pound corresponding to every farthing, and the decimal parts of a year corresponding to every day; the use and application of which are so evident as to require no further observation.

I would here observe, that the present may be considered as the *first* part only of the work which I have in view. The *second* part, should I find leisure to complete it, will contain the doctrine of LIFE ANNUITIES and ASSURANCES.

F. B.

GRAY'S INN, *April* 28, 1808.

INTRODUCTION BY THE EDITOR.

The present volume, styled "The second volume of Baily," was published by the author *first*. But owing to the fact of the present edition of the same containing an additional part with reference to the Doctrine of Life Annuities and Assurances, it is justly styled volume the *second* of Baily.

The First Part of this volume is a faithful reprint of Baily's original work in quarto, "On Interest and Annuities," published by him in 1808, to which is now appended the paper read by him before the Royal Society, on the method of Barret for calculating the value of Life Annuities by means of artificial Tables. This paper was originally published by the author in the year 1813, three years after the publication of his work on the "Doctrine of Life Annuities and Assurances." It is now reproduced to satisfy those who criticised my omission of the same in my new edition of Baily's first volume.

The Second Part of the present work contains a collection of formulæ, in the new Notation, for solving all questions in Compound Interest, and Life Annuities and Assurances relating to Single Lives, Two Lives, and Three Lives, accompanied by a variety of useful Tables, calculated on the basis of the English Life Table, No. 3.

In selecting the formulæ from the various authors on the subject of interest and life annuities, the reader will easily conceive what amount of care was required in transferring the different fanciful notations adopted by each author respectively to one uniform notation. The actuary is fully aware of the great difference prevalent among the various writers, such as Baily, Milne, and a host of others too numerous to tabulate here.

On the Present Notation.

A notation to represent certain expressions is of course arbitrary, and may be laid down at pleasure, but the principal aim ought to be to introduce such symbols as may be *remembered*, and easy not only for reading but also for *writing*. Milne, for instance, used the symbol ⊙ to represent the *annual* premium: because that character in astronomical ephemeris represents the *sun*, which causes an *annual* revolution. Similarly, the probability of a life *dying* is represented by him by *black letter*, a sign for *death;* but he does not continue to carry his system through. He uses Gothic type, Italics, hollow letters, etc., to designate various mathematical expressions, none of which are easily written. It is, in fact, necessary to be an artist in order to do justice to his scheme of notation. Suppose, even, that all his symbols were sufficiently distinguishable in *print*, it must be admitted that they are by no means expedient for manuscript, to say the least, that they present too cabalistic an appearance. Baily adopts the Italic capitals to represent annuities, and Roman capitals to designate the lives; whilst Milne prefers the Italics for the latter, and Roman small capitals for the former. All such distinctions are sufficient in print, but not so in writing. Hardy's proposed notation is equally inconvenient for many reasons, as is well known to all actuaries.

The notation adopted in Part II. of the present, and the whole of the preceding volume—with the exception of a very few symbols —differs very little from that of Dr. Farr, David Jones, and the like of modern writers on the subject of Life Probabilities. A reason will be assigned for each improvement here introduced. An annuity, as is well known, constitutes the total of a series of annual values, each representing the present value of £1 payable at the end of one year, two years, three years, etc., to the end of life of a certain individual aged x. Suppose, then, that a temporary annuity were to be described, it would be but rational to cut off part of the series by a stroke, or a perpendicular line. Thus, from the beginning of the series on the left of the stroke, would form the *temporary* annuity, whilst the remaining part on its right side would form the *deferred* annuity. Now, as a straight stroke would leave it doubtful as to whether the annuity was intended to be temporary or deferred, the best sign to be chosen for that purpose I considered to be a parenthesis; namely, if its horns be turned to the left, thus), it points to the *beginning* of the series, thereby describing a *temporary* annuity; whereas if the horns incline to the right, thus (, the annuity is a *deferred* one, inasmuch as they point to the *end* of the series. For the same reason, two parenthesis, thus ( ), designate a *deferred temporary*, or *intercepted* annuity, as it cuts a certain number of terms out the middle of the series. Hence we have $a_{x)m} =$ a temporary annuity, $a_{x(m} =$ a deferred annuity, $a_{x(m)} =$ a deferred temporary annuity.

${}_{x1}$ and ${}_{1x}$.—This mode of expression has been preferred to $x+1$ and $x-1$, in order to give a better appearance to, as also to avoid the unnecessary extension of numerous equations,—especially as such can easily be remembered. We see such in all Roman dates, on dials, etc. $\text{XI} = 10 + 1 = 11$, and $\text{IX} = 10 - 1 = 9$. This change saves a vast deal of confusion.[1] The sign | has been introduced first by Professor de Morgan to signify a sort of minus, for instance $\text{M}_{x|n} = \text{M}_x - \text{M}_{x+n}$, whilst the sign ‖ introduced here indicates a sort of plus, viz., $a_{xy\|m} = a_{x+m,\, y+m}$. $l_x l_{y\|m} = l_{x+m} \times l_{y+m}$.

The letter c has been selected for "*chance*," *i.e.*, probability of living, in lieu of p, as the latter is often required for "premium." And whereas the "probability of a life living *one* year beyond his present age" is the most frequent in all questions of insurance, the symbol c_x is often used for $c_{x)1}$ to express that probability. A few more changes will be observed, which the reader will easily discern, and he will have no difficulty to find a motive for their adoption. On the whole, I have studied to dispense with double and treble lines, as used by Milne, Hardy, and others. In the present notation, every contingency can be written in one line without the least inconvenience.

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[1] Milne has adopted a somewhat similar symbol, such as ${}_{,}a = l_{x-1}$, $a_{,} = l_{x+1}$.

In the arrangement of the D and N columns, I followed the plan of Dr. Farr, namely, $D_q = N_q$.[1] But as this change in 1, in the arrangement of the Commutation Tables often causes confusion to such students who have accustomed themselves to the arrangement of Davies, Jones, and others (as one is apt to forget which of the two authors is to be increased or diminished with 1), I give here a rule which will always guard the student against committing an error in the use of the formulæ of either. Davies was the first to raise the N column with one, which was followed by all subsequent writers; and it was Farr who restored the N column to its legitimate place. When we bear in mind that the initials of the two respective authors are D and F, one can easily remember that there is one letter in the alphabet intervening between the two initials, namely the letter E. The student, therefore, in case of the N and S columns of each formula relating to Davies' Tables, must always increase the age x with one, in order to come up to F,[2] that is, to use Farr's Tables; and, *vice versa,* he must diminish the age x with one, when applying the formulæ of F to the Tables of D = Davies. This rule, of course, is due to the mere coincidence of D and F, being one letter distant from each other in the alphabetical order, and thereby affording a convenient means for remembrance.

The formulæ contained in the second part of this volume relate both to Compound Interest and Life Annuities and Assurances, or both combined. The greater part of them relate to the first volume of Baily, the rest are selected from different authors, as specified in the Tables of Reference below. Among them will be found a variety of very ingenious theorems by the distinguished actuary Mr. Scratchley, M.A. They are reduced to the greatest simplicity possible, and are exceedingly useful. Not less than fourteen formulæ in Compound Interest and seven in Life Contingencies are claimed by Mr. S. as new and original.[1]

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[1] The reason for this change will be found in my edition of Vol. I. § 39.

[2] This rule holds good also with reference to all formulæ and Tables contained in Part II. of the present work: F being also the initial of the Editor.

TABLE OF REFERENCE, No. I.

§ in Present Work.	§ in Baily.	§ in Milne.	§ in Present Work.	§ in Baily.	§ in Milne.
126	...	231 cor. 3	159	311	391 cor. 2
127[2]	55, 56	......	160	312	391 „ 3
128	...	244 „ 3	161	304	393
129	...	244 „ 4	162	306	394 „ 2
130	...	244 „ 2	163	307	394 „ 3
131	...	244 „ 5	164	...	396
132	...	260	165	317	398
133[3]	94, 96	......	166	318	398 „ 2
134[4]	89	......	167	319	398 „ 3
135	...	267	168	314	400
136	...	277	169	321	402
137	120	253	170	322	403
138	112	254	171	323	403 „ 2
139	98-102	255	172	324	403 „ 3
140[5]	87	256	173	...	405
141	...	257	174	...	407
142	...	258	175	...	409
143	...	280	176	331	411
144	246	360	177	333	412 „ 2
145	256	362	178	334	412 „ 3
146	259	364	179	326	414
147	362	366	180	328	415 „ 2
148	266	368	181	329	415 „ 3
149	264	370	182	281	417
150	269	372	183	290	418
151	276	374	184	291	419 „ 1
152	278	376	185	292	419 „ 2
153	296	379	186	293	419 „ 3
154	...	382	187	283	421
155	298	384	188	285	422 „ 1
156	300	388 cor. 2	189	286	422 „ 2
157	301	388 „ 3	190	287	422 „ 3
158	309	390			

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[1] They are the following:—§§ 3, 10, 11, 12, 13, 16, 19, 22, 23, 25, 28, 31, 32, 33, 41, 42, 44, 45, 46, 47, and 48.

[2] Also Jones, § 144.

[3] Also Simpson, Prob. 19; Dodson, vol. iii., Qu. 74; and Jones, § 170.

[4] Also Jones, § 171.

[5] Also Dodson, vol. iii., Qu. 75; and Jones, § 169.

TABLE OF REFERENCE, No. II.

§ in Present Work.	Philosophical Transactions.	Morgan.	Dodson.	Simpson's Supplement.
144	1791, Prob. 1, p. 248	Prob. 19	Vol. iii. Qu. 32	... ..
145	„ „ 2, p. 253	„ 20	......	......
146	„ „ 3, p. 256	„ 21	......	Prob. 30
147	„ „ 5, p. 261	„ 24	......	„ 40
148	„ „ 7, p. 265	„ 26	......	......
149	„ „ 6, p. 263	„ 25	......	......
150	„ „ 8, p. 267	......	......	... ..
151	„ „ 8, p. 269	......	......	......
152	„ „ 10, p. 272	......	......	......
153	1789, p. 41	„ 18	Vol. iii. Qus. 42-47	„ 38
155	1794, Prob. 6, p. 253	„ 29	......	......
158	1800, „ 6, p. 37	......	......	......
161	1800, „ 5, p. 33	......	......	......
164	1794, „ 5, p. 247	„ 23	......	......
168	1791, „ 4, p. 258	„ 22	Vol. iii. Qu. 48	„ 39
176	1794, „ 4, p. 245 1800, „ 7, p. 40	......	......	......
179	1794, „ 3, p. 242	„ 28	......	......
182	1800, „ 1, p. 22	......	......	......
183	„ „ 3, p. 28	......	......	......
187	„ „ 2, p. 23	......	......	......

From the above Tables of Reference, the reader will perceive that Milne has literally copied the whole series of problems contained in Baily's work—the only difference consisting in the notation. What must strike every reader is, that notwithstanding this fact Milne did not even condescend to name the author from whom he has principally gathered his knowledge on the subject, and to whom he was truly indebted for the publication of his work, which cannot but be regarded as a new edition of Baily with a different notation. Yet, strange to say, there are, nevertheless, some individuals in the present day who give preference to Milne, and pronounce Baily's work as a mere curiosity![1] For my part, I am not aware of a work to equal the latter. The only fault I find in Baily's style is that he is too plain, and too fond of constant repetition, if a fault such can be named.

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[1] For instance, a reviewer in No. 70 of the *Insurance Record*, under the cloak of its editor, assumes rather a high tone of a universal critic, and pronounces a verdict against Baily's work in general, and against the new edition in particular. The former he qualifies as "rather a curiosity than a work of daily reference," though he condescends to admit that "there are still many of the *older* actuaries who use it more or less." From this mode of expression, one has reason to suppose that the reviewer in question is one of the young students for actuaryship.* Assuming this to be the case, there is

* It appears that the reviewer is a Mr. Adler, clerk to an Insurance Office, who fancies himself to be an actuary of no mean capacity.

OF THE ORIGINAL TABLES.

There are at the end of this volume no less than 17 Tables, both for males and females, on the basis of the English Life Table, No. 3, exclusive of the "Key to Premiums." Of these the following are original:—

TABLE V. M column in numbers and logarithms, 3 per cent.
VII. Annuities, 3½, 4½, and 6 per cent.
XII. For calculating the probability of a life x living one or more years.
XIV. Annuities on two joint lives for quinquennial combinations, 3 per cent.
XV. Annuities on the longest of two lives for quinquennial combinations, 3 per cent.
XVI. For calculating temporary and deferred annuities, 3 per cent.
XVII. For converting an *ordinary* assurance into an *endowment* assurance in consideration of bonuses; or, in other words, to make ordinary policies payable during the lifetime of *males.*

All the Tables contained in the present volume are sufficiently defined in their headings, and are described in the two volumes

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scarcely need to take notice of such criticism. He talks of "many modern and more extensive publications," though he does not name even a single one which might surpass Baily's work. The whole catalogue of practical books on insurance is published almost weekly in the *Insurance Record*, and I am at a loss to find among them one to compete with Baily. As for David Jones' book, every actuary is aware, *old* or *young*, that its popularity is owing to the great collection of Tables it contains, and to its cheapness. The only work to range at all with Baily is that of Milne; and from what has been described in the text, the reader will easily judge for himself whether it has any claim to superiority. The only real novelty I can discover in Milne is the original construction of the Carlisle Table, which, in fact, is the true cause of having rendered his work popular.

respectively; but Tables XII., XVI., and XVII. not being described in the body of the work, I do so here.

Use and Application of Table XII.

This Table consists of three columns, both for males and females. Col. I. contains the logarithms of the chance of a life x living one year above his present age; Col. II. contains the logarithms of the chance of a life x living to the highest age of the English Mortality Table, No. 3; while Col. III. forms the complement of Col. I.

To find the logarithm of the chance of a life x living m years above his present age, subtract log. $x + m$ of Col. II. from log. x in the same column; the difference is the logarithm required.

Example.—The logarithm of the chance of a life aged 30 attaining the age of 40, $= 3{\cdot}7180784 - 3{\cdot}7670290 = \bar{1}{\cdot}9510494$ ($= \lambda$ age 30 $- \lambda$ age 40).

To form a series of logarithms of the annual chances of life from age x to age $x + m$, the computor has to start with log. x in Col. I., and add the same consecutively to the age proposed inclusive. The last result of such summation can be checked by Col. II., in the manner described above, and if found to agree, it proves the correctness of the entire series so obtained.

Table XVI.

This Table is likewise formed of three columns, for males and females, on the basis of the English Life Table, No. III., and 3 per cent interest. Col. I. contains the logarithms of the chance of a life x living one year above his present age. Col. II. is composed of Col. I. increased by the logarithm of v, at 3 per cent; or, in other words, each logarithm in Col. I. represents the value of £1 to be received at the end of the *first* year on a life aged x; while Col. III. is composed of Col. I. increased by log. $v^{(109-x)}$.

Note.—The indices in Col. III., though in a *positive* form, are in reality the complements of *negatives* in their character.

To form a series of logarithms expressive of the annual values of which a temporary annuity for m years on a life x is composed, the computor has to start with log. x of Col. II., and add it to the next following logarithm opposite age $x + 1$ in the same column, the sum again added to $\lambda(x+2)$, the next sum to $\lambda(x+3)$, etc. The logarithmic series so obtained expresses the annual values of each year's payment separately, whilst the total of their respective numbers forms the total temporary annuity for m years on that life.

The object of the process thus described is not to facilitate the working of temporary annuities, which are simple enough by means of the D and N columns, but is intended solely for the formation of a table of temporary annuities from year to year. The advantage to be derived from the above process is, that the ultimate logarithm in each series tests the correctness of all the logarithms preceding it; the former being itself easily proved by log. x — log. $(x + m)$ of Col. III.

Example.—Required to form a series of logarithms to represent the several annual values of which the temporary annuity for a term of 10 years on a life aged 20 is to be composed?

Answer.—The logarithm opposite age 20 in Col. II., representing the first year's value, is $\bar{1}\cdot9835502$; this added to the next logarithm of the second year's value, and continuing the process in the same manner, the 10th year's logarithm will be found to be $= \bar{1}\cdot8320276$, $(= 2\cdot5359670 - 2\cdot7039394$ of Col. III.) The logarithms are then to be converted into numbers respectively, and a consecutive summation formed in the same manner as the logarithms themselves; the last result will give the total of the temporary annuity required, and on being checked by the formula $\frac{N_{x|m}}{D_x}$, it will prove the correctness of the whole series of the natural numbers found to the respective logarithms.

In case of *joint lives*, the computor has to add to each logarithm obtained by the process just described, the respective logarithms of the second life involved (out of Col. I.), the sums represent similarly the annual values on the two lives in question. Suppose, for instance, that a life aged 30 is to be coupled with another aged 20. Add $\bar{1}\cdot9956026$ (opposite age 30 in Col. I.) to $\bar{1}\cdot9835502$ (Col. II_{20}); again, $\bar{1}\cdot9955111$ (Col. I_{31}) to $\bar{1}\cdot9670200$ ($= II_{20} + II_{21}$); and so on to the extent proposed.

TABLE XVII.[1]

By means of this Table can be ascertained with facility the age at which a whole life policy may become payable to the assured, if he live so long, in consideration of bonuses.

Example 1.—A person aged 30 effected an assurance on his life for £100, payable at death, and after the policy having been five years in force, a cash bonus was declared of £4,—at what age will his policy become payable, if he live so long, in consideration of such bonus?

Answer.—Find the tabular value in Col. 30 (age at entry), opposite age 35 (age at valuation), and multiply the same by the amount of bonus, the result N_E (the Endowment assurance sought), when compared with the N column, 3 per cent (page 235), will be found to be contained between N_{72} and N_{73}. Consequently the age at which the policy becomes payable is 72.

If, in the above instance, it be required to ascertain the *fraction* of a year, say *quarters*, multiply $N_E - N_{73}$ (the difference of N_{73} and the result found above) by 4, and divide it by D_{72}, the quotient will show the number of quarters to be added to the age of the lesser N minus 1.

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[1] The premiums are calculated on the basis of the English Life Table, No. II., and 3 per cent interest, including 20 per cent loading. For the formula of the Table, see p. 158. It has originally been investigated by the able Mr. Sprague, Actuary to the "Equity and Law Life Office," in Vol. VI. of the *Assurance Magazine*, p. 346, and subsequently much simplified by Mr. Scratchley. The examples given by Mr. Sprague do not seem to agree with the D and N Table of the Experience 4 per cent. This Table has been substituted in lieu of the "Joint Life Table" on the *Equitable Experience*, which was originally intended to be inserted in this volume, owing to the MS. of the latter Table having been accidentally destroyed by fire on the premises of the printers. But it is hoped that the present Table will be found to be of much more service.

If months be required, multiply $N_E - N_{73}$ by 12, and divide the product equally by D_{72}, the quotient will show the number of months to be added to the age of the lesser N minus 1. Thus—

Table XVII. Col. 30, age at valuation 35, = 19142
Amount of Bonus = 4

$N_E = 76568$
$N_{73} = 70622$

$N_E - N_{73} = 5946$
Quarters in a year = 4

$D_{72} = 11776)23784(2{\cdot}02$ Quarters.

Hence the age required is 71 years and 2 quarters.

Or, $N_E - N_{73} = 5946$
Months in a year = 12

11892
5946

$D_{72} = 11776)71352(6{\cdot}06$ Months,

giving as the age required 71 years and 6 months.

Example 2.—Suppose after three more years a second bonus of £2, 10s. be declared on the above policy, the age sought can be found in a similar manner. The tabular result (16917) in Col. 30, opposite age 38 (age at valuation), is to be multiplied by 2·5, the product (42293) to be added to the result found before, and the sum (118861) similarly entered in Col. N (page 235), where it will be found to be contained between N_{69} and N_{70}. Hence the age required is 69. Or, in quarters, = 68 years and 2·25 quarters, or, in months, = 68 years and 7 months.

Writing T for result in Table XVII.,
B for Bonus,
N_m for the lesser N,
N_{m1} for the next greater N,

we shall have

$$\text{For whole years, } N_E = TB,$$

$$\text{For quarters, } N_E = TB + \frac{4(N_E - N_{m1})}{D_m},$$

$$\text{For months, } N_E = TB + \frac{12(N_E - N_{m1})}{D_m}.$$

Whatever be the number and amounts of the bonuses, the values in Table XVII. must always be multiplied by the bonuses respectively, and the products then added together for comparison with the N column in page 235.[1]

The following Tables, described in the present volume in a certain order, are to be found in Vol. I., thus :—

TABLE III. (§ 18). The amount of £1 in any number of years. Vol. I. Table I.

„ IV. (§ 38). The present value of £1 due any number of years.[2] Vol. I. Table III.

„ V. (§ 60). The amount of £1 per annum in any number of years. Vol. I. Table II.

„ VI. (§ 74). The present value of £1 per annum due any number of years. Vol. I. Table IV.

„ VII. (§ 76). The annuity which £1 will purchase for any number of years. Vol. I. Table V.

SUP. TABLE I. (§ 18). The amount of £1 per annum in any number of days. Vol. II. Table III.

„ II. The decimals of £1 corresponding to every farthing; and

„ III. The decimals of a year corresponding to every day are omitted in this edition, as they are found in almost every book of a similar character.

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[1] A similar Table might be constructed for a *reversionary* bonus of £1, in lieu of a *cash* bonus, by employing the formula $\frac{M_{x+n}}{d+p_x}$ instead of $\frac{D_{x+n}}{d+p_x}$, namely, the latter being multiplied into the reversion of £1 on the life aged $x+n$.

[2] The words "Per Annum" in the heading of that Table are to be struck out.

Several footnotes in Vol. I. have reference to the pages in the quarto edition of the present work, and are to be adjusted according to the pagination of the new edition, as follows :—

Vol. I., New Edition.	Quarto Edition.	Vol. II., New Edition.
Page 26 Note 1.	Page 16.	Page 15.
,, 35 ,,	,, 65 Note.	,, 60 Note.
,, 188 ,,	,, 89.	,, 80.
,, 199 ,,	,, 56.	,, 53.
,, 199 Note 2.	,, 89.	,, 80.
,, 199 Note 3.	,, 56.	,, 53.
,, 199 Note 4.	,, 58.	,, 54.
,, 200 Note 1.	,, 55.	,, 50.
,, 203 ,,	,, 46.	,, 43.

NUMBERS AND LOGARITHMS OF v AND d,

At several Rates of Interest.

Per Cent.	v in Numbers.	v in Logarithms.	d in Numbers.	d in Logarithms.	Per Cent.
3	·9708738	$\bar{1}$·9871628	·0291262	$\bar{2}$·4642840	3
3½	·9661836	$\bar{1}$·9850597	·0338164	$\bar{2}$·5291278	3½
4	9615385	$\bar{1}$·9829667	·0384615	$\bar{2}$·5850268	4
4½	·9569378	$\bar{1}$·9808837	·0430622	$\bar{2}$·6340962	4½
5	·9523810	$\bar{1}$ 9788107	·0476191	$\bar{2}$·6777808	5
6	·9433962	$\bar{1}$·9746941	·0566038	$\bar{2}$·7528453	6
7	·9345794	$\bar{1}$·9706162	·0654206	$\bar{2}$·8157143	7
8	·9259259	$\bar{1}$·9665762	·0740741	$\bar{2}$·8696662	8
9	·9174312	$\bar{1}$·9625735	·0825688	$\bar{2}$·9168161	9
10	·9090909	$\bar{1}$·9586073	·0909091	$\bar{2}$·9586073	10

In conclusion, the Editor has to apologize for some errors unavoidably left in the first volume, and he has reason to believe that the *errata* supplied in page 212 of the present volume, including the few specified on next page, are all that can be found. As for this volume, the Editor flatters himself with the hope that it contains no errors whatever.

H. F.

LONDON, *Jan.* 26, 1866.

Additional Corrections to Vol. I.

Page 24, last line but two: for l_{n-x}, *read* l_{x+n-y}.
„ 24, last line: for $l_{n-x}\, l_{n-y}$, *read* $l_{x+n-z}\, l_{y+n-z}$.
„ 25, line 6: for l_{n-x}, *read* l_n.
„ 25, line 10: for l_{n-y}, *read* l_n.
„ 26, line 20: for l_{z+n}, *read* l_{z_1+n}.
„ 40, line 5: at the close of the sentence add "divided by l_x."
„ 95, last line but one: for $-r$, *read* $-d$.

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Raleigh, Samuel, Esq.,	Scottish Widows' Fund,	3
Ratray, William, Esq.,	Victoria Life Office,	1
Scratchley, Arthur, Esq.,	Western Life Office,	1
Shrubb, Sam. Jno., Esq.,	Hercules Fire and Life,	1
Smith, David, Esq.,	North British and Mercantile,	1
Spottiswoode, William, Esq.,	Her Majesty's Printing Office,	1
Sprague, T. B., Esq.,	Equity and Law,	1
Thomson, George, Esq.,	Albion Life Office,	1
Turnbull, Andrew H., Esq.,	City of Glasgow,	1
Wheeler, G. W., Esq.,	Friend-in-Need Life Office,	1
Whytt, A. H., Esq.	Scottish National, London Office,	1
Williams, Henry Jones, Esq.,	Standard Life Office,	1
Williams, J. Hill, Esq.,	English and Scottish Law Life,	2
Wilks, W. G., Esq.,	Liverpool and London,	1
Wilks, John, Esq.,	Standard Life Office,	7
Willich, Charles M., Esq.,	University Life Office,	1
Wilson, J. M., Esq.,	Queen Life Office,	1
Winser, Thomas B., Esq.,	Royal Exchange,	2
Woolhouse, W. S. B., Esq.,	International,	1
Younger, Samuel, Esq.,	English and Irish,	1

THE DOCTRINE

OF

INTEREST AND ANNUITIES.

CHAPTER I.

ON INTEREST IN GENERAL.

§ 1. INTEREST is the premium or sum of money given for the loan or use of another sum of money called the PRINCIPAL, which is lent for a limited time according to some certain RATE (generally estimated at so much per *cent*) agreed upon between the borrower and the lender; and it is always proportional to the time. Thus, if a person borrows £100 for twelve months, and at the end of that time pays *four* pounds for the loan of it, he is said to pay *four* per cent per annum for the use of the money: or, if he pays *five* pounds for the loan of it, he is said to pay *five* per cent per annum for the use of the money: and so for any greater or less sum, or for any longer or shorter period. For, if a person pays *two* pounds for the loan of £100 for *half* a year; or *eight* pounds for the loan of £200 for *one* year, he is still said to pay an interest after the RATE of four per cent per annum for the use of the money.

§ 2. This rate of interest, or the premium which may be made of money, differs according to the commercial and political state of a nation: but in most countries it is established that no individual shall demand *above* a certain rate; which rate is commonly called the Legal Interest:* and if more than this be taken it is *now* called Usury. In the year 1545 (by 37 Henry VIII. c. 9) the legal interest in England (or the legal *usury*, as it was then called,) was fixed at 10 per cent; prior to which time, it does not appear that any limit had been established. We should here observe that the word *usury* was, formerly, always used to express *any* rate or premium (whether great or small) paid for the loan of money; and it is not till the Act of James I. that we meet with the word *interest;* and even there it is adopted as synonymous with *usury.* In all the violent outcry, therefore, which narrow-minded policy or blind religious zeal formerly raised against what was called the crime of usury, we must not suppose that the attack was against *illegal* interest, but against taking *any* interest at all for the use of money. It was this spirit that produced the Act in 1552 (5 and 6 Edward VI. c. 20) which repealed the Act of Henry VIII. above mentioned, and prohibited "any manner of usury or increase to be received or hoped for, above the sum lent:"* an Act which continued in force nineteen years, and was at length repealed by an Act in 1571 (13 Elizabeth, c. 8), which at the same time revived the Statute of Henry VIII. prohibiting all usury (or interest) above 10 per cent. Such, however, was the temper of the times when this Act was made, that the Parliament thought it proper to let their protest go with it; for in this same Statute it is declared that "All usury, being forbidden by the law of God, is sin and detestable,"—a proof of the pliant consciences of our ancient legislators, who could tolerate, in this manner, an acknowledged *crime.* The legal rate of usury continued at 10 per cent till 1624, when (by 21 James I. c. 17) it was made penal to take above 8 per cent.: and even in this Statute it was considered equally proper to declare that "No words in this law contained shall be construed or expounded to allow the practice of usury, in point of *religion* or *conscience.*"† It was afterwards reduced to 6 per cent by an ordinance of the Rump Parliament in 1651, which was confirmed in 1660 (by 12 Charles II. c. 13): and lastly, it was further reduced to 5 per cent in 1714 (by 12 Anne, st. 2, c. 16,)‡ at which rate it now stands. As there are circumstances, however, under which different rates from any of these may be taken, it is necessary that the subject should be treated in its fullest extent.

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* The utility or policy of such a law may be much questioned. Money is a kind of trading commodity which, like every other article of commerce, rises and falls according to the demand for it in the market; and, notwithstanding the existing laws against it, I have known 8, 10, 20 per cent, and even higher rates of interest, currently given for

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the loan of money in times of great scarcity. The transactions of Government agents are protected by particular laws when they undertake to give a rate of interest above the usual legal rate; and the same indulgence has been lately granted to the East India Company; but individuals are not so favoured, and must depend on the honour of those whom they trust if they render any assistance in times of great difficulty and distress.

* The clergy had, at several times prior to this, forbidden any allowance or compensation for the use (*usura*) of money; a measure which was sanctioned by different monarchs both in this and other countries, viz., Charlemagne, Constantine Porphyrogenitus, Henry I., Richard I., Henry VII., &c. &c. This latter sovereign calls them all "dampnable bargayns."

† This is the Act in which the word *interest* is first mentioned; though *usury* is still retained and made use of as synonymous thereto.

‡ The word *usury* is still retained, even in this Act, as the legal term for a fair interest of money.

§ 3. Interest is generally divided into SIMPLE and COMPOUND:[*] and the principal investigations respecting them may be divided into two parts, viz., "The method of determining the *amount* of any given sum at the end of any given time;" and "The method of determining the *present value* of any given sum due at the end of any given time." To the former case belong all questions of *interest* properly so called; and to the latter belongs the subject of *discount*.

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CHAPTER II.

ON SIMPLE INTEREST.

§ 4. SIMPLE INTEREST is that which is reckoned and allowed upon the principal only during the whole time of the loan: and the sum of principal and interest in any given time is called the *Amount*. Thus, the interest of £100 in one year, at 5 per cent per annum, is £5; and the sum of principal and interest at the end of that time, or the Amount, is £105. As the interest of any sum, for any given time, is directly proportional to the principal and that time, we can always (when once the rate is agreed upon) precisely ascertain its value at the end of any given period of time, or on any given sum, by means of the following RULE, viz.:—"As £1 is to the rate of interest,[†] so is any given principal to its interest for one year: and then, as one year is to any given time, so is the interest for one year, just found, to the interest of the given sum for that time." This is a question in the common rule of five, or double rule of three, and is sufficiently explained in most of the elementary treatises on Arithmetic.

§ 5. In order to generalize what has here been laid down, let us make

s = the Amount,
p = the Principal,
n = the Number of Years,[‡]
r = the Rate of Interest;[§]

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[*] The latter, however, is only a different modification of the former, as will more plainly appear in the course of the following work.

[†] Or, as any other sum is to its interest; but it is more convenient to make *one* pound the leading term in this and the other proportions used in the present work.

[‡] When a day, a week, a month, &c., is involved in the calculation, then n becomes a fractional quantity, either proper or improper: thus, if a day, $n = \frac{1}{365}$; if two days, $n=\frac{2}{365}$; if five hundred and twenty days, $n=\frac{520}{365}$; if a month, $n=\frac{1}{12}$; if fifteen months, $n=\frac{15}{12}$; and so on.

[§] The interest on *one pound* at 4 per cent per annum is ·04; at 5 per cent per annum and to find the interest of any given sum for any given time, it will be by the above rule, $1 : r = p : pr =$ the interest of p for one year; and then $1 : n = pr : prn =$ the interest of p for the time n: which being added to p (= the principal) will make the amount $s = p + prn$: and from this theorem others may be deduced for finding any one of the quantities when the rest are given. Thus

$$s = p(1 + rn) = \text{the Amount,}$$

$$p = \frac{s}{1 + rn} = \text{the Principal,}$$

$$n = \frac{s - p}{pr} = \text{the Number of Years,}$$

$$r = \frac{s - p}{pn} = \text{the Rate of Interest.}$$

§ 6. In order to explain the use and application of these formulæ, I shall propose the following questions: —

Example 1. Let it be required to find the amount of £355, 15s. put out to interest at the rate of 4 per cent per annum for four years: Here we make use of the first theorem, where $p = 355{\cdot}75$, $n = 4$, and $r = {\cdot}04$; consequently $s = 355{\cdot}75 \times (1 + 4 \times {\cdot}04) = 412{\cdot}670$, or £412, 13s. 5d.

Example 2. What principal put out to interest at the rate of 5 per cent per annum for three years, will at the end of that time amount to £828? Here we make use of the second theorem, where $s = 828$, $n = 3$, and $r = {\cdot}05$; consequently $p = \frac{828}{1 + {\cdot}05 \times 3} = \frac{828}{1{\cdot}15} = 720$ or £720.

Example 3. Suppose a person returns £517, 10s. for the loan of £450 at the rate of 5 per cent per annum; it is required to determine the time for which it was lent: Here we make use of the third theorem, where $s = 517{\cdot}5$, $p = 450$, and $r = {\cdot}05$; consequently $n = \frac{517{\cdot}5 - 450}{450 \times {\cdot}05} = \frac{67{\cdot}5}{22{\cdot}5} =$ 3 years.

Example 4. If a person returns £610 for the loan of £600 for one month, what is the rate of interest allowed? Here we make use of the fourth theorem, where $s = 610$, $p = 600$, and $n = {\cdot}0833$; consequently $r = \frac{610 - 600}{600 \times {\cdot}0833} = \frac{10}{50} = {\cdot}2$; which being multiplied by 100, gives 20 per *cent.*

§ 7. From what has been said in § 5, it will readily appear that the *interest only* on any given sum, and for any given time and rate, may be

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it is ·05; and so on. These values are called the *Rate* of interest, and being multiplied by 100 give 4, 5, &c., for the rate *per cent;* so that a distinction must be made between the *rate* and the *rate per cent.*

easily calculated; it being in all cases equal to *prn*: Thus, the interest on £160 for a quarter of a year at the rate of 5 per cent per annum, is equal to $160 \times \cdot 05 \times \cdot 25 = 2$ pounds; and by means of this formula the amount of interest may in all cases be accurately determined: but the great and extensive use of this branch of science in all commercial concerns has given rise to the computation of various tables, formed not only for the more ready calculation of the amount of interest on any given sum, but also for the prevention of those mistakes which often occur in the hurry of business. Such tables, adapted to common use, are in the hands of almost every person; but in order that the present work should leave nothing to be desired on this subject, I have annexed a table at the end which shows the interest of £1 for any number of days in the year, and for any term of years not exceeding thirty, according to the several rates of 2, 2½, 3, 3½, 4, 4½, 5, 6, 7, 8, 9, and 10 per cent interest, to eight places of decimals; which will give the value much more accurately than any of the tables above alluded to. This table, therefore, which is the FIRST Table at the end of the present work, consists of the value of *rn* expressed in numbers; where *r* is taken equal to ·02, ·025, ·03, &c., according as the rate is 2, 2½, 3, &c. per cent; and where *n* is taken equal to $\frac{1}{365}$, $\frac{2}{365}$, $\frac{3}{365}$, &c., according to the number of days. Thus, the interest of £1 in 198 days at 5 per cent as expressed in the table, is ·02712329 or about 6½d.; and which being multiplied by any other principal *p* will give the interest of that principal accordingly. The use and application of this table are so obvious that it is needless to enter into any further explanation.

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CHAPTER III.

ON DISCOUNT.

§ 8. DISCOUNT may be considered as the difference between a sum of money due at a future period, and its present value: or, it is an allowance made on a bill or any other debt, not yet become due, in consideration of making immediate payment of such bill or debt. Among bankers, merchants, &c., it is usual to reckon such a sum for *discount* as is equal to the *interest* of the whole bill or debt, calculated from the present time till it becomes due: thus, for the present payment of a bill of £105 due a twelvemonth hence (interest being reckoned at 5 per cent per annum), they deduct a sum equal to the *interest* of £105 for a year, or £5, 5s.;

which makes the present value of such a bill only £99, 15s. But this (however much it may be overlooked by people engaged in business) is certainly neither correct nor just:* for, since the true value of the discount is equal to the difference of the debt and its present worth, it is equal to the interest of that present worth only, instead of the interest on the whole debt: and if I have a bill of £105 due a twelvemonth hence, I ought to receive for the present value of that bill, a sum which, put out to interest (still reckoned at 5 per cent per annum), would at the end of the twelvemonth become equal to £105: but this is evidently £100, and not £99, 15s. as is commonly but erroneously reckoned. In order to find the true discount, therefore, we must make use of the following **Rule**, viz., "As the amount of £1 for the given time and rate is to the given sum or debt; so is the interest of £1 for the given time and rate to the discount of the sum or debt."

§ 9. Now, in order to generalize what has been here laid down, let us make

p = the Present Value,
s = the Sum due,
n = the Number of Years,
r = the Rate of Interest;

and then to find the discount it will be, $(1 + rn) : s = rn : \frac{srn}{1+rn} =$ the true discount; which being subtracted from s (= the sum due) leaves $\frac{s}{1+rn}$ for the present value of such sum. Whence we deduce the following theorems, from which any one of the quantities may be determined when the rest are given:—

$$p=\frac{s}{1+rn} \quad = \text{the Present Value,}$$

$$s=p\,(1+rn) = \text{the Sum due,}$$

$$n=\frac{s-p}{pr} \quad = \text{the Number of Years,}$$

$$r=\frac{s-p}{pn} \quad = \text{the Rate of Interest.}$$

§ 10. To show the use and application of these theorems, I shall propose the following questions:—

Example 1. What is the present value of a bill of £2000 due three years hence, and reckoning interest at 5 per cent per annum? Here we make use of the first theorem, where $s=2000$, $n=3$, and $r=\cdot05$; consequently $p=$

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* I doubt whether it is even *legal*.

$\frac{2000}{1+{\cdot}05\times 3}=\frac{2000}{1{\cdot}15}=1739{\cdot}130$, or £1739, 2s. 7d. Had this bill been discounted in the usual mode, the borrower of the money would have received only £1700 for it.

Example 2. The present value of a sum of money due six months hence is £540, what is the sum that is due, reckoning interest at 4 per cent per annum? Here we make use of the second theorem, where $p=540$, $n={\cdot}5$, and $r={\cdot}04$; consequently $s=540\times(1+{\cdot}04\times{\cdot}5)=540\times 1{\cdot}02=550{\cdot}8$, or £550, 16s.

Example 3. If a person, in getting a bill of £600 discounted, after the rate of 5 per cent interest per annum, receives £580 for the same, how long ought it to run before it is due? Here we make use of the third theorem, where $p=580$, $s=600$, and $r={\cdot}05$; consequently $n=\frac{600-580}{580\times{\cdot}05}$ $=\frac{20}{29}={\cdot}690$ or 252 days.

Example 4. What is the rate of interest allowed, if a person receives £840 in ready money for £1204, 16s. due seven years and a half hence? Here we make use of the fourth theorem, where $p=840$, $s=1204{\cdot}8$ and $n=7{\cdot}5$; consequently $r=\frac{1204{\cdot}8-840}{840\times 7{\cdot}5}=\frac{364{\cdot}8}{6300}={\cdot}0579$, or £5, 15s. 9¾d. per cent.

§ 11. If we compare these theorems with those already given for calculating the interest on any sum, in § 5, we shall find them precisely the same, except that what is there called the *Principal*, is here called the *Present value;* and what is there called the *Amount*, is here called the *Sum due:* and this is exactly what I have endeavoured to demonstrate; for, the *Present value* of a *Sum due* at a future time is the *Principal* that will *Amount* to that sum in the given time.

§ 12. From what has been said it will readily appear that the discount on any given sum and for any given time and rate may be easily calculated; it being in all cases equal to $\frac{srn}{1+rn}$: thus, the discount on £160 for a quarter of a year at the rate of 5 per cent interest per annum is equal to $\frac{160\times{\cdot}05\times{\cdot}25}{1+{\cdot}05\times{\cdot}25}=\frac{2}{1{\cdot}0125}=1{\cdot}975$ or £1, 19s. 6d. But since the inaccurate method of taking the *interest* instead of the *discount* universally prevails, particularly in commercial concerns, the subject is now almost, though very undeservedly, exploded; and tables of discount are rarely to be met with: the reader, however, will find such Table at the end of the present work, being the SECOND TABLE there given, and which will prove useful in his inquiries on this subject. It consists of the value of $\frac{rn}{1+rn}$ expressed in numbers, where r is taken equal to ·02, ·025, ·03, &c., according as the rate of interest is 2, 2½, 3, &c. per cent, and where n is taken equal to $\frac{1}{365}$, $\frac{2}{365}$, $\frac{3}{365}$, &c., according to the number of days: consequently the values there given being multiplied by any other sum s will give the true discount on that other sum accordingly.

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CHAPTER IV.

ON THE AMOUNT OF SUMS AT COMPOUND INTEREST.

§ 13. COMPOUND INTEREST is that which is reckoned not only on the principal sum lent, but also on its interest as it becomes due: so that at every payment, or at the time when the payments become due, there is created a new principal; and for that reason it is called interest upon interest, or compound interest. And although it is not lawful to lend money at compound interest, yet in purchasing annuities, pensions, reversions, leases, &c., it is usual to allow compound interest to the purchaser for his ready money; and therefore the subject is deserving of consideration and attention.

§ 14. In order to show the manner in which any given sum increases when put out to compound interest, let us take the case of £100 at the rate of 5 per cent per annum; which, it is well known, will amount to £105 at the end of the *first* year: now, this may be considered as a new principal put out at the same rate of interest, the amount of which at the end of the *second* year will be found equal to £110, 5s.; which likewise may be considered as a new principal put out to interest, and will amount, at the end of the *third* year, to £115, 15s. 3d.; and so on for any other period. And it is evident that in any given time the amount of principal and interest will be greater as the number of payments are greater; that is, the oftener the interest becomes due and is added to the principal: contrary to what takes place at simple interest, where the amount is the same at the end of any given period whether the payments of interest have been made at once, or at many times.

§ 15. Let us now make

$s =$ the Amount,
$p =$ the Principal,
$n =$ the Number of Years,
$\rho =$ the *annual* Rate of Interest;*

therefore, since £1 put out to interest is increased in one year to $1 + \rho$; in order to find the amount at the end of any other year, it will be as 1 is to $1 + \rho$, so is $1 + \rho$ (or the amount of £1 at the end of the first year) to $(1 + \rho)^2 =$ the amount of £1 at the end of the second year: again, as 1 is to $1 + \rho$, so is $(1 + \rho)^2$, or the amount just found, to $(1 + \rho)^3 =$ the amount of £1 at the end of the third year. That is, the amount of £1 at the end of the

first year is $= (1 + \rho)^1$,
... second year is $= (1 + \rho)^2$,
... third year is $= (1 + \rho)^3$,
... fourth year is $= (1 + \rho)^4$,

and universally it will be found that $(1 + \rho)^n$ is equal to the amount of £1 at the end of the nth year: which expression being multiplied by any other principal, p, will give $p\,(1 + \rho)^n$ for the amount of that other principal in n years. Whence we deduce the following theorems for finding any one of the quantities when the rest are given: Thus

$$s = p\,(1 + \rho)^n = \text{the Amount,}$$
$$p = s\,(1 + \rho)^{-n} = \text{the Principal,}$$
$$n = \frac{\lambda s - \lambda p}{\lambda (1 + \rho)} = \text{the Number of Years,}$$
$$\rho = \left(\frac{s}{p}\right)^{\frac{1}{n}} - 1 = \text{the } \textit{annual} \text{ Rate of Interest.}$$

§ 16. In order to explain the use and application of these theorems, I shall propose the following questions:—

Example 1. Let it be required to find the amount of £355, 15s. put out to interest at the rate of 4 per cent per annum for four years. Here we make use of the first theorem, where $p = 355{\cdot}75$, $n = 4$, and $\rho = {\cdot}04$, consequently $s = 355{\cdot}75 \times (1{\cdot}04)^4 = 355{\cdot}75 \times 1{\cdot}16985856 = 416{\cdot}177$, or £416, 3s. 6d.

Example 2. What principal put out to interest at the rate of 5 per cent per annum for three years, will at the end of that time amount to £828? Here we make use of the second theorem, where $s = 828$, $n = 3$, and $\rho = {\cdot}05$; consequently $p = 828 \times (1{\cdot}05)^{-3} = 828 \times {\cdot}86383760 = 715{\cdot}257$, or £715, 5s. 2d.

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* I have adopted a new character here to denote the true *annual* rate of interest; because, where compound interest is concerned, we must carefully distinguish between the *true* and the *nominal* rate, as I shall more fully explain in §§ 25 and 26.

Example 3. Suppose a person returns £517, 10s. for the loan of £450 at the rate of 5 per cent per annum, it is required to determine the time for which it was lent. Here we make use of the third theorem, where $s = 517{\cdot}5$, $p = 450$, and $\rho = {\cdot}05$; consequently $n = \frac{\lambda\, 517{\cdot}5 - \lambda\, 450}{\lambda\, 1{\cdot}05} = \frac{2{\cdot}7139104 - 2{\cdot}6532125}{{\cdot}0211893} = 2{\cdot}865$, or 2 years and 316 days.

Example 4. If a person returns £810 for the loan of £700 for three years, what is the rate of interest allowed? Here we make use of the fourth theorem, where $s = 810$, $p = 700$, and $n = 3$; consequently $\rho = \left(\frac{810}{700}\right)^{\frac{1}{3}} - 1 = 1{\cdot}04985 - 1 = {\cdot}04985$, or £4, 19s. 8d. *per cent.*

§ 17. From what has been said, it will readily appear that the amount of compound *interest only* on any given sum will be equal to the Amount *minus* the Principal; so that $p\,(1 + \rho)^n - p$ will in every case denote the amount of compound interest only on the given sum. Thus, the compound interest on £355, 15s. for four years at 4 per cent per annum, is $416{\cdot}177 - 355{\cdot}75 = 60{\cdot}427$, or £60, 8s. 6½d.

§ 18. It appears from the first theorem in § 15, that the amount of any given principal at the end of any given year is equal to the amount of £1 in one year raised to the power indicated by the given number of years, and afterwards multiplied by such principal. If, therefore, the geometrical progression $(1+\rho)^1$, $(1+\rho)^2$, $(1+\rho)^3$, &c. (which severally represent the amount of £1 at the end of the 1st, 2d, 3d, &c., years respectively), be converted into numbers, where we may take ρ equal to ·02, ·025, ·03, &c., according as the rate of interest is 2, 2½, 3, &c., per cent per annum, we may form a set of tables which shall be at all times ready for use in the most common cases that arise. Thus, the amount of £1 at *two* per cent per annum compound interest, is, at the end of the first year, equal to $(1{\cdot}02)^1 = 1{\cdot}02$; at the end of the second year it is equal to $(1{\cdot}02)^2 = 1{\cdot}0404$; at the end of the third year it is equal to $(1{\cdot}02)^3 = 1{\cdot}0612$; and so on for the subsequent years.* And the amount of £1 at *two and a half* per cent per annum compound interest, is at the end of the first year equal to $(1{\cdot}025)^1 = 1{\cdot}025$; at the end of the second year it is equal to $(1{\cdot}025)^2 = 1{\cdot}0506$; at the end of the third year it is equal to $(1{\cdot}025)^3 =$

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* These values are best found by the help of logarithms, particularly when the power is a high one. And I cannot omit this opportunity of recommending Didot's stereotype edition of the *Tables Portatives de Logarithmes*, par F. Callet, 8vo, 1795, as they contain the logarithms of numbers from 1 to 108,000, which will be found particularly useful in all questions relative to compound interest: whereas Hutton's Tables (which may be ranked next to these for general use) contain the logarithms of numbers from 1 to 100,000 only.

1·0769; and so on for the subsequent years. In this manner we might proceed with any other rate of interest; and by arranging the values, thus found, in their proper columns, we might enlarge or extend the table at pleasure. It is in this way that the THIRD Table at the end of the present work has been formed, which shows the Amount of £1 at the end of any number of years not exceeding 100; and at the several rates of 2, 2½, 3, 3½, 4, 4½, 5, 6, 7, 8, 9, and 10 per cent interest per annum: and any of the values there found, being multiplied by any other principal, will give the amount of such principal in the given time, and at the given rate. For example, suppose it were required to find the amount of £5 in 13 years at the rate of 5 per cent per annum compound interest; here we multiply 1·8856 (= the value found under 5 per cent, and against 13 years) by 5, which gives 9·4280, or £9, 8s. 7d. for the answer.

§ 19. We may, perhaps, sometimes have to determine the amount of a given sum, for a period of years beyond the limits of this table; in which case we must take the highest value in the table, and consider it as a fresh principal put out to interest for the remainder of the term, the amount of which may be found in the common way. Thus, suppose it were required to find the amount of £1 in 150 years at 5 per cent per annum compound interest: Here, the amount in 100 years would be 131·50125784, which being considered as a new principal, its amount in 50 years more would be $131{\cdot}50125784 \times 11{\cdot}46739978 = 1507{\cdot}97749706$; and in 80 years more (that is, at the end of 230 years) its amount would be $1507{\cdot}97749706 \times 49{\cdot}56144106 = 74737{\cdot}53784035$. The truth of this rule is evident; for $(1+\rho)^{150} = (1+\rho)^{100+50} = (1+\rho)^{100} \times (1+\rho)^{50}$: or universally $(1+\rho)^{n+\nu} = (1+\rho)^{n} \times (1+\rho)^{\nu}$.

§ 20. It should be observed that in calculating the amount s of any principal, for a period of time *shorter* than a year, the value comes out *less* at compound interest than when reckoned at simple interest; and this is always the case when n is a proper fractional quantity: Thus, if it were required to find the amount of £100 at the end of the first *half* year, after the rate of 5 per cent per annum compound interest, we should have $p = 100$, $\rho = {\cdot}05$, but n equal to ½ only; consequently, our formula becomes $s = 100 \times (1{\cdot}05)^{\frac{1}{2}} = 102{\cdot}4695$, or £102, 9s. 4¾d., whereas the amount at simple interest would have been £102, 10s.* So, likewise, if it were required to find the amount of £100 at the end of the first *quarter* of a year, after the same annual rate of interest, we should have $p = 100$, $\rho = {\cdot}05$, as before, but n in this case equal to ¼ only; consequently, $s=100\times(1{\cdot}05)^{\frac{1}{4}}=101{\cdot}2272$, or £101, 4s. 6½d., whereas it would have been £101, 5s. at simple interest. The reason of this is plain, for the interest on any principal put out to compound interest is not supposed to be due till the time that the first payment is either made or supposed to be made; thus, if £105 is supposed to be received at the end of a year for £100, lent after the rate of 5 per cent per annum compound interest, it is manifest that if any part of such interest is paid before the year expires (that is, before the sum is due), it ought to be diminished accordingly, so as to make the *annual* rate of compound interest exactly 5 per cent. For, if £2, 10s. were to be received as the compound interest of £100 at the end of half a year, it is evident that the rate of interest would not be 5 per cent *per annum*, but £5, 1s. 3d.; because we find that there are in this case *two* payments of interest in the year, at each of which the principal is increased 2½ per cent, and which may be considered as a fresh sum put out to interest; in such case we must make the rate of interest $={\cdot}025$, and the number of payments $=2$, which produces $(1{\cdot}025)^2=1{\cdot}05062$ for the true amount at the end of the year. In like manner, if £1, 5s. were to be received for the compound interest on £100 at the end of a quarter of a year, it is evident that the rate of interest would not be 5 per cent *per annum*, but £5, 1s. 11d.; for we find that in this case there are *four* payments in a year, at each of which the principal is increased 1¼ per cent, and which may be considered as a fresh sum at each payment, put out to interest: in this case we must make the rate of interest $={\cdot}0125$, and the number of payments $=4$, which produces $(1{\cdot}0125)^4=1{\cdot}05095$ for the true amount at the end of a year.

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* It is evident that $1 + \frac{r}{2}$ must in all cases be greater than $(1+r)^{\frac{1}{2}}$; for $1 : (1+r)^{\frac{1}{2}} = (1+r)^{\frac{1}{2}} : 1+r$, but (by Euclid 25·5) $2+r$ is greater than $2(1+r)^{\frac{1}{2}}$; therefore $1 + \frac{r}{2}$ is greater than $(1+r)^{\frac{1}{2}}$.

§ 21. Since, then, it appears that the advantage arising from a sum of money put out to compound interest for smaller portions of a year, increases in a certain degree at the end of that period, it may be useful to know the quantity and extreme limit of this increase, and which may be determined in the following manner:—Let the simple interest of £1 for a year be denoted by r; then, supposing the payments of interest to be made *yearly*, the amount of £1 at the end of the year will be expressed by $(1+r)$. If the payments of interest be supposed to be made *half-yearly* (each payment in this case being one-*half* of the annual simple interest) then the amount of £1 at the end of the year will be expressed by $(1+\frac{r}{2})^2$, which being expanded is equal to $1+r+\frac{r^2}{4}$, and which exceeds the former expression by $\frac{r^2}{4}$, or ¼ of the square of r. If the payments of interest be supposed to be made *quarterly* (each payment in this case being one *quarter* of the annual simple interest), then the amount of

£1 at the end of the year will be expressed by $(1+\frac{r}{4})^4$, which being expanded is equal to $1+r+\frac{3r^2}{8}+\frac{r^3}{16}+\frac{r^4}{256}$, and which exceeds the first expression by $\frac{3r^2}{8}+\frac{r^3}{16}+\frac{r^4}{256}$, or by $\frac{3}{8}$ of the square of r, nearly. And, universally, if the payments of interest be supposed to be made m times in a year (each payment in this case being the mth part of the annual simple interest) then the amount of £1 at the end of the year will be expressed by $(1+\frac{r}{m})^m$, which being expanded by the binomial theorem, as in the preceding cases, is equal to $1+r+\frac{m-1}{2m}\cdot r^2+\frac{m-1}{2m}\cdot\frac{m-2}{3m}\cdot r^3+\frac{m-1}{2m}\cdot\frac{m-2}{3m}\cdot\frac{m-3}{4m}\cdot r^4+$ &c., and which exceeds the first expression by the quantity $\frac{m-1}{2m}\cdot r^2+\frac{m-1}{2m}\cdot\frac{m-2}{3m}\cdot r^3+\frac{m-1}{2m}\cdot\frac{m-2}{3m}\cdot\frac{m-3}{4m}\cdot r^4$ + &c., or by $\frac{m-1}{2m}$th part of the square of r, nearly.[*]

§ 22. Now, when m is infinite, the quantities $m-1$, $m-2$, $m-3$, &c., become severally equal to m;[†] consequently, the expression $(1+\frac{r}{m})^m$ is in this case equal to $1+r+\frac{r^2}{1.2}+\frac{r^3}{1.2.3}+\frac{r^4}{1.2.3.4}+$ &c., which is the limit of the value to which £1 will increase by making the number of payments infinitely great, and the interest received at each time infinitely small; and which exceeds the first expression by the quantity $\frac{r^2}{1.2}+\frac{r^3}{1.2.3}+\frac{r^4}{1.2.3.4}+$ &c., or by half the square of r nearly. But the whole of this series is easily determined, it being equal to the number which has r for its *Neperean*[‡] logarithm, or $r\times{\cdot}4342944819032518$, &c., for its logarithm in Briggs' system. For example, suppose it were required to find the sum to which £100 would increase at the end of a year, if it were put out to interest at the rate of 5 per cent, in such manner that the interest which might *momently* become due on the same should be continually added to the principal, and in this manner increase together to the end of the given term: In this case we multiply ·05 by ·4342944819, the product of which is ·021714724095, the natural number to which logarithm is 1·05127; and this, multiplied by 100, will give 105·127, or £105, 2s. 6½d. for the amount required, which exceeds the annual rate of simple interest by $\frac{1}{2} \cdot r^2$, or by half the square of such rate, nearly.

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[*] For, in all these cases, the quantities involving the third, fourth, and all the higher powers of r are very small in comparison with those involving the first and second powers, and they may be safely omitted in this view of the subject.

[†] Because the numbers 1, 2, 3, &c., which are subtracted from m, will make no sensible difference in its value. See Euler's *Introd. in Anal. Inf.*, vol. i. c. 7, § 116.

[‡] The terms of every art or science should be clear, definite, and explicit; and though they may not always be sufficiently precise, yet they should never tend to convey any false ideas on the subject. By using the term *Hyperbolic* logarithm, an idea is immediately entertained that this is the only system of logarithms that can be expressed by the hyperbola; whereas, not only the common system, but every other system whatever, may be expressed by means of that curve; and the only difference is, that in the former, or hyperbolic logarithm, the asymptotes of the curve are at right angles to each other; but in the latter, or common logarithm, they form an angle of 25°.44′.25½″. These are

§ 23. From this it will appear that the advantages which may be made by lending out money for very small portions of a year, instead of lending it out for a year, or half a year at a time, are so small as not to be worth attending to during the short interval of one year. But it is evident that this advantage will increase as the number of years increases: and since the amount of £1 at the end of any given number of years n is expressed by the formula $(1+\frac{r}{m})^{mn}$ (or by the amount of £1 in one year raised to the power indicated by the number of years), where m is taken equal to 1, 2, 4, &c., according as the payments are supposed to be made yearly, half-yearly, quarterly, &c., we may readily determine the *difference* between this value and that of $(1+r)^n$, it being in all cases equal to $(1+\frac{r}{m})^{mn} - (1+r)^n$.

§ 24. When m is infinite, the formula $(1+\frac{r}{m})^{mn}$ may be expressed by the series $1 + rn + \frac{r^2n^2}{1.2} + \frac{r^3n^3}{1.2.3} + \frac{r^4n^4}{1.2.3.4} + \&c.$;* the sum of which is equal to the number that has rn for its Neperean logarithm, or $rn \times {}$·4342944819, &c., for its logarithm in Briggs' system.† Thus the amount of £100 in

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generally called Briggs' logarithms, after the name of their inventor; and the former, for the same reason, I have here called the Neperean logarithm. For the value of the series see the note to § 24.

[*] See Euler's *Introd. in Anal. Inf.*, vol. i. c. 7, § 117, or Simpson's *Fluxions*, vol. ii. § 11. Prob. 1.

[†] It is shown by all the writers on logarithms, that the sum of the series $1+\frac{x}{1}+\frac{x^2}{1.2}+\frac{x^3}{1.2.3}+\frac{x^4}{1.2.3.4}+\&c.$, is equal to the number which has x for its Neperean logarithm; and that any Neperean logarithm may be converted into a corresponding vulgar logarithm, by multiplying it by the decimal ·4342944819, &c. When x is equal to 1, then the sum of the series is equal to 2·7182818284, &c., the Neperean logarithm of which is unity. See Euler's *Introd.* ut supra. La Grange, *Théorie des Fonctions*, p. 16. Bonnycastle's *Trigonometry*, p. 329. Hutton's *Mathematical Dictionary* (art. *Logarithms*, in Sup.), and many other works.

50 years put out to interest at the rate of 5 per cent, in such manner that it might be received *momently* and continually added to the principal, will be equal to the natural number corresponding to $\cdot 05 \times 50 \times \cdot 4342944819$, &c., in Briggs' logarithms: that is, equal to 12·18250.

§ 25. Since then $(1+\frac{r}{m})^m$ is the general expression for the amount of £1 at the end of a year on the supposition that the payments of interest are made m times in a year, each payment being the mth part of r, which is the rate of simple interest, it follows that the true *annual* rate is expressed by $(1+\frac{r}{m})^m-1$. If, therefore, we make this equal to ρ, then will r denote the *nominal* rate of simple interest, and ρ the *true* rate of compound interest per annum: consequently $1+\rho$ will be the amount of £1 at the end of a year at the given annual rate of compound interest whatever be the value of m; and when $m=1$, then ρ and r will be equal to each other.

§ 26. It will be necessary, therefore, in all cases of compound interest, that we should attend to this observation, and carefully distinguish between the *nominal* rate of interest and such rate as is *actually* made during the space of one year. Thus, a person who receives £2, 10s. half-yearly, and another who receives £35 every seven years, on £100 laid out,* may both be said to receive after the rate of 5 per cent per annum for their money; and if simple interest only were concerned, there would be no difference in the calculations for the two cases: but in questions relating to compound interest they are totally different, as I shall now more fully explain. It is true that we may reject the new character ρ, and lay aside all consideration of the *annual* rate of interest: for, by making n equal to the number of *payments* of interest, and r equal to the interest on £1 *received at every such time*, the general formula $p(1+r)^n$ will accurately express the true amount in such case: but it is more convenient, and very much simplifies the operations (particularly in the doctrine of ANNUITIES) to make n equal to the number of *years*, and ρ equal to the true *annual* rate of interest.

§ 27. It will be the more necessary to attend to the remarks which have just been made, as they furnish us with an easy method of deducing from the Tables above alluded to, the amount of any sum at the end of any given term, on the supposition that the payments of interest are made *half-yearly*, *quarterly*, &c.; each half-yearly or quarterly payment being one *half* or one *quarter*, &c., of the *nominal* rate: or on the supposition that the payments of interest are made *biennially*, *triennially*, &c.; each payment being *double*, *treble*, &c., of the *nominal* rate, respectively. For, as the Tables* are calculated on the supposition that the payments of interest become due and are paid annually, it follows from what has just been said, that in order to obtain the amount of £1 at the end of any given year, supposing the payments of interest to be made *half-yearly*, we must take the amount in the tables at *double* the number of years, and at one *half* the rate of interest: and if the payments be made *quarterly*, we but take the amount in the tables at *quadruple* the number of years, and one *quarter* the rate of interest. So likewise, if the payments of interest be made *biennially*, we must take the amount in the tables at one *half* the number of years, and at *double* the rate of interest: and if the payments be made *triennially*, we must take the amount in the tables at one *third* the number of years, and at *treble* the rate of interest: and so on for any longer or shorter term. For example, the amount of £1 in *six* years at the rate of *eight* per cent interest payable *yearly* is, by the theorem, equal to $(1{\cdot}08)^6$, which by the tables is found to be 1·586874: if the payments be made *half-yearly*, as above mentioned, the value will be equal to the amount of £1 in *twelve* years, at *four* per cent, or $(1{\cdot}04)^{12}$, which by the tables is 1·601032: and if the payments be made *quarterly*, the value will in such case be equal to the amount of £1 in *twenty-four* years, at *two* per cent, or $(1{\cdot}02)^{24}$, which by the tables is 1·608437. So likewise the amount of £1 in *eighteen* years at the rate of *three* per cent interest payable *yearly* is, by the theorem, equal to $(1{\cdot}03)^{18}$, which by the tables is found to be 1·702433: if the payments be made *biennially*, as above mentioned, the value will be equal to the amount of £1 in *nine* years, at *six* per cent, or $(1{\cdot}06)^9$, which by the tables is 1·689479: and if the payments be made *triennially*, the value will be equal to the amount of £1 in *six* years, at *nine* per cent, or $(1{\cdot}09)^6$, which by the tables is 1·677100: and so on for any longer or shorter period. And universally we shall find that $(1+\frac{r}{m})^{mn}$ will denote the amount of £1 at the end of n years, on the supposition that the payments of interest are made m times in a year, each payment being the mth part of r, which is called the nominal rate. These remarks, however, apply only when we have recourse to the tables for the solution of such cases: for if they are solved agreeably to the theorem already laid down, we shall find it will lead us to the true answer.

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* Of the former kinds are dividends arising from the public funds; and of the latter, are fines paid septennially for the renewing of leases.

§ 28. The best method of obtaining these values, when recourse cannot be had to the tables, is by means of logarithms: for it is evident that the natural number corresponding to $nm \times \lambda(1+\frac{r}{m})$ is the amount required. I shall, however, point out another way by which the same results may be obtained; and that is to take the natural number corresponding to $n \times \lambda(1+\frac{r}{m})^m$: but since the finding of the value of $(1+\frac{r}{m})$ (or of $1+\rho$, see § 25) in order to determine its logarithm, may be attended with trouble and inconvenience, I have subjoined the following table for the most practical cases, viz., those of yearly, half-yearly, and quarterly payments, to which I have added that of momently payments, in order that the reader may see the extreme limit of the increase in these subdivisions.

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* I allude to Table III., given at the end of this work; and the similar tables of other writers.

§ 29. This table, therefore, consists of the expansion of the binomial $(1+\frac{r}{m})^m$ in numbers; where r is taken equal to ·02, ·025, ·03, &c., according as the nominal rate is 2, 2½, 3, &c., per cent; and where m is taken equal to 1, 2, 4, and ∞, according as the payments are supposed to be made yearly, half-yearly, quarterly, or momently.* The first column indicates the value of r, or the nominal rate of simple interest: the second column denotes the periods at which the interest becomes due, and is payable; where y, h, q, and m respectively signify yearly, half-yearly, quarterly, and momently payments: the third column expresses the value of $1+\rho$, or the amount of £1 in a year at the several rates annexed thereto, and according as the payments of interest are made yearly, half-yearly, &c.; from which, by subtracting unity, we obtain the *true* rate of compound interest per annum: the fourth column contains the logarithms of the numbers in the third column, to 15 places of figures, which will be found very useful for determining the amount of £1 at the end of any number of years, according to the several rates of interest mentioned in the table, and according as such interest is payable yearly, half-yearly, &c.

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* When the interest is payable momently, the formula $(1+\frac{r}{m})^{mn}$ will be expressed by that number which has rn for its Neperean logarithm, or $rn \times$ ·4342944819, &c., for its logarithm in Briggs' system. See § 24.

Nominal Rate of Interest. r	Payable.	Amount of £1 in One Year. $1+\rho$	Logarithms of such Amount. $\lambda(1+\rho)$
2 per cent.	*y*	1·020000	·008 6001 7176 1918
	h	1·020100	·008 6427 4756 5285
	q	1·020150	·008 6642 4702 6031
	m	1·020201	·008 6858 8963 8065
2½ per cent.	*y*	1·025000	·010 7238 6539 1773
	h	1·025156	·010 7900 6377 3412
	q	1·025235	·010 8235 7350 3700
	m	1·025315	·010 8573 6204 7581
3 per cent.	*y*	1·030000	·012 8372 2470 5172
	h	1·030225	·012 9320 8449 8463
	q	1·030339	·012 9802 1925 2588
	m	1·030454	·013 0288 3445 7098
3½ per cent.	*y*	1·035000	·014 9403 4979 2937
	h	1·035306	·015 0688 3579 4515
	q	1·035462	·015 1341 9092 0507
	m	1·035620	·015 2003 0686 6614
4 per cent.	*y*	1·040000	·017 0333 3929 8780
	h	1·040400	·017 2003 4352 3835
	q	1·040604	·017 2854 9513 0570
	m	1·040811	·017 3717 7927 6130
4½ per cent.	*y*	1·045000	·019 1162 9044 7073
	h	1·045506	·019 3266 3335 8759
	q	1·045765	·019 4341 3848 1315
	m	1·046028	·019 5432 5168 5646
5 per cent.	*y*	1·050000	·021 1892 9906 9938
	h	1·050625	·021 4477 3078 3546
	q	1·050946	·021 5801 2754 6825
	m	1·051271	·021 7147 2409 5163
6 per cent.	*y*	1·060000	·025 3058 6526 4770
	h	1·060900	·025 6744 4941 0344
	q	1·061364	·025 8641 6899 6927
	m	1·061837	·026 0576 6891 4195
7 per cent.	*y*	1·070000	·029 3837 7768 5210
	h	1·071225	·029 8806 9958 5873
	q	1·071859	·030 1376 7158 9030
	m	1·072508	·030 4006 1373 3228
8 per cent.	*y*	1·080000	·033 4237 5548 6950
	h	1·081600	·034 0666 7859 7561
	q	1·082432	·034 4006 8704 7670
	m	1·083287	·034 7435 5855 2260
9 per cent.	*y*	1·090000	·037 4264 9794 0624
	h	1·092025	·038 2325 8089 4146
	q	1·093083	·038 6532 6671 7518
	m	1·094175	·039 0865 0337 1293
10 per cent.	*y*	1·100000	·041 3926 8515 8225
	h	1·102500	·042 3785 9813 9876
	q	1·103813	·042 8954 6156 7092
	m	1·105171	·043 4294 4819 0325

§ 30. In order to explain the use of this table, let us take the following example, viz., to find the amount of £1 in 25 years, at the nominal rate of 5 per cent interest, such interest, however, to be paid quarterly. Here we shall find that the value set down against the letter *q* in 5 per cent is 1·050946, which is the amount of £1 in a year, and the logarithm to which is found in the fourth column to be ·021 5801 2754 6825; now this being multiplied by 25 will produce ·539 5031 8867 06, the natural number to which logarithm is 3·4634, whereas the amount in yearly payments would be 3·3864 only, as appears by Table III.

§ 31. That the reader may see the difference between the amount of £1 in any given number of years improved at a given annual rate of interest, and the amount of £1 in the same time when improved at the same nominal rate of interest payable half-yearly, quarterly, &c., I have subjoined another table, which shows the amount of £1 in 1, 5, 25, 50, and 100 years, on the supposition that money may be improved at the nominal rate of 2, 3, 4, 5, 6, 7, 8, 9, and 10 per cent interest, payable yearly, half-yearly, quarterly, and momently, where *y*, *h*, *q*, and *m* denote the same as in the last table.

Nominal rate of Interest.	Payable.	The Amount of £1 in				
		1 Year.	5 Years.	25 Years.	50 Years.	100 Years.
2 per cent.	*y*	1·02000	1·10408	1·64061	2·69159	7·24465
	h	1·02010	1·10462	1·64463	2·70481	7·31602
	q	1·02015	1·10490	1·64667	2·71152	7·35232
	m	1·02020	1·10517	1·64872	2·71828	7·38906
3 per cent.	*y*	1·03000	1·15927	2·09378	4·38391	19·21863
	h	1·03022	1·16054	2·10524	4·43204	19·64303
	q	1·03034	1·16119	2·11108	4·45667	19·86195
	m	1·03045	1·16183	2·11700	4·48169	20·08553
4 per cent.	*y*	1·04005	1·21665	2·66584	7·10668	50·50495
	h	1·04040	1·21899	2·69159	7·24465	52·48490
	q	1·04060	1·22019	2·70481	7·31602	53·52411
	m	1·04081	1·22140	2·71828	7·38906	54·59815
5 per cent.	*y*	1·05000	1·27628	3·38634	11·46740	131·50126
	h	1·05062	1·28008	3·43711	11·81372	139·56389
	q	1·05095	1·28204	3·46340	11·99517	143·88409
	m	1·05127	1·28402	3·49034	12·18249	148·41314
6 per cent.	*y*	1·06000	1·33823	4·29187	18·42015	339·30202
	h	1·06090	1·34392	4·38391	19·21863	369·35580
	q	1·06136	1·34685	4·43204	19·64303	385·84862
	m	1·06184	1·34986	4·48169	20·08553	403·42883
7 per cent.	*y*	1·07000	1·40255	5·42743	29·45703	867·71633
	h	1·07122	1·41060	5·58493	31·19141	972·90391
	q	1·07186	1·41478	5·66816	32·12799	1032·20776
	m	1·07251	1·41907	5·75460	33·11545	1096·63320
8 per cent.	*y*	1·08000	1·46933	6·84847	46·90161	2199·76126
	h	1·08160	1·48024	7·10668	50·50495	2550·74976
	q	1·08243	1·48595	7·24465	52·48490	2754·66456
	m	1·08329	1·49182	7·38906	54·59815	2980·95794
9 per cent.	*y*	1·09000	1·53862	8·62308	74·35752	5529·04079
	h	1·09202	1·55297	9·03264	81·58851	6656·68600
	q	1·09308	1·56051	9·25405	85·63738	7333·75950
	m	1·09417	1·56831	9·48774	90·01713	8103·08407
10 per cent.	*y*	1·10000	1·61051	10·83471	117·39085	13780·61234
	h	1·10250	1·62889	11·46740	131·50126	17292·58207
	q	1·10381	1·63862	11·81372	139·56389	19478·07892
	m	1·10517	1·64872	12·18249	148·41314	22026·46566

§ 32. From this table it may be seen that the amount of £1 in 25 years at 5 per cent interest payable yearly is 3·38634; but if such interest is payable half-yearly it will amount to 3·43711; and if payable quarterly it will amount to 3·46340, in the same time: and that in no case will it amount to more than 3·49034. This table will therefore serve to show the *limits* of the values arising within the terms and rates there given.

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CHAPTER V.

ON THE PRESENT VALUE OF SUMS AT COMPOUND INTEREST.

§ 33. From what has been said in the preceding chapters, it will readily appear that the method of finding the *Present value* of a sum of money put out to compound interest will be the inverse method of that which has been there adopted for determining the Amount of any given sum in any given time: and that this present value will vary according to the number of payments of interest between the present time and the time when the sum becomes due.

§ 34. Now, in order to generalize these observations, let us make

p = the Present Value,
s = the Sum due,
n = the Number of Years,
ρ = the *annual* Rate of Interest;

then since £1 is to its amount at the end of the first year, as any other sum is to its amount in the same time; inversely it will be that, as $(1+\rho)$, or the amount of £1 at the end of the first year, is to £1, its present value; so is £1 to $\frac{1}{1+\rho}=$ the present value of £1 due at the end of the first year: again, as $1+\rho$ is to 1, so is $\frac{1}{1+\rho}$, or the present value just found, to $\frac{1}{(1+\rho)^2}=$ the present value of £1 due at the end of the second year. And by proceeding in this way it will be found that the present value of £1 due at the end of the

first year is $=(1+\rho)^{-1}=$
... second year is $=(1+\rho)^{-2}=$
... third year is $=(1+\rho)^{-3}=$
... fourth year is $=(1+\rho)^{-4}=$

or that universally $(1+\rho)^{-n}$ is equal to the present value of £1 due at the end of the nth year; which being multiplied by any other sum s will give $s\,(1+\rho)^{-n}$ for the present value of that other sum, due at the end of n years: and from this theorem others may be deduced for finding any one of the quantities when the rest are given: Thus,

$$p=s(1+\rho)^{-n} = \text{the Present Value,}$$
$$s=p(1+\rho)^{n} = \text{the Sum due,}$$
$$n=\frac{\lambda s-\lambda p}{\lambda(1+\rho)} = \text{the Number of Years,}$$
$$\rho=\left(\frac{s}{p}\right)^{\frac{1}{n}}-1 = \text{the } \textit{annual} \text{ Rate of Interest.}$$

§ 35. To show the use and application of these theorems, I shall propose the following questions:—

Example 1. What is the present value of £3000 due 20 years hence, and reckoning compound interest at the rate of 5 per cent per annum? Here we make use of the first theorem, where $s=3000$, $n=20$, and $\rho=\cdot05$; consequently $p=3000\times(1\cdot05)^{-20}=3000\times\cdot37688948=1130\cdot66844$, or £1130, 13s. 4d.

Example 2. A principal of £500 is put out to compound interest at the rate of 6 per cent per annum, to what sum will it amount in 10 years? Here we make use of the second theorem, where $p=500$, $n=10$, and $\rho=\cdot06$; consequently $s=500\times(1\cdot06)^{10}=500\times1\cdot79084770=895\cdot423850$, or £895, 8s. 6d.

Example 3. In what number of years will a principal of £493, 10s. amount to £1000 at the rate of 4 per cent per annum compound interest? Here we make use of the third theorem, where $p=493\cdot5$, $s=1000$, and $\rho=.04$; consequently $n=\frac{\mathrm{L}1000-\mathrm{L}493\cdot5}{\mathrm{L}1\cdot04}=\frac{3-2\cdot6932872}{\cdot0170333}=\frac{\cdot3067128}{\cdot0170333}=$ $18\cdot007$, or about 18 years.

Example 4. If a principal of £5000 amount to £8000 at the end of 9 years, what is the rate of compound interest per annum? Here we make use of the fourth theorem, where $p=5000$, $s=8000$, and $n=9$; consequently $\rho=\left(\frac{8000}{5000}\right)^{\frac{1}{9}}-1=1\cdot05361-1=\cdot05361$, or £5, 7s. 3d. *per cent.*

§ 36. It we compare these theorems with those already given in § 15, for calculating the quantities concerned in the *Amounts* of any given sum, we shall find them precisely the same, except that what is here called the *Principal*, is there called the *Present Value;* and what is there called the *Amount*, is here called the *Sum* due: whence we draw the same conclusions that I have already pointed out; namely, the Present value of a Sum due at any future time, is the Principal that will Amount to the given sum in that time. (See § 11.)

§ 37. Since the Discount of a given sum may be considered as the difference between its value at a future period and its present value, it follows that, when compound interest is allowed, the true discount of the sum s, to be received at a future period, will be expressed by $s-s(1+\rho)^{-n}$. Thus, the discount on £3000 for 20 years at the rate of 5 per cent per annum compound interest, is equal to $3000-1130{\cdot}668=1869{\cdot}332$ or £1869, 6s. 7¾d.

§ 38. It appears from what has been said in § 34, that the present value of any given sum which is to be received at the end of any given year, is equal to the product of such sum and the reciprocal of the amount of £1 as expressed in § 18. If, therefore, the geometrical progression $(1+\rho)^{-1}$, $(1+\rho)^{-2}$, $(1+\rho)^{-3}$, $(1+\rho)^{-4}$, &c., which respectively represent the present value of £1 at the end of the 1st, 2d, 3d, 4th, &c., year, be converted into numbers, where we may take ρ equal to ·02, ·025, ·03, &c., according as the rate of interest is 2, 2½, 3, &c., per cent per annum, we may form a set of tables that shall be at all times ready for use in the most common cases that arise. Thus, the present value of £1, due at the end of the first year, interest at *two* per cent per annum, is equal to $(1{\cdot}02)^{-1} = {\cdot}9804$; and the present value of £1 due at the end of the second year, at the same rate of interest per annum, is equal to $(1{\cdot}02)^{-2} = {\cdot}9612$; and if due at the end of the third year it is equal to $(1{\cdot}02)^{-3} = {\cdot}9423$, &c.* In the same manner, if the rate of interest were *two and a half* per cent per annum, the respective present values of £1 at the end of the first, second, third, &c., years, would be $(1{\cdot}025)^{-1} = {\cdot}9756$, $(1{\cdot}025)^{-2} = {\cdot}9518$, $(1{\cdot}025)^{-3} = 9286$, &c.; and so on, for any other rate of interest, and for any other terms. And by arranging the values, thus found, in proper columns, we might enlarge or extend the table at pleasure. It is in this manner that the Fourth Table at the end of the present work has been formed, which shows the Present value of £1 due at the end of any number of years not exceeding 100; and at the several rates of 2, 2½, 3, 3½, 4, 4½, 5, 6, 7, 8, 9, and 10 per cent interest per annum; and any of the values there found being multiplied by any other sum due, will give the present value of such other sum, at the given rate, and due at the end of the given time. For example, suppose it were required to determine the present value of £5 due at the end of 13 years, compound interest being allowed at the rate of 5 per cent per annum; here we multiply ·53032135 (=the value found under 5 per cent, and against 13 years) by 5, which produces 2·6516, or £2, 13s. for the answer.

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* These values are best found by the help of logarithms, particularly if the power be a high one: but it must be observed that in operating with a *negative* index, we must make the tabular logarithm negative; which is done by subtracting unity from the characteristic, and taking the arithmetical complement of the decimal part of the logarithm: and this is called taking the Co. Log. Thus the Log. of 1·05 is equal to ·0211893, and the Cog. Log. of the same is $\bar{1}{\cdot}9788107$, and which is also the Log. of $\frac{1}{1{\cdot}05}$ or $(1{\cdot}05)^{-1}$. In multiplying a logarithm with a negative index by any affirmative number (as in the raising of powers) the product will be negative; but since what is carried from the decimal part of the logarithm must be accounted affirmative, the difference only will be the index of the product. Therefore we shall probably find it most convenient, in working with logarithms having a negative index, to multiply separately the decimal part and the index, each by the given factor, and to take the difference of the *integers* thence arising for the index of the product.

§ 39. When we have to determine the present value of a given sum for a period of years beyond the limits of this table, we may proceed in the same manner as I have already pointed out in § 19, with respect to Table III. Thus the present value of £1, due at the end of 150 years at 5 per cent per annum compound interest, is equal to $\cdot 00760449 \times \cdot 08720373 = \cdot 00066314$.

§ 40. It should be observed here, that in calculating the present value of any given sum for a period of time *shorter* than a year, the value comes out *more* at compound interest than when reckoned at simple interest; and this is always the case when n is a proper fractional quantity; thus, if it were required to find the present value of £100 due at the end of half a year, at the rate of 5 per cent per annum compound interest, we should have $s = 100$, and $\rho = \cdot 05$, but $n = \frac{1}{2}$ only; consequently the formula will become $p = 100 \times (1 \cdot 05)^{-\frac{1}{2}} = 100 \times \cdot 97590 = 97 \cdot 59$, or £97, 11s. 9¾d.; whereas the present value at simple interest would have been only £97, 11s. 3d. And by a similar method of deduction it will be found that the present value of £100 due at the end of a quarter of a year, at the rate of 5 per cent per annum compound interest will be equal to $100 \times (1 \cdot 05)^{-\frac{1}{4}} = 100 \times \cdot 98788 = 98 \cdot 788$, or £98, 15s. 9d.; whereas the present value of the same at simple interest would have been only £98, 15s. 4d.

§ 41. But if we suppose (as is most usual) that, in the former case where the payments of interest are made half-yearly, each payment is one half of the nominal annual rate; and that in the latter case where the payments are made quarterly, each payment is one quarter of the nominal annual rate; then we shall find, by a similar method of reasoning to that which has been adopted in § 20, that the present value of £1 due a year hence, interest at 5 per cent, payable *half-yearly*, will be expressed by

$(1{\cdot}025)^{-2} = {\cdot}95181$; and that its present value, at an interest of 5 per cent payable *quarterly*, will be expressed by $(1{\cdot}0125)^{-4} = {\cdot}95152$.

§ 42. Since then it appears that the present value of any given sum, due a twelvemonth hence, decreases in a certain degree on the supposition that money is put out to interest for smaller portions of time than a year, it may be useful to know the quantity and extreme limit of this decrease, and which may be determined in the following manner:—Let the simple interest of £1 for a year be denoted by r; then, supposing the payments of interest to be made *yearly*, the present value of £1 due at the end of the year will be expressed by $(1+r)^{-1}$, which being expanded, either by the binomial theorem or by division, is found equal to $1-r+r^2-r^3+\&c.$ If the payments of interest be supposed to be made *half-yearly* (each payment in this case being one *half* of the nominal annual rate), then the present value of £1 due at the end of the year will be expressed by $(1+\frac{r}{2})^{-2}$, which, by the binomial theorem, is equal to $1-r+\frac{3r^2}{4}-\frac{r^3}{2}+\&c.$, and which is less than the former expression by $\frac{r^2}{4}-\frac{r^3}{2}+\&c.$, or by $\frac{1}{4}$ of the square of r, nearly. If the payments of interest be supposed to be made *quarterly* (each payment in this case being one *quarter* of the nominal annual rate), then the present value of £1 due at the end of the year will be expressed by $(1+\frac{r}{4})^{-4}$, which, by the binomial theorem, is equal to $1-r+\frac{5r^2}{8}-\frac{5r^3}{16}+\&c.$, and which is less than the first expression by the quantity $\frac{3r^2}{8}-\frac{11r^3}{16}+\&c.$, or by $\frac{3}{8}$ of the square of r, nearly. And, universally, if the payments of interest be supposed to be made m times in a year (each payment in this case being the mth part of the annual simple interest), then the present value of £1 due at the end of the year will be expressed by $(1+\frac{r}{m})^{-m}$, which, by the binomial theorem, is equal to $1-r+\frac{m+1}{2m}\cdot r^2-\frac{m+1}{2m}\cdot\frac{m+2}{3m}\cdot r^3+\&c.$, and which is less than the first expression by $\frac{m-1}{2m}\cdot r^2-\frac{m-1}{2m}\cdot\frac{5m+2}{3m}\cdot r^3+\&c.$, or by the $\frac{m-1}{2m}$th part of the square of r, nearly. Now when m is infinite, the quantities $m-1$, $m-2$, $m-3$, &c., become severally equal to m (see § 22); consequently the expression $(1+\frac{r}{m})^{-m}$ is in this case equal to $1-r+\frac{r^2}{1.2}-\frac{r^3}{1.2.3}+\frac{r^4}{1.2.3.4}-\&c.$, which is the limit of the decrease of the present value of £1 due at the end of a year, arising from our making the number of payments infinitely great, and the interest received at each time infinitely small; and which is less than the first expression by the quantity $\frac{r^2}{2}-\frac{5r^3}{6}+$ &c., or by ½ the square of r, nearly. But the whole of this series is easily determined, it being equal to the number which has $-r$ for its Neperean logarithm, or $-r\times{\cdot}4342944819$, &c., for its logarithm in Briggs' system.* For example, suppose it were required to find the limit of the decrease of £100 due a twelvemonth hence, if it were put out to interest in such a manner that the interest, which might momently become due on the same, should be continually added to the principal, and in this manner increase together: Here we multiply ·4342944819, &c., by $-{\cdot}05$, the product of which is equal to $-{\cdot}021714724095=\bar{1}{\cdot}978285275905$,† the natural number to which logarithm is ·95123; and this, multiplied by 100, gives 95·123, or £95, 2s. 5½d. for the present value required.

§ 43. It will appear from this that the decrease in the present value of any sum due at the end of one year, is not considerable, even on the supposition that money may be improved at an interest payable *momently* during that period; but such decrease will become greater according as the number of years increases. And since the present value of £1 due at the end of n years is expressed by the formula $(1+\frac{r}{m})^{-mn}$,‡ we may readily determine the *difference* between this value and that of $(1+r)^{-n}$, it being in all cases equal to $(1+r)^{-n}-(1+\frac{r}{m})^{-mn}$.

§ 44. When m is infinite, the formula $(1+\frac{r}{m})^{-mn}$ is expressed by the series $1-nr+\frac{n^2r^2}{1.2}-\frac{n^3r^3}{1.2.3}+\frac{n^4r^4}{1.2.3.4}-$ &c.,§ the sum of which is equal to the number that has $-nr$ for its Neperean logarithm, or $-nr\times{\cdot}4342944819$, &c., for its logarithm in Briggs' system. Thus, let it be required to find the present value of £100 due 50 years hence, on the supposition that money may be put out to interest at the rate of 5 per cent in such manner that it may be received *momently* and added to the principal; here we must multiply $-2{\cdot}5$ by ·4342944819, &c., which produces $-1{\cdot}085736205=\bar{2}{\cdot}914263795$, the natural number to which logarithm is

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* See the note to § 24.

† As the tabular logarithms are all positive, they may be rendered wholly negative by taking the Co. Log., as already observed in the note to § 38.

‡ Or the reciprocal of the amount of £1 in the same time; see § 23.

§ See Euler's *Introd. in Anal. Inf.*, vol. i. c. 7, § 117. Also the notes to § 24, *supra*.

·08208. It may be remarked here, that in finding the present value of any sum of money at compound interest, we must be careful to distinguish the *nominal* from the *true* annual rate, agreeably to what I have already observed in §§ 25 and 26.

§ 45. The investigations which have just been made, point out a method whereby we may in most cases deduce, from the tables above alluded to, the present value of any sum due at the end of any given year, on the supposition that the payments of interest are made *half-yearly*, *quarterly*, &c., each half-yearly or quarterly payment being one *half*, or one *quarter*, &c., of the *nominal* annual rate; or on the supposition that the payments of interest become due and are paid *biennially*, *triennially*, &c., each payment being respectively *double*, *treble*, &c., of the *nominal* annual rate. For, since the tables* are calculated on the supposition that the payments of interest become due, and are paid annually, it follows from the theorems above deduced, that in order to obtain the present value of £1 due at the end of any year, supposing the payments of interest to be made *half-yearly*, we must take the present value in the tables at *double* the number of years, and at *half* the rate of interest; and if the payments be made *quarterly*, we must take the present value in the tables at *quadruple* the number of years, and at a *quarter* of the rate of interest; and so on for any other periodical payment, as I have already remarked respecting the amount of sums, in § 27. For example, the present value of £1 due at the end of *six* years, at the rate of *eight* per cent interest payable *yearly*, is by the theorem equal to $(1{\cdot}08)^{-6}$, which by the table is found to be ·630170; if the payments of interest be made *half-yearly*, as above mentioned, the value will be equal to the present value of £1 due at the end of *twelve* years, at *four* per cent interest, or $(1{\cdot}04)^{-12}$, which by the table is found to be ·624597; if the payments be made *quarterly*, the value will be equal to the present value of £1 due *twenty-four* years hence, at *two* per cent interest, or $(1{\cdot}02)^{-24}$, which by the table is found to be ·621721. In a similar manner we might proceed where the payments of interest are *biennial*, *triennial*, &c.; but as the method of deduction is the same as in the Amount of sums, already treated of in § 27, it will be needless to insist further on this subject. For, universally, we shall find that $(1+\frac{r}{m})^{-mn}$ will denote the present value of £1 due at the end of n years, on the supposition that the payments of interest are made m times in a year, each payment being the mth part of r, which is the *nominal* annual rate; but where the *true* annual rate is $(1+\frac{r}{m})^{m}-1$, or ρ, as I have before observed in § 25. These remarks, indeed, apply only when we have recourse to the tables for the solution of such cases; for if they be solved agreeably to the theorem already laid down, we shall find that it will lead us to the true answer.

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* I allude to Table IV., given at the end of the present work, and to the similar tables of other writers.

§ 46. The best method of obtaining these values, when recourse cannot be had to the table, is by means of logarithms; for it is evident that the natural number corresponding to $-mn \times \lambda(1+\frac{r}{m})$ is the present value required. I shall, however, point out another method whereby the same results may be obtained, and that is by taking the natural number corresponding to $n \times \lambda(1+\frac{r}{m})^{-m}$; but as the finding of the value of $(1+\frac{r}{m})^{-m}$, or the reciprocal of $1+\rho$, in order to determine its logarithm, may be attended with trouble and inconvenience, I have subjoined the following table for the most practical cases, viz., those of yearly, half-yearly, and quarterly payments, to which I have added that of momently payments, in order that the reader may see the extreme limit of the decrease in these subdivisions.

§ 47. The following table, therefore, consists of the expansion of the binomial $(1+\frac{r}{m})^{-m}$ in numbers, where r is taken equal to ·02, ·025, ·03, &c., according as the nominal rate is 2, 2½, 3, &c., per cent, and where m is taken equal to 1, 2, 4, and ∞, according as the payments of interest are supposed to be made yearly, half-yearly, quarterly, and momently.* The first column indicates the value of r, or the nominal rate of simple interest; the second column denotes the periods at which the interest becomes due, and is payable, where y, h, q, and m respectively signify yearly, half-yearly, quarterly, and momently payments; the third column expresses the value of $\frac{1}{1+\rho}$, or the present value of £1 due at the end of a year, according to such annual rates; and the fourth column contains the logarithms of the numbers in the third column to 15 places of figures, which will be found very useful for determining the present value of £1 due at the end of any number of years, according to the several rates of interest mentioned in the table, and according as such interest is payable yearly, half-yearly, &c.

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* When the interest is payable momently, the formula $(1+\frac{r}{m})^{-mn}$ will be expressed by that number which has $-rn$ for its Neperean logarithm, or $-rn \times$ ·4342944819, &c., for its logarithm in Briggs' system; see § 44.

Nominal Rate of Interest.	Payable.	Present value of £1 in One Year.	Logarithm of such Present Value.
2 per cent.	*y*	·980392	$\bar{1}$·991 3998 2823 8082
	h	·980296	$\bar{1}$·991 3572 5243 4715
	q	·980247	$\bar{1}$·991 3357 5297 3969
	m	·980199	$\bar{1}$·991 3141 1036 1935
2½ per cent.	*y*	·975610	$\bar{1}$·989 2761 3460 8227
	h	·975461	$\bar{1}$·989 2099 3622 6588
	q	·975386	$\bar{1}$·989 1764 2649 6300
	m	·975310	$\bar{1}$·989 1426 3795 2419
3 per cent.	*y*	·970874	$\bar{1}$·987 1627 7529 4828
	h	·970662	$\bar{1}$·987 0679 1550 1537
	q	·970554	$\bar{1}$·987 0197 8074 7412
	m	·970445	$\bar{1}$·986 9711 6554 2902
3½ per cent.	*y*	·966184	$\bar{1}$·985 0596 5020 7063
	h	·965898	$\bar{1}$·984 9311 6420 5485
	q	·965752	$\bar{1}$·984 8658 0907 9493
	m	·965605	$\bar{1}$·984 7996 9313 3386
4 per cent.	*y*	·961538	$\bar{1}$·982 9666 6070 1220
	h	·961169	$\bar{1}$·982 7996 5647 6165
	q	·960980	$\bar{1}$·982 7145 0486 9430
	m	·960789	$\bar{1}$·982 6282 2072 3870
4½ per cent.	*y*	·956938	$\bar{1}$·980 8837 0955 2927
	h	·956474	$\bar{1}$·980 6733 6664 1241
	q	·956238	$\bar{1}$·980 5658 6151 8685
	m	·955997	$\bar{1}$·980 4567 4831 4354
5 per cent.	*y*	·952381	$\bar{1}$·978 8107 0093 0062
	h	·951814	$\bar{1}$·978 5522 6921 6454
	q	·951524	$\bar{1}$·978 4198 7245 3175
	m	·951230	$\bar{1}$·978 2852 7590 4837
6 per cent.	*y*	·943396	$\bar{1}$·974 6941 3473 5230
	h	·942596	$\bar{1}$·974 3255 5058 9656
	q	·942184	$\bar{1}$·974 1358 3100 3073
	m	·941764	$\bar{1}$·973 9423 5108 5805
7 per cent.	*y*	·934579	$\bar{1}$·970 6162 2231 4790
	h	·933511	$\bar{1}$·970 1193 0041 4127
	q	·932958	$\bar{1}$·969 8623 2841 0970
	m	·932394	$\bar{1}$·969 5993 8626 6772
8 per cent.	*y*	·925926	$\bar{1}$·966 5762 4451 3050
	h	·924556	$\bar{1}$·965 9333 2140 2439
	q	·923845	$\bar{1}$·965 5993 1295 2330
	m	·923116	$\bar{1}$·965 2564 4144 7740
9 per cent.	*y*	·917431	$\bar{1}$·962 5735 0205 9376
	h	·915730	$\bar{1}$·961 7674 1910 5854
	q	·914843	$\bar{1}$·961 3467 3328 2482
	m	·913931	$\bar{1}$·960 9134 9662 8707
10 per cent.	*y*	·909091	$\bar{1}$·958 6073 1484 1775
	h	·907029	$\bar{1}$·957 6214 0186 0124
	q	·905950	$\bar{1}$·957 1045 3843 2908
	m	·904837	$\bar{1}$·956 5705 5180 9675

§ 48. In order to show the use and application of this table, let us take the following example: suppose it were required to find the present value of £1 due at the end of 25 years, compound interest being allowed at the nominal rate of 5 per cent, payable quarterly: here we shall find that the value set down against the letter *q* in 5 per cent, is ·951524 which is the present value of £1 due at the end of one year, according to such annual rate, and the logarithm to which is found in the fourth column to be $\bar{1}$·978 4198 7245 3175; now this being multiplied by 25, will produce $\bar{1}$·460 4968 1132 94, the natural number to which logarithm is ·288733; whereas the present value in yearly payments would have been ·295303.

§ 49. With a view of showing the difference between the present value of £1 due at the end of a given number of years, improved at a given annual rate of interest, and the present value of £1 due at the same time when improved at the same nominal rate of interest payable half-yearly, quarterly, &c., I have subjoined another table which shows the present value of £1 due at the end of 1, 5, 25, 50, and 100 years, on the supposition that money may be improved at the nominal rate of 2, 3, 4, 5, 6, 7, 8, 9, and 10 per cent interest, payable yearly, half-yearly, quarterly, and momently; where *y*, *h*, *q*, and *m* denote the same as in the last table.

Nominal Rate of Interest.	Payable.	Present Value of £1 in				
		1 Year.	5 Years.	25 Years.	50 Years.	100 Years.
2 per cent.	*y*	·98039	·90573	·60953	·37153	·13803
	h	·98030	·90529	·60804	·36971	·13669
	q	·98025	·90506	·60729	·36880	·13601
	m	·98020	·90484	·60653	·36788	·13533
3 per cent.	*y*	·97087	·86261	·47761	·22811	·05203
	h	·97066	·86167	·47500	·22563	·05091
	q	·97055	·86119	·47369	·22438	·05035
	m	·97044	·86071	·47237	·22313	·04979
4 per cent.	*y*	·96154	·82193	·37512	·14071	·01980
	h	·96117	·82035	·37153	·13803	·01905
	q	·96098	·81954	·36971	·13669	·01868
	m	·96079	·81873	·36788	·13533	·01832
5 per cent.	*y*	·95238	·78353	·29530	·08720	·00760
	h	·95181	·78120	·29094	·08465	·00717
	q	·95152	·78001	·28873	·08337	·00695
	m	·95123	·77880	·28650	·08208	·00674
6 per cent.	*y*	·94340	·74726	·23300	·05429	·00295
	h	·94260	·74409	·22811	·05203	·00271
	q	·94218	·74247	·22563	·05091	·00259
	m	·94176	·74082	·22313	·04979	·00248
7 per cent.	*y*	·93458	·71299	·18425	·03395	·00115
	h	·93351	·70892	·17905	·03206	·00103
	q	·93296	·70682	·17642	·03113	·00097
	m	·93239	·70469	·17377	·03020	·00092
8 per cent.	*y*	·92593	·68058	·14602	·02132	·00045
	h	·92456	·67556	·14071	·01980	·00039
	q	·92384	·67297	·13803	·01905	·00036
	m	·92312	·67032	·13533	·01832	·00033
9 per cent.	*y*	·91743	·64993	·11597	·01345	·00018
	h	·91573	·64393	·11071	·01226	·00015
	q	·91484	·64082	·10806	·01168	·00014
	m	·91393	·63763	·10540	·01111	·00012
10 per cent.	*y*	·90909	·62092	·09230	·00852	·00007
	h	·90703	·61391	·08720	·00760	·00006
	q	·90595	·61027	·08465	·00717	·00005
	m	·90484	·60653	·08208	·00674	·00004

§ 50. From this table it appears the present value of £1 due at the end of 25 years, at 5 per cent interest payable yearly, is ·29530; but if such interest is payable half-yearly, the value will be ·29094; and if payable quarterly, it will be ·28873; but in no case will it be less than ·28650; whence we readily observe the limits of the values arising within the terms and rates here given.

§ 51. When n is infinite, the several values of $(1+\rho)^{-n}$ become equal to each other, or vanish altogether, whatever rate of interest is employed; and from a comparison of the several values against any two rates of interest in the table, it will be found that in any corresponding time the *difference* between such two values will first increase from nothing (when $n=0$) to some certain term or value of n, where the difference is greatest; and after that, the difference between such corresponding values will get less and less as n increases, till at last, when n is infinite, it vanishes altogether. For example, if we compare the several values set down against the two rates of 4 and 5 per cent each, payable yearly, we shall find that the *difference* between the present values of £1, due at the end of 1 year, is ·00916; at the end of 5 years, it is ·03840; at the end of 25 years, it is ·07982; at the end of 50 years, it is ·05351; at the end of 100 years, it is ·01220; and were the number of years infinitely great, the difference would at length entirely vanish; so that the maximum or greatest difference takes place when n is about 25. This term or value of n, however, will vary according as the two rates of interest vary, or as the periods of the payments of the same nominal rate of interest vary; and in these cases, its general expression will be $n=\frac{\lambda\left(\frac{\lambda(1+\rho)}{\lambda(1+r)}\right)}{\lambda(1+\rho)-\lambda(1+r)}$, where r denotes the least, and ρ the greatest of the two *annual* rates of interest assumed.* Hence we may deduce the value of n in comparing the values arising from any yearly, half-yearly, or quarterly payment of interest. For the difference between the present value of £1 improved at 5 per cent interest payable yearly, and the present value of £1 improved at the same nominal rate payable half-yearly, is greatest when

$$n=\frac{\lambda\left(\frac{\lambda\,1{\cdot}050625}{\lambda\,1{\cdot}05}\right)}{\lambda\,1{\cdot}050625-\lambda\,1{\cdot}05}=\frac{\lambda\left(\frac{{\cdot}021447731}{{\cdot}021189299}\right)}{{\cdot}021447731-{\cdot}021189299}=$$

$$\frac{\lambda{\cdot}021447731-\lambda{\cdot}021189299}{{\cdot}021447731-{\cdot}021189299}=\frac{\bar{2}{\cdot}3313814-\bar{2}{\cdot}3261166}{{\cdot}000258432}=\frac{{\cdot}005264800}{{\cdot}000258432}=$$

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* In order to find the time when $(1+r)^{-n}-(1+\rho)^{-n}$ is a maximum, we must make its differential equal to nothing. Consequently, as n is the variable quantity, we shall have $-(1+r)^{-n}\times\lambda(1+r)d.n+(1+\rho)^{-n}\times\lambda(1+\rho)\,d.n=0$; whence $n=\frac{\lambda\left(\frac{\lambda(1+\rho)}{\lambda(1+r)}\right)}{\lambda(1+\rho)-\lambda(1+r)}$.

20·372, or 20 years and 136 days. And the difference between the present value of £1 improved at 5 per cent interest payable yearly, and the present value of £1 improved at the same nominal rate payable quarterly, is greatest when $n = \dfrac{\lambda\left(\dfrac{\lambda 1{\cdot}050946}{\lambda 1{\cdot}05}\right)}{\lambda 1{\cdot}050946 - \lambda 1{\cdot}05} = \dfrac{\lambda\left(\dfrac{{\cdot}021580128}{{\cdot}021189299}\right)}{{\cdot}021580128 - {\cdot}021189299} =$ $\dfrac{\lambda{\cdot}021580128 - \lambda{\cdot}021189299}{{\cdot}021580128 - {\cdot}021189299} = \dfrac{\bar{2}{\cdot}3340540 - \bar{2}{\cdot}3261166}{{\cdot}000390828} = \dfrac{{\cdot}007937400}{{\cdot}000390828} =$ 20·309, or 20 years and 113 days. Again, the difference between the present value of £1 improved at 5 per cent interest per annum, and the present value of £1 improved at 10 per cent interest per annum, is greatest when $n = \dfrac{\lambda\left(\dfrac{\lambda 1{\cdot}10}{\lambda 1{\cdot}05}\right)}{\lambda 1{\cdot}10 - \lambda 1{\cdot}05} = \dfrac{\lambda\left(\dfrac{{\cdot}041392685}{{\cdot}021189299}\right)}{{\cdot}041392685 - {\cdot}021189299}$ $= \dfrac{{\cdot}290807100}{{\cdot}020203386} = 14{\cdot}394$, or 14 years and 144 days. These observations respecting the *difference* in the present values of any sum due at the end of any given period, according to the different rates of interest concerned, may be more fully illustrated and clearly explained by means of the logarithmic curve, as I shall show in the Appendix (Note E).

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CHAPTER VI.

ON ANNUITIES AT SIMPLE INTEREST.

§ 52. ANNUITY is a term applied to any periodical income arising from money lent, or from houses, lands, salaries, pensions, &c., payable from time to time either annually or at any other interval. Annuities may be divided into such as are certain; and such as depend on some contingency, as the continuance of one or more lives, and which are therefore called Life annuities. They are likewise divided into such as are in possession and such as are in reversion: the former signifying such as have already commenced; and the latter, such as will not commence till some particular event has happened, or till some given period of time has elapsed.

§ 53. The principal questions relating to the doctrine of annuities may be divided into two parts; viz., those relating to the *Amount*, and those relating to the *Present Value* of such annuities: and these will evidently differ according as Simple or Compound Interest is used in the calculation. The value of annuities, however, at Simple interest, is of so little use in practice, that I shall be very brief in my observations on this part of the subject.

§ 54. In order to find the total *Amount* of an annuity forborne during any given period, it is evident that we must find the amount of each year's annuity put out to interest to the end of the given term; and the sum of these several amounts will be the total amount to which the annuity will have increased in that time. Let us therefore make

s = the Amount of the Annuity,
a = the Annuity,
n = the Number of Years,
r = the Rate of Interest,

and let the annuity be £1 per annum: then, since £1 will amount to $1+r$, at the end of one year; to $1+2r$, at the end of two years; to $1+3r$, at the end of three years; and universally to $1+nr$, at the end of n years (see § 5); it follows that the amount of the last year's annuity will be 1 only; that the amount of the last year's annuity but one, will be $1+r$; that the amount of the last year's annuity but two, will be $1+2r$; that the amount of the last year's annuity but three, will be $1+3r$; and universally that the amount of the last year's annuity, but $(n-1)$, will be $1+(n-1).r$:[*] consequently the sum of all these amounts, or the series $1+(1+r)+(1+2r)+(1+3r)+\ldots\ldots$ to $(1+\overline{n-1}.r)$, will be the total amount of the annuity forborne n years. But the sum of this arithmetical series whose first term is 1, difference r, and number of terms n, is easily found to be $n+\frac{n(n-1)}{2}.r$; and which being multiplied by any other annuity a will produce $a(n+\frac{n(n-1)}{2}.r)$ for the total amount of that other annuity forborne n years. Whence we deduce the following theorems, from which any one of the quantities may be determined when the rest are given. Thus,

$$s=a(n+\frac{n\,(n-1)}{2}\,.\,r) \qquad = \text{the Amount,}$$

$$a=\frac{2s}{2n+n\,(n-1)r} \qquad = \text{the Annuity,}$$

$$n=\frac{\sqrt{(2-r)^2+8r.\frac{s}{a}}-(2+r)}{2r} = \text{the Number of Years,}$$

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[*] This is the amount of the *first* year's annuity in the time n; for such annuity will not be received till the end of the first year, or beginning of the second year; therefore its amount must be computed for the *remainder* of the term, or for $(n-1)$ years only.

$$r=\frac{2\,\frac{s}{a}-n)}{n\,(n-1)} \qquad = \text{the Rate of Interest.}$$

§ 55. As questions of this kind are so readily solved after what has gone before, I shall not detain the reader with any examples of the use and application of these formulæ; but shall only observe that if the annuity is payable half-yearly or quarterly, the formulæ will still be correct provided n be always equal to the number of *payments*, and r equal to the interest on £1 at the time the first payment becomes due.

§ 56. The method of determining the *Present Value* of a similar annuity will readily follow after what has just been laid down; for in this case we must find the present value of each year's annuity as it becomes due, and the sum of these several values will be the present value of the whole annuity. Now the present value of £1 to be received at the end of a year is $\frac{1}{1+r}$; and the present value of £1 to be received at the end of two years is $\frac{1}{1+2r}$; and the present value of £1 to be received at the end of three years is $\frac{1}{1+3r}$; and universally the present value of £1 to be received at the end of n years is $\frac{1}{1+nr}$ (see § 9): consequently the sum of all these values, or the series $\frac{1}{1+r}+\frac{1}{1+2r}+\frac{1}{1+3r}+\ldots\frac{1}{1+nr}$, will be the total present value of the annuity to continue n years; and which being multiplied by any other annuity a will produce the present value of such other annuity; or the sum which, put out to interest, will provide for the several payments of the annuity as they become due.

§ 57. The summation of this series of the reciprocals of an arithmetical progression is somewhat difficult; and I shall not detain the reader with an investigation of the method of determining its value: particularly as it is a subject more of speculation than of any real use. Indeed, it has been asserted with much reason by Sir Samuel Moreland, Dr. Dodson, and other writers on this subject, that the present value of annuities cannot be truly and accurately deduced according to simple interest; and in fact, different authors have laid down different methods for determining such value, when simple interest is concerned. The major part of them have adopted the following mode, viz., to find the *amount* of the annuity in the given time, as in § 54, and then to *discount* this amount by the rule in § 9: Thus, since the amount of an annuity a in n years is equal to $a\,(n+$

$\frac{n(n-1)}{2}.r)=s$, they say it follows that this amount is to its present value in the same proportion that the amount of £1 in that time is to *its* present value; or that $(1+nr):1=s:\frac{s}{1+nr}=$ the present value of the annuity. This rule, however, brings out the value too great; for, it is founded on the supposition that the whole of the annuities are to be paid off at the end of the given time in one payment; or that the several annuities are paid yearly out of the principal, and that no part of the interest is appropriated to that purpose till the end of the given term. Mr. Simpson has given a new method, which is, to divide s, or the amount of the annuity in the given time, by $1+nr+\frac{n(n-1)}{2}.r^2$, instead of $1+nr$ as above mentioned: for he says, that the amount of the annuity is to its present value, not in the proportion of the amount of £1 in the given time to its present value, but as the amount of £1 (the interest of which is also improved at simple interest)* is to its present value. This rule, however, brings out the value too little,† as it is founded on the supposition that the interest of the principal, or present value, is improved also at simple interest; and that thus, increasing together to the end of the given term, they amount to s, or the value of all the annuities with their interest up to that time. Neither of these methods, however, is correct; because, such a sum exactly ought to be taken from the principal every year, as together with its interest will amount to the given annuity. But since it is both legal and customary, when money is paid in discharge of part of a debt, to deduct the interest then due, from such payment, and to apply only the remaining part to the discharge of the principal, it follows that in computing the present value of annuities, &c., the rules founded upon the principles of compound interest will give their legal value. And as this method will be found much more useful, as well as more frequent in practice, I shall leave the present subject as it is, and proceed to the consideration of annuities at compound interest.

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* The interest of £1 being considered as an annuity, its amount in n years (by writing r for a in the formula in § 54) will be equal to $nr+\frac{n(n-1)}{2}.r^2$; to which the principal of £1 being added, it follows that $1+nr+\frac{n(n-1)}{2}.r^2$ will be the whole amount of £1 in the time n.

† Even less than when compound interest is reckoned.

CHAPTER VII.

ON THE AMOUNT OF ANNUITIES AT COMPOUND INTEREST PAYABLE YEARLY.

§ 58. THE method of calculating the amount of Annuities, at the end of any given term, improved annually at compound interest, will easily follow after what has been said respecting those at simple interest: for we have only to determine the amount of each year's annuity put out to compound interest to the end of the given term, and the sum of all these values will be the amount of the given annuity in that time. If, therefore, we make

$s =$ the Amount,
$a =$ the Annuity,
$n =$ the Number of Years,
$\rho =$ the *Annual* Rate of Interest,

and the annuity, in the first instance, equal to £1 per annum; then since £1 will amount to $(1+\rho)$ in one year; to $(1+\rho)^2$ in two years; to $(1+\rho)^3$ in three years; and universally to $(1+\rho)^n$ in n years (see § 15); it follows (as in the case of annuities at simple interest) that the amount of the last year's annuity will be simply 1; that the amount of the last year's annuity but one, will be $(1+\rho)$; that the amount of the last year's annuity but two, will be $(1+\rho)^2$; that the amount of the last year's annuity but three, will be $(1+\rho)^3$; and universally that the amount of the last year's annuity but $(n-1)$ will be $(1+\rho)^{n-1}$:* consequently the sum of all these values for n years, or the series $1+(1+\rho)+(1+\rho)^2+(1+\rho)^3+\ldots.(1+\rho)^{n-1}$, will be the total amount of the annuity forborne n years, and improved at compound interest during that term. But this series is a geometrical progression whose first term is 1, and ratio $(1+\rho)$; consequently the sum of this progression for n terms is found (by the common rules given for summing a geometrical series)† to be equal to $\frac{(1+\rho)^n-1}{\rho}$: and this expression being multiplied by any other annuity a will produce $a \times \frac{(1+\rho)^n-1}{\rho}=s$, for the amount of that other annuity in n years. From which theorem others may be deduced for determining any one of the quantities when the rest are given: thus,

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* Or rather the amount of the *first* year's annuity will at the end of n years be $(1+\rho)^{n-1}$; because such annuity will not be received till the *end* of that year, and therefore can be improved at interest during $(n-1)$ years only. It may here be observed that in calculating the value of any periodical annuity, the first payment of such annuity is in all cases supposed to be made at the *end* of the given period. See § 106.

† See Dodson's *Mathem. Repos.*, vol. ii. p. 63. Maseres's *Treatise on Life Annuities*, p. 91. Price's *Observ. on Rev. Pay.*, vol. i. p. 258, ed. 1803. Euler's *Algebra*, vol. i. p. 260. Bonnycastle's *Algebra*, p. 165: and most treatises on that science.

$$s=a.\frac{(1+\rho)^n-1}{\rho} \qquad = \text{the Amount,}$$

$$a=s.\frac{\rho}{(1+\rho)^n-1} \qquad = \text{the Annuity,}$$

$$n=\frac{\lambda.(1+\frac{s}{a}.\rho)}{\lambda.(1+\rho)} \qquad = \text{the Number of Years,}$$

$$\left.\begin{array}{l} \rho=\dfrac{[12+(n+1)\beta].\beta}{12+2(n+1)\beta} \\ \text{where } \beta=\left(\dfrac{s}{an}\right)^{\frac{2}{n-1}}-1 \end{array}\right\} = \text{the \textit{Annual} Rate of Interest.*}$$

§ 59. In order to show the use and application of these formulæ, I shall propose the following questions:—

Example 1. To what sum will an annuity of £1212 per annum increase in 76 years, at the rate of 4 per cent per annum compound interest? Here we make use of the first theorem, where $a=1212$, $n=76$, and $\rho=\cdot04$; consequently $s=1212\times\frac{(1\cdot04)^{76}-1}{\cdot04}=1212\times467\cdot57662125=$ $566702\cdot865$, or £566,702, 17s. 4d. From this example it may be seen that an annuity of £1,212,000 per annum, would amount, in the same time, to £566,702,865. Now, in the year 1732, the annual income of the Sinking Fund, taken on an average of 5 years, was £1,212,000; therefore, had that annuity been improved at 4 per cent interest to the present time, it would have amounted to a sum sufficient to pay off the whole National Debt, as it now stands in 1808.

Example 2. What annuity, improved at the rate of 8 per cent per annum compound interest, will at the end of 10 years amount to £3000? Here we make use of the second theorem, where $s=3000$, $n=10$, and $\rho=\cdot08$; consequently $a=3000\times\frac{\cdot08}{(1\cdot08)^{10}-1}=207\cdot088$, or £207, 1s. 9d.

Example 3. In what number of years will an annuity of £5586 per annum amount to £500,000, at the rate of 3 per cent interest per annum? Here we make use of the third theorem, where $s=500{,}000$, $a=5586$, and $\rho=\cdot03$; consequently $n=\frac{\lambda(1+\frac{500000}{5586}\times\cdot03)}{\lambda1\cdot03}=\frac{\cdot5664711}{\cdot0128372}=44\cdot127$, or 44 years and 46 days. From this example it appears that an annuity of £5,586,000 per annum will, if improved at the same rate, amount to

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* The method of deducing this formula will be explained in the Appendix (Note A); to which the reader is referred for other approximations to the value of ρ: and likewise for a method of *correcting* such approximations to any degree of accuracy.

£500,000,000 in the same time. In the year 1802, when the new Sinking Fund was established, it appeared that its income at that time might be estimated at £5,585,572; and the calculations that were laid before the House of Commons on that occasion, went to demonstrate that if this annuity were improved at 3, 3¼, 3½, 3¾, or 4 per cent interest, it would amount, in less than 45 years, to more than the whole funded debt, which was estimated at that time at £488,987,656; and the truth of this principle is proved by the present example.

Example 4. If an annuity of £34, 8s. per annum, forborne during 70 years, amount to £33,296, 17s. 4d., what is the rate of interest allowed? Here we make use of the last formula, where $s=33296{\cdot}8666$, $a=34{\cdot}4$, and $n=70$; consequently $\beta=\left(\frac{33296\cdot8666}{34\cdot4\times70}\right)^{\frac{2}{69}}-1=(13{\cdot}827602)^{\frac{2}{69}}-1=$ $\cdot07911$, and $\rho=\frac{(12+71\times\cdot07911)\times\cdot07911}{12+2\times71\times\cdot07911}=\frac{17\cdot61681\times\cdot07911}{23\cdot23362}=$ $\cdot059985$; which, being multiplied by 100, produces 5·9985, or about £6 for the rate per *cent.**

§ 60. It appears from what has been said in § 58, that the amount of any given annuity is equal to such annuity multiplied by the quantity $\frac{(1+\rho)^n-1}{\rho}$, which quantity is the expression for the amount of an annuity of £1 per annum in n years; if, therefore, we convert this expression into numbers, where n may be taken equal to the number of years, and ρ equal to ·02, ·025, ·03, &c., according as the annual rate is 2, 2½, 3, &c., per cent, we may form a set of tables which will be at all times ready for use in the most practical cases. Thus, supposing the rate of interest to be *two* per cent per annum, the amount of an annuity of £1 at the end of the first year will be equal to $\frac{1\cdot02-1}{\cdot02}=1$; at the end of the second year it will be equal to $\frac{(1\cdot02)^2-1}{\cdot02}=2{\cdot}0200$; at the end of the third year it will be equal to $\frac{(1\cdot02)^3-1}{\cdot02}=3{\cdot}0604$, and so on for subsequent years. And at the rate of *two and a half* per cent interest per annum, the amount of an annuity of £1 in one year would be $\frac{1\cdot025-1}{\cdot025}=1$; at the end of two years it would be equal to $\frac{(1\cdot025)^2-1}{\cdot025}=2{\cdot}0250$; at the end of three years it will be $\frac{(1·025)^3-1}{·025}=3·0756$; and so on for the succeeding years. By proceeding in this way with other rates of interest, and by arranging the values thus found into proper columns, we may enlarge or extend the table at pleasure. It is in this manner that the FIFTH Table at the end of the present work has been formed, which shows the amount of an annuity of £1 per annum at the end of any number of years not exceeding 100, at the several annual rates of 2, 2½, 3, 3½, 4, 4½, 5, 6, 7, 8, 9, and 10 per cent;* and any of the values there found being multiplied by any other annuity, will give the amount of such other annuity in the given time and at the given rate. For example, suppose it were required to find the amount of an annuity of £1212 per annum in 76 years at the annual rate of 4 per cent interest; here we multiply 467·5766 2125 by 1212, which produces 566702·865, as already found by the first example.

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* The answer to this question, according to Dr. Halley's formula, would come out 5·87, or £5, 17s. 5d. per cent, as may be seen in the Preface to Sherwin's *Mathem. Tables*, or in the *Scriptores Logarithmici*, vol. v. p. 267. I make this observation to show the superior accuracy of the present formula to the one there given; for since the true rate is exactly ·06, it appears that this formula will bring out ρ within ·000015 of its real value. This, however, will be more fully shown in the Appendix (Note A).

§ 61. It must be particularly observed here that in all cases of Annuities, when n denotes the number of *years*, ρ will denote the *true* rate of interest per annum; in opposition to r which I shall always take to express the *nominal* rate only. We may, however, in every case, express ρ in terms of r, in the given formula, by substituting its equal $(1+\frac{r}{m})^m-1$ (see § 25). Consequently the amount of an annuity of £1 payable yearly, is in all cases, whatever be the rate of interest, equal to $\frac{(1+\rho)^n-1}{\rho}=$

$$\frac{(1+\frac{r}{m})^{mn}-1}{(1+\frac{r}{m})^m-1}.$$

It is true that by making n equal to the number of *payments* of interest, and r equal to the rate of interest *received at every such time*, we may avoid the introduction of the new character ρ: but it is more convenient (as I have before observed in § 26) to consider these quantities in the manner I have here laid down.

§ 62. This last formula, $\frac{(1+\frac{r}{m})^{mn}-1}{(1+\frac{r}{m})^m-1}$, will enable us to determine the amount of any YEARLY annuity* in the time n, on the supposition that the money arising from the same may be improved at an interest payable *half yearly*, *quarterly*, &c., each payment of interest being one *half*, or one *quarter*, &c., of the nominal annual rate. Or, on the supposition that the money arising from the same may be improved at an interest payable only *biennially*, *triennially*, &c., each payment of interest being, in this case, *double* or *treble*, &c., of the nominal annual rate. For, since m denotes the number of payments of interest in one year, it will in half-yearly payments be equal to 2; in quarterly payments, equal to 4; in biennial payments, equal to $\frac{1}{2}$; in triennial payments, equal to $\frac{1}{3}$; and so on. From which it may be seen that the amount of any given YEARLY annuity will increase at the end of any given term in a certain degree according to the number of payments of interest which are supposed to be made on the money arising from the same. The quantity and extreme limit of this increase in some of the most common rates of interest, will be shown hereafter. See § 70.

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* It is evident, from the principles already laid down, that this table might be formed by adding together each year's annuity as found by Table III. Thus, the amount of an annuity of £1 per annum in 5 years at 5 per cent is

1·00000000
1·05000000
1·10250000
1·15762500
1·21550625

5·52563125

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CHAPTER VIII.

ON THE AMOUNT OF ANNUITIES AT COMPOUND INTEREST PAYABLE HALF-YEARLY, QUARTERLY, ETC.

§ 63. I HAVE hitherto considered only such annuities as are payable YEARLY, and have shown that the amount of these will differ according as the money arising from the same may be improved at any yearly, half-yearly, quarterly, &c., rate of interest. I shall now proceed to treat of such annuities as are payable at any other interval of time than whole years, whether they be HALF-YEARLY, QUARTERLY, &c.; or BIENNIAL, TRIENNIAL, &c.: and after I have pointed out the difference in the value of such annuities, I shall show that each of them will vary in their amount at the end of any given time, according as the money arising from the same is improvable as above mentioned. I am aware that it has been usual hitherto for writers on this subject to consider the interest of money as payable at the same periods or intervals as the annuity: these two cases, however, are totally independent of each other, as I shall now proceed to explain.

§ 64. The amount of an annuity of £1 per annum payable HALF-YEARLY, each payment being one half of the yearly annuity, is equal to the amount of an annuity which is the HALF of £1 in DOUBLE the number of payments: that is, it will be equal to $\frac{1}{2}+\frac{1}{2}(1+\rho)^{\frac{1}{2}}+\frac{1}{2}(1+\rho)+\frac{1}{2}(1+\rho)^{\frac{3}{2}}+\frac{1}{2}(1+\rho)^{2}+\ldots\frac{1}{2}(1+\rho)^{n-\frac{1}{2}}$; a series, which in n years, will consist of $2n$ terms. But this series is evidently a geometrical progression whose first term is $\frac{1}{2}$, and common ratio $(1+\rho)^{\frac{1}{2}}$, consequently its sum for $2n$ terms is readily found equal to $\frac{1}{2}\times\frac{(1+\rho)^{n}-1}{(1+\rho)^{\frac{1}{2}}-1}$.* And the amount of an annuity of £1 per annum payable QUARTERLY, each payment being one quarter of the yearly annuity, is equal to the amount of an annuity which is the QUARTER of £1 in QUADRUPLE the number of payments: that is, it will be equal to $\frac{1}{4}+\frac{1}{4}(1+\rho)^{\frac{1}{4}}+\frac{1}{4}(1+\rho)^{\frac{1}{2}}+\frac{1}{4}(1+\rho)^{\frac{3}{4}}+\frac{1}{4}(1+\rho)^{1}+\frac{1}{4}(1+\rho)^{\frac{5}{4}}+\ldots\ldots\frac{1}{4}(1+\rho)^{n-\frac{1}{4}}$; a series which in n years will consist of $4n$ terms; and the sum of which for that period is readily found equal to $\frac{1}{4}\times\frac{(1+\rho)^{n}-1}{(1+\rho)^{\frac{1}{4}}-1}$. And the amount of an annuity of £1 per annum payable BIENNIALLY, each payment being double the yearly annuity, is equal to the amount of an annuity which is the DOUBLE of £1 in HALF the number of payments: that is, it will be equal to $2+2(1+\rho)^{2}+2(1+\rho)^{4}+2(1+\rho)^{6}+\ldots\ldots\ldots 2(1+\rho)^{n-2}$; a series which in n years will consist of $\frac{1}{2}n$ terms; and the sum of which for that period is readily found equal to $2\times\frac{(1+\rho)^{n}-1}{(1+\rho)^{2}-1}$. And universally, the amount of an annuity of £1 per annum payable μ times in a year, each payment being $\frac{1}{\mu}$th of the yearly annuity, is equal to the amount of an annuity which is $\frac{1}{\mu}$th of £1 in μn payments: that is, it is equal to $\frac{1}{\mu}+\frac{1}{\mu}(1+\rho)^{\frac{1}{\mu}}+\frac{1}{\mu}(1+\rho)^{\frac{2}{\mu}}+\frac{1}{\mu}(1+\rho)^{\frac{3}{\mu}}+\frac{1}{\mu}(1+\rho)^{\frac{4}{\mu}}+\ldots\ldots\frac{1}{\mu}(1+\rho)^{n-\frac{1}{\mu}}$; a series which, in n years, will consist of μn terms; and the sum of which in that time is equal to $\frac{1}{\mu}\times\frac{(1+\rho)^{n}-1}{(1+\rho)^{\frac{1}{\mu}}-1}$: and this is a general formula for the amount of an annuity of £1 under such circumstances.

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* I call YEARLY annuities such as are paid only ONCE a year; in opposition to HALF-YEARLY, QUARTERLY, &c., annuities, which are paid TWO, FOUR, &c., times in a year; each payment of the annuity being one HALF, QUARTER, &c. of the whole yearly annuity. And these cases must be carefully distinguished from each other without any regard to the periodical payment of *interest*.

§ 65. It must be observed that in deducing this formula I have had regard to the *true* rate only of annual interest; but we may easily convert it into one that will contain the *nominal* rate, by substituting for ρ its equal $(1+\frac{r}{m})^{m}-1$ (see § 25): which substitution being made we shall have $\frac{1}{\mu}\times\frac{(1+\rho)^{n}-1}{(1+\rho)^{\frac{1}{\mu}}-1}=\frac{1}{\mu}\times\frac{(1+\frac{r}{m})^{mn}-1}{(1+\frac{r}{m})^{\frac{m}{\mu}}-1}$: where ρ denotes the true rate of annual interest, and r the nominal rate of interest payable m times in a year, each payment of interest being the mth part of r.

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* See the authors quoted in the note to § 58.

§ 66. When m is infinite, that is, when the payments of *interest* are supposed to be made momently, we shall have $(1+\frac{r}{m})^{m}=\mathrm{R}$; where R denotes the natural number which has r for its Neperean logarithm, or $r\times{\cdot}4342944819$, &c., for its logarithm in Briggs' system: and we shall also in such case have $(1+\frac{r}{m})^{mn}=\mathrm{RN}$; where RN denotes the natural number which has rn for its Neperean logarithm, or $rn\times{\cdot}4342944819$, &c., for its logarithm in Briggs' system (see §§ 22 and 24). When μ is infinite, that is, when the payments of the *annuity* are supposed to be made momently, we shall have $\mu\times[(1+\frac{r}{m})^{\frac{m}{\mu}}-1]$ equal to the Neperean logarithm of $(1+\frac{r}{m})^{m}$:* and when both m and μ are infinite, that is, when $m=\mu$, the expression $\mu\times[(1+\frac{r}{m})^{\frac{m}{\mu}}-1]$ will become equal to r.

§ 67. From the general formula above mentioned I have deduced the following theorems for some of the most practical cases; namely, for showing the amount of annuities, which ANNUITIES are payable YEARLY, HALF-YEARLY, QUARTERLY, and MOMENTLY; and each on the supposition that the money arising from the same may be improved at an interest, which *interest* is payable *yearly*, *half-yearly*, *quarterly*, and *momently* as above mentioned. It may be observed here that y, h, q, and m denote the same as in §§ 29 or 47: and that the *momently* theorems are added, in order that the extreme limit of the amount in such cases might be readily deduced. I would also observe that I use the initial letters Nλ to denote the Neperean logarithm of the quantity immediately following them.

If the annuity is payable YEARLY

$$y=\frac{(1+r)^{n}-1}{r}\qquad h=\frac{(1+\frac{r}{2})^{2n}-1}{(1+\frac{r}{2})^{2}-1}\qquad q=\frac{(1+\frac{r}{4})^{4n}-1}{(1+\frac{r}{4})^{4}-1}\qquad m=\frac{\mathrm{RN}-1}{\mathrm{R}-1}$$

If the annuity is payable HALF-YEARLY

$$y=\tfrac{1}{2}\times\frac{(1+r)^{n}-1}{(1+r)^{\frac{1}{2}}-1}\qquad h=\frac{(1+\frac{r}{2})^{2n}-1}{r}\qquad q=\tfrac{1}{2}\times\frac{(1+\frac{r}{4})^{4n}-1}{(1+\frac{r}{4})^{2}-1}\qquad m=\tfrac{1}{2}\times\frac{\mathrm{RN}-1}{\mathrm{R}^{\frac{1}{2}}-1}$$

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* See Euler's *Introd. in Anal. Inf.*, vol. i. chap. 7, § 119.

If the annuity is payable QUARTERLY

$$y=\tfrac{1}{4}\times\frac{(1+r)^n-1}{(1+r)^{\frac{1}{4}}-1}\qquad h=\tfrac{1}{4}\times\frac{\left(1+\frac{r}{2}\right)^{2n}-1}{\left(1+\frac{r}{2}\right)^{\frac{1}{2}}-1}\qquad q=\frac{\left(1+\frac{r}{4}\right)^{4n}-1}{r}\qquad m=\tfrac{1}{4}\times\frac{\mathrm{RN}-1}{\mathrm{R}^{\frac{1}{4}}-1}$$

If the annuity is payable MOMENTLY

$$y=\frac{(1+r)^n-1}{\mathrm{N}\lambda(1+r)}\qquad h=\frac{\left(1+\frac{r}{2}\right)^{2n}-1}{\mathrm{N}\lambda\left(1+\frac{r}{2}\right)^{2}}\qquad q=\frac{\left(1+\frac{r}{4}\right)^{4n}-1}{\mathrm{N}\lambda\left(1+\frac{r}{4}\right)^{4}}\qquad m=\frac{\mathrm{RN}-1}{r}$$

§ 68. From an inspection of these theorems, or of the general formula in § 64, it will be seen that the amount of any annuity payable YEARLY is to the amount of the same annuity payable μ times in a year (the *annual* interest being still the same) in the ratio of 1 to $\frac{1}{\mu}\cdot\frac{\rho}{(1+\rho)^{\frac{1}{\mu}}-1}$. Therefore, having found the amount of an annuity payable yearly, we may easily find the amount of the same annuity payable μ times in a year (at the same *annual* rate of interest) by multiplying such amount by the constant quantity $\frac{1}{\mu}\cdot\frac{\rho}{(1+\rho)^{\frac{1}{\mu}}-1}$: Thus, the amount of an annuity of £1 per annum payable YEARLY, for 25 years at 5 per cent per annum, is by the Tables equal to 47·727099; whence the amount of the same annuity payable HALF-YEARLY (the annual interest ρ being the same) is equal to $47{\cdot}727099\times\frac{1}{2}\cdot\frac{{\cdot}05}{(1{\cdot}05)^{\frac{1}{2}}-1}=47{\cdot}727099\times 1{\cdot}01235=48{\cdot}31653$; and the amount of the same annuity payable QUARTERLY (the annual interest ρ being still the same) is equal to $47{\cdot}727099\times\frac{1}{4}\cdot\frac{{\cdot}05}{(1{\cdot}05)^{\frac{1}{4}}-1}=47{\cdot}727099\times 1{\cdot}01856=48{\cdot}61291$.

§ 69. When the periods of the payment of the annuity and of the payments of interest are equal to each other; that is, when the annuity is payable half-yearly and the money arising from the same can be improved at an interest payable half-yearly; or when the annuity is payable quarterly and the money arising from the same can be improved at an interest payable also quarterly; or universally when $\mu = m$, then the denominator in the general formula becomes equal to r. This particular case is worthy of observation, not only as being that which is most usually assumed in practice, but as it affords us an opportunity, in many instances, of deducing from the tables (which show the amounts of annuities payable YEARLY) the amounts of the same annuities payable HALF-YEARLY, QUARTERLY, &c.; or BIENNIALLY, TRIENNIALLY, &c. For, under such circumstances, the amount of any HALF-YEARLY annuity at any given rate of interest is equal to *half* the amount of the same annuity in *double* the number of years, at *half* the given rate; and the amount of any QUARTERLY annuity at any given rate of interest is, under such circumstances, equal to *quarter* the amount of the same annuity in *quadruple* the number of years, at a *quarter* the given rate; and the amount of any BIENNIAL annuity at any given rate of interest is, under such circumstances, equal to *double* the same annuity in *half* the number of years, and at *double* the given rate; and so on for any other periodical annuity. Thus, the amount of an annuity of £1 per annum payable YEARLY for *ten* years, and interest at *eight* per cent, is equal to $\frac{(1{\cdot}08)^{10}-1}{{\cdot}08}$, which by the tables is found to be 14 4866; if such annuity be payable HALF-YEARLY (the interest also being payable half-yearly), it will be equal to *half* the amount of an annuity of £1 per annum in *twenty* years, at *four* per cent, or $\frac{1}{2}\times\frac{(1{\cdot}04)^{20}-1}{{\cdot}04}$, which by the tables is $\frac{1}{2}\times 29{\cdot}7781=14{\cdot}8890$: and if such annuity be payable QUARTERLY (the interest also being payable quarterly), it will be equal to *quarter* the amount of an annuity of £1 per annum in *forty* years at *two* per cent, or $\frac{1}{4}\times\frac{(1{\cdot}02)^{40}-1}{{\cdot}02}$, which by the tables is $\frac{1}{4}\times 60{\cdot}4020=15{\cdot}1005$. These remarks, however, are of use only when we have recourse to the tables for the solution of such questions; for if we resolve them agreeably to the formula given in § 58, we shall find that it will lead us to the true answer; in fact, the tables are too limited to afford us the requisite assistance in many practical cases; and I have pointed out the above method of deducing the values, merely that we may be enabled to take advantage of it whenever an opportunity does occur.

§ 70. In order that the reader may see at one view the real difference which arises in the amounts of annuities payable YEARLY, HALF-YEARLY, QUARTERLY, and MOMENTLY, according as such payments can be improved at interest payable *yearly*, *half-yearly*, *quarterly*, and *momently*, I have subjoined the following table, which may be considered as so many examples of the theorems in § 67, at the several nominal rates of 2, 5, and 10 per cent interest; and for 5, 25, 50, and 100 years. These mean and extreme cases will enable us to discover the limits of the difference which may arise in any intermediate cases.

Nominal rate of Interest.	Payments of the Annuity.	Paymt. of Interest.	5 Years.	25 Years.	50 Years.	100 Years.
2 per Cent.	Yearly.	*y*	5·2040	32·0303	84·5794	312·2323
		h	5·2051	32·0712	84·8166	314·2296
		q	5·2058	32·0927	84·9388	315·2518
		m	5·2061	32·1128	85·0578	316·2690
	Half-yearly.	*y*	5·2299	32·1897	85·0002	313·7857
		h	5·2311	32·2316	85·2407	315·8008
		q	5·2317	32·2527	85·3624	316·8239
		m	5·2323	32·2741	85·4852	317·8580
	Quarterly.	*y*	5·2429	32·2695	85·2112	314·5645
		h	5·2441	32·3119	85·4531	316·5878
		q	5·2448	32·3334	85·5758	317·6162
		m	5·2454	32·3550	85·6993	318·6542
	Momently.	*y*	5·2559	32·3495	85·4224	315·3444
		h	5·2572	32·3925	85·6662	317·3771
		q	5·2579	32·4142	85·7896	318·4097
		m	5·2585	32·4361	85·9141	319·4527
5 per Cent.	Yearly.	*y*	5·5256	47·7271	209·3480	2610·0252
		h	5·5325	48·1404	213·6044	2737·0650
		q	5·5361	48·3538	215·8228	2804·6539
		m	5·5397	48·5720	218·1051	2875·1685
	Half-yearly.	*y*	5·5939	48·3164	211·9329	2642·2524
		h	5·6017	48·7422	216·2743	2771·2776
		q	5·6057	48·9621	218·5376	2839·9326
		m	5·6098	49·1869	220·8661	2911·5661
	Quarterly.	*y*	5·6282	48·6129	213·2340	2658·4731
		h	5·6365	49·0450	217·6180	2788·4945
		q	5·6407	49·2681	219·9034	2857·6820
		m	5·6451	49·4963	222·2552	2929·8775
	Momently.	*y*	5·6627	48·9105	214·5391	2674·7455
		h	5·6714	49·3489	218·9666	2805·7758
		q	5·6759	49·5754	221·2749	2875·5053
		m	5·6805	49·8069	223·6500	2948·2634
10 per Cent.	Yearly.	*y*	6·1051	98·3471	1163·9085	137796·1234
		h	6·1356	102·1209	1273·1830	168698·3900
		q	6·1516	104·1653	1334·7452	187617·0000
		m	6·1683	106·3269	1401·6532	209425·4300
	Half-yearly.	*y*	6·2541	100·7472	1192·3133	141159·0000
		h	6·2889	104·6740	1305·0126	172915·8400
		q	6·3073	106·8021	1368·5325	192366·2200
		m	6·3264	109·0526	1437·5843	214794·0500
	Quarterly.	*y*	6·3295	101·9618	1206·6883	142860·9200
		h	6·3666	105·9664	1321·1261	175050·9300
		q	6·3862	108·1372	1385·6390	194770·8100
		m	6·4065	110·4331	1455·7830	217513·1500
	Momently.	*y*	6·4055	103·1865	1221·1814	144576·7700
		h	6·4449	107·2696	1337·3724	177203·5700
		q	6·4657	109·4833	1402·8881	197195·4100
		m	6·4872	111·8249	1474·1314	220254·6600

Thus, the amount of an annuity of £1 per annum for 25 years, payable YEARLY, at 5 per cent interest payable also *yearly*, is 47·7271; but if the interest is payable *half-yearly*, the amount is 48·1404; and if the annuity is also payable HALF-YEARLY, the amount is 48·7422; but in no case will the amount be more than 49·8069, even if the annuity and interest were payable at every moment.

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CHAPTER IX.

ON THE PRESENT VALUE OF ANNUITIES AT COMPOUND INTEREST PAYABLE YEARLY.

§ 71. THE *present value* of an annuity is such a sum as, put out to interest, will enable us to provide for the several payments of the annuity as they become due. In order to obtain such value, we must find the present value of these several payments, and the sum of them will be the total present value of the annuity. It will be readily seen that the mode of investigating this subject is similar to that which has been adopted in the last chapter for determining the amount of annuities; whence some repetition will apparently occur.

§ 72. In order to generalize the subject, let us make

p = the Present Value,
a = the Annuity,
n = the Number of Years,
ρ = the *Annual* Rate of Interest;

and let the annuity be in the first instance equal to £1 per annum; then since the present value of £1 due at the end of the first year is $(1+\rho)^{-1}$; and, when due at the end of two years, is $(1+\rho)^{-2}$; and, when due at the end of three years, is $(1+\rho)^{-3}$; and, universally, since the present value of £1 due at the end of n years is $(1+\rho)^{-n}$ (see § 34), it follows that the sum of all these values for n years, or the series $(1+\rho)^{-1}+(1+\rho)^{-2}+(1+\rho)^{-3}+\ldots\ldots(1+\rho)^{-n}$, will express the total present value of an annuity of £1 per annum for n years. But this series is evidently a geometrical progression, whose first term and common ratio are each equal to $(1+\rho)^{-1}$, therefore the sum of it for n terms, or the present value of all the yearly annuities is found (by the common rules for summing a geometrical series)* equal to $\dfrac{1-(1+\rho)^{-n}}{\rho}$; and this expression, being multiplied by any other annuity a, will produce $a\times\frac{1-(1+\rho)^{-n}}{\rho}=p$, for the present value of that other annuity for n years. From which theorem others may be deduced for finding any one of the quantities when the rest are given: Thus,

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* See the authors quoted in the note to § 58.

$$p=a.\frac{1-(1+\rho)^{-n}}{\rho} \quad = \text{the Present Value,}$$

$$a=p.\frac{\rho}{1-(1+\rho)^{-n}} \quad = \text{the Annuity,}$$

$$n=\frac{\text{Co.}\,\lambda(1-\frac{p}{a}.\rho)}{\lambda(1+\rho)} \quad = \text{the Number of Years,}$$

$$\left.\begin{aligned}\rho&=\frac{[12-(n-1)\beta].\beta}{12-2(n-1)\beta}\\ \text{where }\beta&=\left(\frac{an}{p}\right)^{\frac{2}{n+1}}-1\end{aligned}\right\} = \text{the \textit{Annual} Rate of Interest.*}$$

§ 73. In order to show the use and application of these formulæ, let us resolve the following questions:—

Example 1. What is the present value of an annuity of £20 per annum for 40 years, at the rate of 6 per cent interest per annum? Here we make use of the first theorem, where $a=20$, $n=40$, and $\rho=\cdot06$; consequently, $p=20\times\frac{1-(1\cdot06)^{-40}}{\cdot06}=20\times15\cdot046297=300\cdot925940$, or £300, 18s. 6d.

Example 2. If a lease for 55¼ years be purchased for £100, what rent ought to be received in order that the purchaser may make 5½ per cent interest per annum of his money? Here we make use of the second theorem, where $p=100$, $n=55\frac{1}{4}$, and $\rho=\cdot055$; consequently, $a=100\times\frac{\cdot055}{1-(1\cdot055)^{-55\cdot25}}=100\times\frac{\cdot055}{\cdot9480896}=5\cdot801$ or £5, 16s.

Example 3. If Government should be disposed to exchange 5 per cent stock at par for terminable annuities of 7 per cent per annum; for what number of years ought such annuities to be granted in order that the proprietors may make 5½ per cent interest per annum of their money? Here we make use of the third theorem, where $p=100$, $a=7$, and $\rho=\cdot055$; consequently $n=\frac{\text{Co.}\,\lambda(1-\frac{100}{7}\times\cdot055)}{\lambda1\cdot055}=\frac{\text{Co.}\,\lambda\cdot2142857}{\lambda1\cdot055}=\frac{\cdot6690067}{\cdot0232525}=28\cdot771$, or rather more than 28¾ years.

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* The method of deducing this formula will be explained in the Appendix (Note B), to which the reader is referred for other approximations to the value of ρ; and likewise for a method of correcting such approximations to any degree of accuracy.

Example 4. An annuity of £20 per annum for 21 years is sold for £220; it is required to find the rate of interest allowed to the purchaser. Here we make use of the fourth formula, where $p=220$, $a=20$, and $n=21$; consequently $\beta=\left(\frac{20\times 21}{220}\right)^{\frac{1}{11}}-1=(1{\cdot}9090909)^{\frac{1}{11}}-1={\cdot}060546$, and $\rho=\frac{(12-20\times{\cdot}060546)\times{\cdot}060546}{12-2\times 20\times{\cdot}060546}=\frac{10{\cdot}789080\times{\cdot}060546}{9{\cdot}578160}={\cdot}06820$; which being multiplied by 100 produces 6·820, or £6, 16s. 5d. for the rate per *cent.**

§ 74. It appears from what has been said in § 72, that the present value of any given annuity is equal to such annuity multiplied by $\frac{1-(1+\rho)^{-n}}{\rho}$, which is the expression for the present value of an annuity of £1 per annum for n years. If, therefore, we convert this expression into numbers, where n may be taken equal to the number of years, and ρ equal to ·02, ·025, ·03, &c., according as the true rate is 2, 2½, 3, &c., per cent per annum, we may form a set of tables which will be at all times ready for use in the most practical cases. Thus, supposing interest to be *two* per cent per annum, the present value of £1 per annum for *one* year will be equal to $\frac{1-(1{\cdot}02)^{-1}}{{\cdot}02}={\cdot}980392$; for *two* years, it will be equal to $\frac{1-(1{\cdot}02)^{-2}}{{\cdot}02}=1{\cdot}941561$; for *three* years, it will be equal to $\frac{1-(1.02)^{-3}}{{\cdot}02}=2{\cdot}883883$; and so on for any other term, or for any other rate of interest, as I have already observed respecting the amounts of annuities (see § 60). By arranging the values thus found into proper columns, we may enlarge or extend the table at pleasure: and it is in this manner that the SIXTH table, at the end of the present work, has been formed; which shows the present value of £1 per annum for any number of years not exceeding 100, and at the several rates of 2, 2½, 3, 3½, 4, 4½, 5, 6, 7, 8, 9, and 10 per cent interest per annum. And by multiplying any of the values there given by any other annuity, we shall obtain the present value of such other annuity in the given time as before demonstrated. For example, suppose it were required to find the present value of an annuity of £20 per annum for 40 years, at the annual rate of 6 per cent interest: here we multiply 15·046296 by 20, which produces 300·925940, as already found by the first example.

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* The *true* rate in this question is ·06814, as stated in the preface to the third edition of Sherwin's *Mathematical Tables*, 1741; see also *Scriptores Logarithmici*, vol. v. page 312. It may be determined by the method of correction given in the Appendix, and which is alluded to in the note to § 72.

§ 75. The present value of an annuity of *one pound* per annum is commonly called the *Number of years' purchase* that any annuity is worth for that time and rate: thus, in the last case, the present value of *any* annuity for 40 years at 6 per cent interest is rather more than 15 years purchase; and the present value of *any* annuity for 30 years at 8 per cent interest is about 11¼ years' purchase. This term is applied not only to immediate annuities, but to such as are in reversion, or that depend on any contingency, &c., and its value is always $\frac{p}{a}$.

§ 76. In questions relating to this subject it is oftentimes required to determine *the annuity which a given sum will purchase* for a given number of years, and at a given rate of interest. In order to facilitate the solution of such questions, the SEVENTH table has been formed, which shows the annuity which £1 will purchase for any time and rate therein mentioned. From which values we may easily ascertain the annuity that any other sum will purchase, by simple multiplication as in all the preceding tables. It is deduced from the second formula in § 72, and consists of the reciprocals of the values in Table VI. Thus, the annuity which £1 will purchase for 12 years at 5 per cent interest is equal to $\frac{1}{8{\cdot}863251 64}=$ ·11282541, and if this were multiplied by 300 it would give 33·84762300, or £33, 16s. 11½d., as the annuity which £300 would purchase for the same time and rate.

§ 77. We must be careful to observe in all cases of annuities, whether we are investigating the amounts or the present values, that when n denotes the number of years, ρ will denote the *true* rate of interest in opposition to r which denotes the *nominal* rate only (see §§ 25, 61). But ρ may always be expressed in terms of r by substituting its equal $(1+\frac{r}{m})^m-1$ in the given formula: consequently the present value of an annuity of £1 payable yearly for n years will be expressed by $\frac{1-(1+\rho)^{-n}}{\rho}=$

$$\frac{1-(1+\frac{r}{m})^{-mn}}{(1+\frac{r}{m})^m-1}.$$

This last formula will enable us to determine the present value of any YEARLY annuity for the time n, on the supposition that money can be improved at an interest payable *half-yearly*, *quarterly*, &c., each payment of interest being one *half*, or one *quarter* of the nominal annual rate: Or, on the supposition that money may be improved at an interest payable *biennially*, *triennially*, &c., each payment of interest being *double*, *treble*, &c., the nominal annual rate. For, since m denotes the number of payments of interest, it will in half-yearly payments be equal to 2; in quarterly payments, equal to 4; in biennial payments, it will be equal to $\frac{1}{2}$; and in triennial payments, equal to $\frac{1}{3}$; and so on. From which it will appear that the present value of any given YEARLY annuity, for any given term, will decrease in a certain degree according to the number of payments of interest which are supposed to be made. The quantity of this decrease in some of the most common rates of nominal interest will be shown hereafter (see § 82).

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CHAPTER X.

ON THE PRESENT VALUE OF ANNUITIES AT COMPOUND INTEREST PAYABLE HALF-YEARLY, QUARTERLY, ETC.

§ 78. I SHALL now endeavour to prove that the present value of annuities for any given term will differ not only as the payments of *interest* are supposed to be made yearly, half-yearly, &c., but also as the payments of the *annuity* are supposed to be made YEARLY, HALF-YEARLY, QUARTERLY, &c.; or BIENNIALLY, TRIENNIALLY, &c.: and what I have to deliver on this head will be readily understood after what has been said respecting the amounts of such annuities in § 64, and by referring to the method of deduction laid down in § 72. For the present value of an annuity of £1 per annum payable HALF-YEARLY, each payment being one-half of the yearly annuity, is equal to the present value of an annuity which is the HALF of £1 in DOUBLE the number of payments: that is, it will be equal to $\frac{1}{2}(1+\rho)^{-\frac{1}{2}}+\frac{1}{2}(1+\rho)^{-1}+\frac{1}{2}(1+\rho)^{-\frac{3}{2}}+\frac{1}{2}(1+\rho)^{-2}+\ldots\ldots\frac{1}{2}(1+\rho)^{-n}$, a series which in n years will consist of $2n$ terms. But this series is a geometrical progression, whose first term is $\frac{1}{2}(1+\rho)^{-\frac{1}{2}}$ and common ratio $(1+\rho)^{-\frac{1}{2}}$; consequently its sum for $2n$ terms will be equal to $\frac{1}{2}\times\frac{1-(1+\rho)^{-n}}{(1+\rho)^{\frac{1}{2}}-1}$.* And the present value of an annuity of £1 per annum payable QUARTERLY, each payment being one-quarter of the yearly annuity, is equal to the present value of an annuity which is one QUARTER of £1 in QUADRUPLE the number of payments: that is, it will be equal to $\frac{1}{4}(1+\rho)^{-\frac{1}{4}}+\frac{1}{4}(1+\rho)^{-\frac{1}{2}}+\frac{1}{4}(1+\rho)^{-\frac{3}{4}}+\frac{1}{4}(1+\rho)^{-1}+\frac{1}{4}(1+\rho)^{-\frac{5}{4}}+\ldots\ldots\frac{1}{4}(1+\rho)^{-n}$, a series which in n years will consist of $4n$ terms; and the sum of which will be found equal to $\frac{1}{4} \times \frac{1-(1+\rho)^{-n}}{(1+\rho)^{\frac{1}{4}}-1}$. So likewise the present value of an annuity of £1 per annum payable BIENNIALLY, each payment being double the yearly annuity, is equal to the present value of an annuity which is the DOUBLE of £1 in HALF the number of payments: that is, it will be equal to $2(1+\rho)^{-2}+2(1+\rho)^{-4}+2(1+\rho)^{-6}+\ldots\ldots 2(1+\rho)^{-n}$, a series which in n years will consist of $\frac{n}{2}$ terms; and the sum of which for that period is found to be equal to $2 \times \frac{1-(1+\rho)^{-n}}{(1+\rho)^{2}-1}$. And, universally, the present value of an annuity of £1 per annum payable μ times in a year, each payment being $\frac{1}{\mu}$th of the yearly annuity, is equal to the present value of an annuity which is $\frac{1}{\mu}$th of £1 for μn terms: that is, it is equal to $\frac{1}{\mu}(1+\rho)^{-\frac{1}{\mu}}+\frac{1}{\mu}(1+\rho)^{-\frac{2}{\mu}}+\frac{1}{\mu}(1+\rho)^{-\frac{3}{\mu}}+\ldots\ldots \frac{1}{\mu}(1+\rho)^{-n}$, a series which in n years will consist of μn terms, and the sum of which is equal to $\frac{1}{\mu} \times \frac{1-(1+\rho)^{-n}}{(1+\rho)^{\frac{1}{\mu}}-1}$; and this is a general formula for the present value of an annuity of £1 per annum under such circumstances.

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* See the authors quoted in the note to § 58.

§ 79. But this formula being contained in terms of ρ, which denotes the *true* rate of interest, may be converted into one that shall contain r, or the *nominal* rate of interest, by the method laid down in § 77. For, by adopting the substitution there alluded to, we shall find the formula denoting the present value of an annuity of £1 per annum payable μ times in a year, each payment being $\frac{1}{\mu}$th of the yearly annuity, will become

$$\frac{1}{\mu} \times \frac{1-(1+\rho)^{-n}}{(1+\rho)^{\frac{1}{\mu}}-1} = \frac{1}{\mu} \times \frac{1-\left(1+\frac{r}{m}\right)^{-mn}}{\left(1+\frac{r}{m}\right)^{\frac{m}{\mu}}-1};$$

where ρ denotes the true rate, and r the nominal rate of interest payable m times in a year, each payment of interest being the mth part of r.

§ 80. It is from this last formula that I have deduced the following theorems for some of the most practical cases, namely, for showing the present value of annuities, which ANNUITIES are payable YEARLY, HALF-YEARLY, QUARTERLY, and MOMENTLY; and each on the supposition that money may be improved at interest, which *interest* is payable also *yearly*, *half-yearly*, *quarterly*, and *momently*. It should be previously observed that R and RN stand for the natural numbers which have r and rn respectively for their Neperean logarithm, as already explained in § 66; and that y, h, q, and m denote the same as in §§ 29, 47, and 67.

If the annuity is payable Yearly

$$y=\frac{1-(1+r)^{-n}}{r} \qquad h=\frac{1-(1+\frac{r}{2})^{-2n}}{(1+\frac{r}{2})^{2}-1} \qquad q=\frac{1-(1+\frac{r}{4})^{-4n}}{(1+\frac{r}{4})^{4}-1} \qquad m=\frac{1-(\mathrm{RN})^{-1}}{\mathrm{R}-1}$$

If the annuity is payable Half-yearly

$$y=\tfrac{1}{2}\times\frac{1-(1+r)^{-n}}{(1+r)^{\frac{1}{2}}-1} \qquad h=\frac{1-(1+\frac{r}{2})^{-2n}}{r} \qquad q=\tfrac{1}{2}\times\frac{1-(1+\frac{r}{4})^{-4n}}{(1+\frac{r}{4})^{2}-1} \qquad m=\tfrac{1}{2}\times\frac{1-(\mathrm{RN})^{-1}}{\mathrm{R}^{\frac{1}{2}}-1}$$

If the annuity is payable Quarterly

$$y=\tfrac{1}{4}\times\frac{1-(1+r)^{-n}}{(1+r)^{\frac{1}{4}}-1} \qquad h=\tfrac{1}{4}\times\frac{1-(1+\frac{r}{2})^{-2n}}{(1+\frac{r}{2})^{\frac{1}{2}}-1} \qquad q=\frac{1-(1+\frac{r}{4})^{-4n}}{r} \qquad m=\tfrac{1}{4}\times\frac{1-(\mathrm{RN})^{-1}}{\mathrm{R}^{\frac{1}{4}}-1}$$

If the annuity is payable Momently

$$y=\frac{1-(1+r)^{-n}}{\mathrm{N}\lambda(1+r)} \qquad h=\frac{1-(1+\frac{r}{2})^{-2n}}{\mathrm{N}\lambda(1+\frac{r}{2})^{2}} \qquad q=\frac{1-(1+\frac{r}{4})^{-4n}}{\mathrm{N}\lambda(1+\frac{r}{4})^{4}} \qquad m=\frac{1-(\mathrm{RN})^{-1}}{r}$$

§ 81. From an inspection of these theorems, or of the general formula in § 78, it will be seen that the present value of an annuity payable YEARLY will, when payable μ times in a year, be invariably increased by the constant ratio $\frac{1}{\mu}\cdot\frac{\rho}{(1+r)^{\frac{1}{\mu}}-1}$, the annual interest being still the same. Thus, the present value of an annuity of £1 per annum, payable YEARLY, for 25 years at 5 per cent per annum, is by the Tables equal to 14·093945; whence the present value of the same annuity payable HALF-YEARLY (the annual interest ρ being the same) is equal to 14·093945 × 1·01235 = 14·268005: and the present value of the same annuity payable QUARTERLY (the annual interest ρ being still the same) is equal to 14·093945 × 1·01856 = 14·355529. See § 68, where the same observation is made respecting the amounts of annuities payable at different periods.*

§ 82. In order that the reader may be enabled to see at one view the real difference which arises in the present value of annuities payable YEARLY, HALF-YEARLY, QUARTERLY, and MOMENTLY, according as money can be improved at an interest receivable *yearly*, *half-yearly*, *quarterly*, and *momently*, I have subjoined the following Table, which may be considered as so many examples of the theorems in § 80, at the several nominal rates of 2, 5, and 10 per cent interest; and for 5, 25, 50, 100, and ∞ years.† These mean and extreme cases will enable us to discover the limits of the difference which may arise in any intermediate case.

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* I presume that it was an inattention to this distinction between the periodical payments of the *annuity*, and of the *interest*, which led Dr. Price and Mr. Morgan to attack M. de Moivre's method of deducing the value of annuities payable YEARLY, HALF-YEARLY, QUARTERLY, &c. The works of that profound analyst, however, suffer little from such incautious remarks. See *Phil. Trans.*, vol. lxvi. p. 112, or Price's *Obs. on Rev. Pay.*, vol. i. p. 241, 263; and Mr. Morgan's note in p. 251 of the same volume; ed. 1803: also p. 99 of my *Tables for the Purchasing and Renewing of Leases*, 2d edition.

† This last column will be explained in the following chapter "On Perpetuities."

Nominal Rate of Interest.	Payments of the Annuity.	Payment of Interest.	5 Years.	25 Years.	50 Years.	100 Years.	Perpetuity.
2 per Cent.	Yearly.	*y*	4·7135	19·5235	31·4236	43·0984	50·0000
		h	4·7121	19·5006	31·3576	42·9509	49·7512
		q	4·7115	19·4895	31·3252	42·8778	49·6278
		m	4·7107	19·4774	31·2910	42·8023	49·5017
	Half-yearly.	*y*	4·7369	19·6206	31·5800	43·3128	50·2488
		h	4·7356	19·5981	31·5144	43·1657	50·0000
		q	4·7350	19·5867	31·4814	43·0917	49·8753
		m	4·7344	19·5753	31·4483	43·0174	49·7504
	Quarterly.	*y*	4·7487	19·6693	31·6583	43·4203	50·3735
		h	4·7474	19·6469	31·5930	43·2732	50·1246
		q	4·7469	19·6356	31·5601	43·1994	50·0000
		m	4·7462	19·6243	31·5270	43·1251	49·8750
	Momently.	*y*	4·7604	19·7180	31·7368	43·5279	50·4984
		h	4·7593	19·6959	31·6717	43·3811	50·2496
		q	4·7587	19·6847	31·6390	43·3073	50·1249
		m	4·7581	19·6735	31·6060	43·2332	50·0000
5 per Cent.	Yearly.	*y*	4·3295	14·0939	18·2559	19·8479	20·0000
		h	4·3220	14·0061	18·0810	19·6116	19·7531
		q	4·3182	13·9614	17·9925	19·4925	19·6289
		m	4·3143	13·9161	17·9032	19·3727	19·5042
	Half-yearly.	*y*	4·3829	14·2679	18·4813	20·0930	20·2469
		h	4·3760	14·1812	18·3071	19·8567	20·0000
		q	4·3725	14·1370	18·2188	19·7376	19·8758
		m	4·3689	14·0923	18·1298	19·6180	19·7511
	Quarterly.	*y*	4·4098	14·3555	18·5948	20·2163	20·3712
		h	4·4032	14·2693	18·4208	19·9801	20·1242
		q	4·3998	14·2253	18·3327	19·8610	20·0000
		m	4·3964	14·1809	18·2438	19·7414	19·8753
	Momently.	*y*	4·4368	14·4434	18·7086	20·3401	20·4959
		h	4·4305	14·3577	18·5349	20·1039	20·2490
		q	4·4273	14·3141	18·4470	19·9849	20·1247
		m	4·4240	14·2699	18·3583	19·8652	20·0000
10 per Cent.	Yearly.	*y*	3·7908	9·0770	9·9148	9·9993	10·0000
		h	3·7667	8·9053	9·6819	9·7555	9·7561
		q	3·7541	8·8173	9·5637	9·6322	9·6327
		m	3·7412	8·7278	9·4443	9·5079	9·5083
	Half-yearly.	*y*	3·8833	9·2986	10·1568	10·2433	10·2440
		h	3·8609	9·1280	9·9240	9·9994	10·0000
		q	3·8492	9·0405	9·8058	9·8760	9·8765
		m	3·8371	8·9516	9·6864	9·7516	9·7521
	Quarterly.	*y*	3·9301	9·4107	10·2792	10·3668	10·3675
		h	3·9085	9·2407	10·0465	10·1229	10·1235
		q	3·8973	9·1535	9·9283	9·9995	10·0000
		m	3·8857	9·0649	9·8090	9·8751	9·8755
	Momently.	*y*	3·9773	9·5237	10·4027	10·4913	10·4921
		h	3·9566	9·3543	10·1700	10·2474	10·2480
		q	3·9458	9·2675	10·0519	10·1240	10·1245
		m	3·9347	9·1792	9·9326	9·9996	10·0000

§ 83. When the periods of the payments of the annuity and of the payments of interest correspond, that is, when $\mu=m$, then the denominator of the general formula $\frac{1}{\mu}\times\frac{1-(1+\frac{r}{m})^{-mn}}{(1+\frac{r}{m})^{\frac{m}{\mu}}-1}$ in § 79, becomes equal to r. This particular case is worthy of observation, not only as being the most usual in practice, but as it furnishes us with a method of deducing, in many instances, from the common tables of the present value of annuities payable YEARLY, the present value of the same annuities payable HALF-YEARLY, QUARTERLY, &c.; or BIENNIALLY, TRIENNIALLY, &c. For, under such circumstances, the present value of any HALF-YEARLY annuity, at any given rate of interest, is equal to HALF the present value of the same annuity payable a DOUBLE number of times at HALF the given rate: and the present value of any QUARTERLY annuity, at any given rate of interest, is equal to a QUARTER the present value of the same annuity payable a QUADRUPLE number of times at a QUARTER the given rate. So likewise, the present value of any BIENNIAL annuity, at any given rate of interest, is equal to DOUBLE the present value of the same annuity payable HALF the number of times, at DOUBLE the given rate: and so on for any other periodical annuity. Thus, the present value of an annuity of £1 per annum payable YEARLY, for *ten* years, interest at *eight* per cent, is equal to $\frac{1-(1\cdot08)^{-10}}{\cdot08}$, which by the tables is found to be 6·710081: if such annuity be payable HALF-YEARLY (the interest also being payable half-yearly) it will be equal to *half* the present value of the same annuity for *twenty* years, at *four* per cent, or $\frac{1}{2}\times\frac{1-(1\cdot04)^{-20}}{\cdot04}$, which by the tables is $\frac{1}{2}\times 13\cdot590326=6\cdot795163$: and if such annuity be payable QUARTERLY (the interest also being payable quarterly) it will be equal to a *quarter* the present value of the same annuity for *forty* years, at *two* per cent, or to $\frac{1}{4}\times\frac{1-(1\cdot02)^{-40}}{\cdot02}$, which by the tables is $\frac{1}{4}\times 27\cdot355479=6\cdot83887$. These remarks, however, are of use only when we have recourse to the tables for the solution of such questions: for if we resolve them agreeably to the formula given in § 72, we shall find that it will express the true answer. Indeed, the tables are so limited that we cannot always avail ourselves of their assistance.

§ 84. Another remark may likewise be made in this particular case where $\mu=m$; which is, that when n is infinite (or the annuity is a perpetuity) the present values of such annuities, whether payable YEARLY, HALF-YEARLY, QUARTERLY, or at any other interval of time, become ultimately equal to each other. For, the formula will in every such case become equal to $\frac{1}{r}$ as I shall more particularly explain in the following chapter. See § 90.

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CHAPTER XI.

ON PERPETUITIES.

§ 85. WHEN an annuity, instead of terminating at the end of a given period, is to continue for ever, it is called a *Perpetuity;* and in every such case, the quantity $(1+\rho)^{-n}$, in the general formula in § 72, entirely vanishes, or becomes equal to nothing. For n being infinitely great, the fraction $\frac{1}{(1+\rho)^n}$ is infinitely small, and may be safely neglected; consequently, the formula expressing the present value of the perpetuity of any annuity will be $p=a\times\frac{1}{\rho}$. Or the present value of an annuity of £1 per annum for ever is equal to $(1+\rho)^{-1}+(1+\rho)^{-2}+(1+\rho)^{-3}+\ldots\ldots$ *ad infinitum;* the sum of which, agreeably to the common method of summing an infinite geometrical series,* is readily found to be $=\frac{1}{\rho}$. Consequently, the present value of the perpetuity of any other annuity will be $p=a\times\frac{1}{\rho}$ as before; whence we deduce the following theorems, from which any one of the quantities may be found when the rest are given. Thus,

$p=\frac{a}{\rho}=$ the Present Value,

$a=p\rho=$ the Annuity,

$\rho=\frac{a}{p}=$ the *Annual* Rate of Interest.

The method of applying these theorems to any practical case that may arise is so extremely easy, that I shall not detain the reader with any examples of such application.

§ 86. In the above theorems, ρ denotes the *true* rate of annual interest, but they may be easily converted into others involving the *nominal* rate, by substituting for ρ its equal $(1+\frac{r}{m})^{m}-1$, agreeably to what has been said in § 77. Whence the present value of the perpetuity of £1 per annum will also be expressed by $\frac{1}{(1+\frac{r}{m})^{m}-1}$, where m varies according to the number of payments of *interest* in the year.

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* The sum of any infinite geometrical series decreasing is equal to the square of the first term divided by the difference between the first and second. See Bernoulli *de Seriebus infinitis,* Cor. to Prop. 8.

§ 87. It should also be observed that this theorem gives the value of the perpetuity of an annuity of £1 payable ANNUALLY; but if the annuity is payable at any other intervals, such as μ times in a year, its present value will be universally expressed by $\frac{1}{\mu}\times\frac{1}{(1+\frac{r}{m})^{\frac{m}{\mu}}-1}$, agreeably to what has been already said in §§ 78 and 79. For, since the quantity affected with n vanishes in all those cases, the general formula there given, $\frac{1}{\mu}\times\frac{1-(1+\frac{r}{m})^{-mn}}{(1+\frac{r}{m})^{\frac{m}{\mu}}-1}$, becomes equal to $\frac{1}{\mu}\times\frac{1}{(1+\frac{r}{m})^{\frac{m}{\mu}}-1}$.

§ 88. Whence, by making the numerators of the different theorems in § 80 equal to unity, we may obtain so many different formulæ for expressing the present value of perpetuities payable at the different intervals therein mentioned, and on the supposition that money may be improved in the several ways there alluded to. For example, let the annuity be £1 per annum payable YEARLY at any given *annual* rate of interest, then will the present value of the perpetuity be expressed by $\frac{1}{r}$. But if the annuity be payable HALF-YEARLY, the *annual* rate of interest being the same, the present value of the perpetuity will be $\frac{1}{2}\times\frac{1}{(1+r)^{\frac{1}{2}}-1}$. And if payable QUARTERLY, the *annual* rate of interest being still the same, its present value will be $\frac{1}{4}\times\frac{1}{(1+r)^{\frac{1}{4}}-1}$. In like manner, if the interest is payable *half-yearly*, the annuity being payable YEARLY, the present value of the perpetuity will be expressed by $\frac{1}{(1+\frac{r}{2})^{2}-1}$; and if the interest be payable *quarterly*, the annuity being still payable YEARLY, its value will be expressed by $\frac{1}{\left(1+\frac{r}{4}\right)^{4}-1}$

§ 89. In order that the reader may see the difference which arises in the present value of perpetuities according to the circumstances here alluded to, I have inserted the value of the same in numbers in the last column of the table in § 82, from which the limits of the differences which arise in these cases may be easily ascertained.

§ 90. It is worthy of remark here, that in the particular case where $\mu = m$ (that is, when the periods of the payments of the annuity and the payments of interest correspond), the present values of the perpetuities of an annuity, whether payable YEARLY, HALF-YEARLY, QUARTERLY, or at any other interval of time, become ultimately equal to each other; for the formula will in every such case become equal to $\frac{1}{r}$ (see § 84). But when n is any finite quantity, there will be a difference in the present value of such annuities, as may be readily seen from the table just alluded to; and this difference will first increase from nothing, when $n = o$, to some certain term or value of n where the difference is a maximum, and will afterwards decrease till the present values of such annuities become ultimately equal, as above mentioned. This term, or value of n, when the difference is greatest, will vary according as the real and nominal rates of interest vary, or rather, according to the number of payments of interest in the year; that is, if $\frac{1-(1+r)^{-n}}{r}$ denote the present value of an annuity of £1 per annum, the interest and annuity being supposed to be payable yearly; and if $\frac{1-(1+\frac{r}{m})^{-mn}}{r}$ denote the present value of an annuity of £1 per annum, the interest and annuity being supposed to be payable m times in a year, each payment being the mth part of the nominal rate and annuity, then the value of n, when the difference is a maximum, will depend on the value of r and m; and its general expression* will be $\frac{\lambda\left(\frac{\lambda(1+\frac{r}{m})^{m}}{\lambda(1+r)}\right)}{\lambda\left(1+\frac{r}{m}\right)^{m}-\lambda(1+r)}$. For example, the difference between the present value of an annuity of £1 per annum payable yearly, interest being reckoned at 5 per cent, payable also yearly, and the present value of the same annuity payable half-yearly, the interest being also payable half-yearly, is greatest when $n=\frac{\lambda\left(\frac{\lambda(1\cdot025)^2}{\lambda(1\cdot05)}\right)}{\lambda(1\cdot025)^2-\lambda(1\cdot05)}=\frac{\lambda\left(\frac{\cdot0214478}{\cdot0211893}\right)}{\cdot0214478-\cdot0211893}=\frac{\cdot0052661}{\cdot0002585}=20\cdot372$, or 20 years and 136 days; but if the annuity and interest are both payable quarterly, the difference is greatest when $n=\frac{\lambda\left(\frac{\lambda(1\cdot0125)^4}{\lambda(1\cdot05)}\right)}{\lambda(1\cdot0125)^4-\lambda(1\cdot05)}=\frac{\lambda\left(\frac{\cdot0215801}{\cdot0211893}\right)}{\cdot0215801-\cdot0211893}=\frac{\cdot0079367}{\cdot0003908}=20.309$, or 20 years and 113 days. In all cases, however, where the annuity is not payable at the same periods as the interest, the difference is greatest when n is infinite.

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* This value of n is deduced in the same manner as I have already pointed out in the note to § 51. In fact, we have only to substitute $(1+\frac{r}{m})^{m}$ for $(1+\rho)$ there given.

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CHAPTER XII.

ON REVERSIONS.

§ 91. When an annuity is not to be entered upon immediately, but after a certain number of years, it is called a *Reversion;* and its present value[*] is such a sum as, put out to interest, will provide for the several payments of the annuity as they become due. In order to obtain such value, we must find the present value of each reversionary payment, and the sum of them will be the total present value of the reversion. With this view let us make

p = the Present Value,
a = the Annuity,
n = the Number of Years during which the annuity is to continue,
ν = the Number of Years during which the annuity is deferred,
ρ = the *annual* Rate of Interest.

And let the annuity in the first instance be £1 per annum: then, since the present value of £1 due at the end of the first year *after*[†] the term ν is (by § 34) equal to $(1+\rho)^{-(\nu+1)}$, and since the present value of £1 due at the end of the second year after the term ν is equal to $(1+\rho)^{-(\nu+2)}$, and universally since the present value of £1 due at the end of n years after the term ν is equal to $(1+\rho)^{-(\ +n)}$, it follows that the sum of all these values for n years, or the series $(1+\rho)^{-(\nu+1)}+(1+\rho)^{-(\nu+2)}+(1+\rho)^{-(\nu+3)}+\ldots(1+\rho)^{-(\nu+n)}$, will express the total present value of an annuity of £1 per annum for n years, the title to which, however, is not to commence till after ν years. But this series is evidently a geometrical progression whose first term is $(1+\rho)^{-(\nu+1)}$, common ratio $(1+\rho)^{-1}$, and number of terms n; consequently its sum, or the present value of the reversion, is equal to $(1+\rho)^{-\nu}\times\frac{1-(1+\rho)^{-n}}{\rho}=\frac{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}{\rho}$;* and this expression, being multiplied by any other annuity a, will give $a\times\frac{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}{\rho}=p$ for the present value of the reversion of that other annuity. Whence we deduce the following theorems, from which any one of the quantities may be found when the rest are given: Thus

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[*] The *amounts* of annuities in reversion, for any given term, are the same as the amounts of similar annuities for the same term which commence immediately.

[†] It must be observed that ν denotes the number of years during which the annuity is deferred; at the expiration of which, *the right of entering upon* the annuity commences: the first payment of such annuity, however, does not take place till the end of $(\nu+1)$ years.

$$p=a\,.\,\frac{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}{\rho}\qquad=\text{ the Present Value,}$$

$$a=p\,.\,\frac{\rho}{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}\qquad=\text{ the Annuity,}$$

$$n=\frac{\text{Co.}\lambda\left[1-\frac{p}{a}\,.\,\rho\,.\,(1+\rho)^{\nu}\right]}{\lambda(1+\rho)}\qquad=\text{ the Years of the Annuity,}$$

$$\nu=\frac{\lambda[1-(1+\rho)^{-n}]-\lambda(\frac{p}{a}\,.\,\rho)}{\lambda(1+\rho)}\qquad=\text{ the Years deferred,}$$

$$\left.\begin{array}{l}\rho=\dfrac{[12\delta-(n^2-1)\beta]\,.\,\beta}{12\delta-2(n^2-1)\beta}\\ \text{where }\delta=2\nu+n+1\text{, and }\beta=\left(\dfrac{an}{p}\right)^{\frac{2}{\delta}}-1\end{array}\right|=\text{ the \textit{Annual} Rate of Interest.†}$$

§ 92. In order to show the use and application of these theorems, I shall propose the following questions.

Example 1. What sum ought to be given for the reversion of an annuity of £50 per annum, for 7 years after the next 14 years, in order that the purchaser may make 6 per cent interest of his money? Here we make use of the first theorem, where $a=50$, $n=7$, $\nu=14$, and $\rho=\cdot06$; consequently $p=50\times\frac{(1\cdot06)^{-14}-(1\cdot06)^{-21}}{\cdot06}=50\times\frac{\cdot442301\,-\,\cdot294155}{\cdot06}=$ $50\times2\cdot4691=123\cdot455$ or £123, 9s. 1d.

Example 2. If the reversion of an estate for 14 years after the next 26 years, cost £2000, what rent ought it to produce in order that the purchaser may make 5 per cent interest of his money? Here we make use of the second theorem, where $p=2000$, $n=14$, $\nu=26$, and $\rho=\cdot05$; consequently, $a=2000\times\frac{\cdot05}{(1\cdot05)^{-26}-(1\cdot05)^{-40}}=2000\times\frac{\cdot05}{\cdot281241-\cdot142046}=$ $2000\times\cdot359208=718\cdot416$ or £718, 8s. 4d.

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* See the authors quoted in the note to § 58.

† The method of deducing this formula will be explained in the Appendix (Note C), to which the reader is referred for other approximations to the value of ρ: and likewise for a method of correcting such approximations to any degree of accuracy.

Example 3. Suppose a debt of £816, 18s. 9d. were proposed to be paid off, by making over an annuity of £175 per annum, which, however, is not to commence till the end of 9 years; how many years must the creditor enjoy such an annuity in order to have his debt cleared, and to be allowed 6 per cent interest for his money? Here we make use of the third theorem, where $p=816\cdot9375$, $a=175$, $\nu=9$, and $\rho=\cdot06$; consequently $\frac{p}{a}=\frac{816\cdot9375}{175}=4\cdot6682$, and $n=\frac{\text{Co.}\lambda[1-4\cdot6682\times\cdot06\times(1\cdot06)^{9}]}{\lambda1\cdot06}$ $=\frac{\text{Co.}\lambda\cdot526790}{\lambda1\cdot06}=\frac{\cdot2783622}{\cdot0253059}=11$ years.

Example 4. If an annuity of £40 per annum for 35 years be purchased for £315, interest being reckoned at 5 per cent, how long will it be before the annuity commences? Here we make use of the fourth theorem, where $p=315$, $a=40$, $n=35$, and $\rho=\cdot05$; consequently $\frac{p}{a}=\frac{315}{40}=7\cdot875$ and $\nu=\frac{\lambda[1-(1\cdot05)^{-35}]-\lambda(7\cdot875\times\cdot05)}{\lambda1\cdot05}=\frac{\lambda\cdot818710-\lambda\cdot393750}{\lambda1\cdot05}=$ $\frac{\bar{1}\cdot9131301-\bar{1}\cdot5952206}{0\cdot0211893}=\frac{\cdot3179095}{\cdot0211893}=15$ years.

Example 5. If £40 be paid down for an annuity of £20 per annum for 10 years after the next 21 years, what is the rate of interest allowed? Here we make use of the fifth formula, where $p=40$, $a=20$, $n=10$, and $\nu=21$; consequently $\delta=42+10+1=53$, $\beta=\left(\frac{20\times10}{40}\right)^{\frac{2}{53}}-1=(5)^{\frac{2}{53}}-1$ $=\cdot062616$, and $\rho=\frac{[12\times53-(10\times10-1)\times\cdot062616]\times\cdot062616}{12\times53-2\times(10\times10-1)\times\cdot062616}$ $=\cdot063238.$*

§ 93. It appears from what has been said in § 91, that the present value of the reversion of an annuity of £1 per annum, is equal to the present value of the same annuity for the time n, discounted for the time ν, or equal to $(1+\rho)^{-\nu}\times\frac{1-(1+\rho)^{-n}}{\rho}$: there is a method, however, of deducing the same value, different from what is there laid down. For it is evident that the present value of the reversion of an annuity for n years after the term ν, will be equal to the present value of the same annuity for the whole term $\nu+n$ years, *minus* the present value of the annuity for ν years: or, in the case of an annuity of £1 per annum, from which all others are deduced, it will be equal to $\frac{1-(1+\rho)^{-(\nu+n)}}{\rho}-\frac{1-(1+\rho)^{-\nu}}{\rho}=$ $\frac{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}{\rho}$; which is the very same formula as the one above given. Now, if this expression be converted into numbers, as I have already observed respecting the formulæ in §§ 60 and 74, we may form a set of tables which would at all times be ready for use: but since it admits of such an infinite variety of combinations of the quantities ν and n, which cause a material alteration in the results, an infinite variety of tables must be formed to suit every case. And there is less occasion for forming tables of this kind, since we may deduce the present value of reversions with great ease, from the present value of annuities as exhibited in Table VI. For we have only to find the present value of the annuity for the time ν, and the present value of the same annuity for the time $\nu+n$, and then subtracting the one from the other the difference will be the present value of the reversion. For example, the present value of the reversion of an annuity of £1 per annum for 7 years, after the next 14 years, interest at 6 per cent per annum, is equal to the present value of an annuity for 21 years, *minus* the present value of an annuity for 14 years, or equal to $11{\cdot}76407661-9{\cdot}29498392=2{\cdot}46909269$, which is the same value as is deduced by the method adopted in the first example.

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* By Dr. Halley's theorem, the answer to this question comes out ·063233, as may be seen in the preface to Sherwin's *Mathem. Tables*, or in *Scriptores Logarithmici*, vol. v. p. 350; the exact rate is ·063236.

§ 94. In all cases above given, I have had regard only to the *true* annual rate of interest; but where the interest is payable *half-yearly*, *quarterly*, &c., the theorems may be readily expressed in terms of r, which denotes the nominal rate of interest, by making the substitution alluded to in § 77: whereby we shall have the formula $\frac{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}{\rho}=$ $\frac{\left(1+\frac{r}{m}\right)^{-m\nu}-\left(1+\frac{r}{m}\right)^{-m(\nu+n)}}{\left(1+\frac{r}{m}\right)^{m}-1}$. I have also supposed that in all the cases above given, the annuity is payable YEARLY; but where such annuity is payable HALF-YEARLY, QUARTERLY, &c., we shall (agreeably to the method laid down in § 78) have the formula $\frac{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}{\rho}=\frac{1}{\mu}\times$ $\frac{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}{(1+\rho)^{\frac{1}{\mu}}-1}$. Or, universally, the present value of the reversion of an annuity of £1 per annum for n years, after ν years, payable μ times in a year, and the interest being payable m times in a year, is equal to $\frac{1}{\mu}\times\frac{(1+\frac{r}{m})^{-m\nu}-(1+\frac{r}{m})^{-m(\nu+n)}}{(1+\frac{r}{m})^{\frac{m}{\mu}}-1}$, which will serve as a general formula in all such cases.

§ 95. When the reversion is a *perpetuity* (that is, when n is infinite), the quantity $(1+\rho)^{-(\nu+n)}$ in the general formula in § 91 vanishes: consequently the present value of the reversion of the perpetuity of any annuity after ν years, is universally $p=a\times\frac{(1+\rho)^{-\nu}}{\rho}$. Whence we deduce the following theorems, from which any one of the quantities may be found when the rest are given; thus

$$p=a.\frac{(1+\rho)^{-\nu}}{\rho} \quad = \text{the Present Value,}$$

$$a=p\,\rho(1+\rho)^{\nu} \quad = \text{the Annuity,}$$

$$\nu=\frac{\text{Co. }\lambda(\frac{p}{a}\,.\,\rho)}{\lambda(1+\rho)} \quad = \text{the Years deferred,}$$

$$\left.\begin{array}{l} \rho=\dfrac{[6+(5\nu+1)\beta].\beta}{6+4(2\nu+1)\beta} \\ \text{where } \beta=\left(\dfrac{p+a\nu}{p}\right)^{\frac{1}{\nu}}-1 \end{array}\right\} = \text{the } \textit{Annual} \text{ Rate of Interest.*}$$

§ 96. In order to show the use and application of these theorems, I shall propose the following questions:—

Example 1. What is the present value of the perpetuity of an annuity of £3 per annum after 5¾ years, interest being reckoned at 4 per cent, payable half-yearly? Here we have $a=3$, $\nu=5{\cdot}75$, and $\rho={\cdot}0404$; consequently $p=\frac{3\times(1{\cdot}0404)^{-5{\cdot}75}}{{\cdot}0404}=\frac{2{\cdot}3890170}{{\cdot}0404}=59{\cdot}134$. On the 5th April 1802, a loan of 25 millions was contracted for; and the terms agreed upon were, that the subscribers should receive for every £100 sterling advanced, the sum of £65 Consols, £60 Reduced, and £6, 19s. 3d. in a Deferred stock, the interest on which was not to commence till the 5th January 1808. As the average price of Consols on that day was about 75, it appears from this example that the price of the Deferred stock might be fairly estimated at about 59⅛.

Example 2. If a person purchases the reversion of a fee-simple estate after 20 years, for £500, what rent ought it to produce in order that he may make 6 per cent interest per annum of his money? Here we have $p=500$, $\nu=20$, and $\rho=\cdot06$; consequently $a=500\times\cdot06\times(1\cdot06)^{20}=500\times\cdot06\times3\cdot207135=96\cdot214$ or £96, 4s. 3d. per annum.

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* See the investigation of this formula in the Appendix (Note D).

Example 3. The reversion of an estate in fee simple producing £60 per annum, is made over for the discharge of a debt of £577, 4s. 5d.; how soon ought the creditor to take possession, if he be allowed 5 per cent interest for his debt? Here we have $p=577\cdot221$, $a=60$, and $\rho=\cdot05$; consequently $\frac{p}{a}=9\cdot62034$, and $\nu=\frac{\text{Co. }\lambda(9\cdot62034\times\cdot05)}{\lambda1\cdot05}=\frac{\cdot3178396}{\cdot0211893}=15$ years.

Example 4. If £350 has been paid for the reversion of an annuity of £70 per annum for ever after 25 years, what rate of interest does the purchaser make of his money? Here we have $p=350$, $a=70$, and $\nu=25$; consequently $\beta=\left(\frac{350+70\times25}{350}\right)^{\frac{1}{25}}-1=(6)^{\frac{1}{25}}-1=\cdot074301$, and $\rho=\frac{(6+126\times\cdot074301)\times\cdot074301}{6+204\times\cdot074301}=\frac{1\cdot141406}{21\cdot157404}=\cdot053948$, or £5, 7s. 11d. *per cent* per annum: and this is sufficiently exact for most purposes, since the true rate is ·05387, or about £5, 7s. 9d. per cent: as may be found by the method of correction laid down in the Appendix (Note D).

§ 97. The above theorems relate only to the case of YEARLY annuities, and on the supposition of a given rate of *annual* interest: but it is easy to apply, in these cases, the observations just made in § 94; by which it will appear that the present value of the reversion of an annuity of £1 per annum for ever after ν years, payable μ times in a year, and at the nominal rate of interest r payable m times in a year, will be universally expressed by the formula

$$\frac{1}{\mu}\times\frac{(1+\rho)^{-\nu}}{(1+\rho)^{\frac{1}{\mu}}-1}=\frac{1}{\mu}\times\frac{\left(1+\frac{r}{m}\right)^{-m\nu}}{\left(1+\frac{r}{m}\right)^{\frac{m}{\mu}}-1}.$$

§ 98. As it may occasionally be useful to see at one view the present value of the reversion of a perpetuity of £1 per annum after any given term, at different rates of interest, I have calculated the following table for that purpose: which shows the present value of such perpetuity after any number of years not exceeding 50, and at the several rates of 3, 4, 5, 6, 8, and 10 per cent per annum; whence we may obtain the present value of the reversion of any other annuity, by multiplication only; as already explained respecting all the preceding tables:—

After these Years.	3 per Cent.	4 per Cent.	5 per Cent.	6 per Cent.	8 per Cent.	10 per Cent.
1	32·3624	24·0385	19·0476	15·7232	11·5741	9·0909
2	31·4198	23·1139	18·1406	14·8332	10·7167	8·2645
3	30·5047	22·2249	17·2768	13·9936	9·9229	7·5131
4	29·6162	21·3701	16·4540	13·2015	9·1879	6·8301
5	28·7536	20·5482	15·6705	12·4542	8·5073	6·2092
6	27·9161	19·7579	14·9243	11·7493	7·8771	5·6447
7	27·1030	18·9979	14·2136	11·0842	7·2936	5·1316
8	26·3136	18·2673	13·5368	10·4568	6·7534	4·6651
9	25·5472	17·5647	12·8922	9·8650	6·2531	4·2410
10	24·8031	16·8891	12·2783	9·3065	5·7899	3·8554
11	24·0807	16·2395	11·6936	8·7797	5·3610	3·5049
12	23·3793	15·6149	11·1367	8·2828	4·9639	3·1863
13	22·6983	15·0144	10·6064	7·8139	4·5962	2·8966
14	22·0372	14·4369	10·1014	7·3716	4·2558	2·6333
15	21·3954	13·8816	9·6203	6·9544	3·9405	2·3939
16	20·7722	13·3477	9·1622	6·5607	3·6486	2·1763
17	20·1672	12·8343	8·7259	6·1893	3·3784	1·9784
18	19·5798	12·3407	8·3104	5·8390	3·1281	1·7986
19	19·0095	11·8661	7·9147	5·5085	2·8964	1·6351
20	18·4558	11·4097	7·5378	5·1967	2·6819	1·4864
21	17·9183	10·9708	7·1788	4·9025	2·4832	1·3513
22	17·3964	10·5489	6·8370	4·6250	2·2993	1·2285
23	16·8897	10·1432	6·5114	4·3632	2·1289	1·1168
24	16·3978	9·7530	6·2014	4·1162	1·9712	1·0153
25	15·9202	9·3779	5·9061	3·8832	1·8252	·9230
26	15·4565	9·0172	5·6248	3·6634	1·6900	·8391
27	15·0063	8·6704	5·3570	3·4561	1·5648	·7628
28	14·5692	8·3369	5·1019	3·2604	1·4489	·6934
29	14·1448	9·0163	4·8589	3·0759	1·3416	·6304
30	13·7329	7·7080	4·6275	2·9018	1·2422	·5731
31	13·3329	7·4115	4·4072	2·7375	1·1502	·5210
32	12·9445	7·1264	4·1973	2·5826	1·0650	·4736
33	12·5675	6·8524	3·9975	2·4364	·9861	·4306
34	12·2015	6·5888	3·8071	2·2985	·9131	·3914
35	11·8461	6·3354	3·6258	2·1684	·8454	·3558
36	11·5011	6·0917	3·4531	2·0457	·7828	·3235
37	11·1661	5·8574	3·2887	1·9298	·7248	·2941
38	10·8408	5·6321	3·1321	1·8206	·6711	·2673
39	10·5251	5·4155	2·9830	1·7175	·6214	·2430
40	10·2185	5·2072	2·8409	1·6203	·5754	·2209
41	9·9209	5·0069	2·7056	1·5387	·5328	·2009
42	9·6319	4·8144	2·5768	1·4421	·4933	·1826
43	9·3514	4·6292	2·4541	1·3604	·4568	·1660
44	9·0790	4·4512	2·3372	1·2834	·4229	·1509
45	8·8146	4·2800	2·2259	1·2108	·3916	·1372
46	8·5579	4·1153	2·1199	1·1422	·3626	·1247
47	8·3086	3·9571	2·0190	1·0776	·3357	·1134
48	8·0666	3·8049	1·9228	1·0166	·3109	·1031
49	7·8316	3·6585	1·8313	·9590	·2878	·0937
50	7·6035	3·5178	1·7441	·9047	·2665	·0852

CHAPTER XIII.

ON THE RENEWAL OF LEASES.

§ 99. To the subject of reversions, naturally belongs that of the *Renewal of Leases;*[*] where the principal question is to determine the sum or *Fine* which ought to be given for renewing any number of years lapsed in a lease: and the method of doing this will readily occur after what has just been said. For, if $n+v$ denote the number of years for which the lease was originally granted (of which I shall take v to denote the number of years *remaining* in the lease, and n to denote the number of years *lapsed*), then will the difference between the present value of an annuity of £1 per annum for the term $n+v$, and the present value of a similar annuity for the term v, denote the number of years' purchase that ought to be given for the renewal of the term n which is lapsed; and this, multiplied by the estimated *clear*[†] annual rent of the estate, will give the sum or fine required. Thus the fine which ought to be paid down in order to get renewed 7 years, which are lapsed in a lease of 20 years, of an estate estimated at the clear improved rent of £50 per annum, and reckoning interest at 5 per cent, is equal to $12{\cdot}4622-9{\cdot}3936=3{\cdot}0686$, or rather more than 3 years' purchase; which being multiplied by 50 will produce 153·430, or £153, 8s. 7d. for the fine required.

§ 100. Many corporations, societies, and colleges grant their leases for certain periods, the most usual of which are for 10, 20, 21, and 40 years: and it is customary for them to renew any number of years lapsed in such leases, on payment of a fine which is agreed upon by the parties; the quit-rent remaining the same. Such fine is sometimes the subject of much dispute and controversy; and in order to guide the public in renewing any estates held by this tenure, I have formed the four following tables, which respectively represent the *number of years' purchase* that ought to be given for renewing any number of years lapsed in a lease originally granted for 10, 20, 21, and 40 years, and at the several rates of interest therein mentioned: and any one of the values there found being multiplied by the *net* annual rent of the estate, will give the sum that ought to be paid for renewing the number of years, against which such value is set down. Thus, if it were required to know what sum ought to be given as a fine for renewing 7 years lapsed in a lease of 20 years, allowing the tenant 6 per cent interest for his money, and estimating the net rack-rent of the estate at £120 per annum; it will be found by multiplying 2·6172 by 120, which produces 314·064, or £314, 1s. 3d. for the fine required.

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[*] See this subject more fully treated on in my *Tables for the Purchasing and Renewing of Leases,* 8vo, 2d edit., 1807.

[†] That is, the net *surplus* rent, after deducting the reserved rent (if any) and all taxes and other annual charges.

Table for Renewing any number of years lapsed in a Lease for TEN years.

Years.	3 per Cent	4 per Cent.	5 per Cent	6 per Cent.	8 per Cent.	10 per Cent.
1	·7441	·6756	·6139	·5584	·4632	·3855
2	1·5105	1·3782	1·2585	1·1503	·9634	·8096
3	2·2999	2·1088	1·9354	1·7777	1·5037	1·2761
4	3·1130	2·8688	2·6460	2·4428	2·0872	1·7893
5	3·9505	3·6591	3·3923	3·1477	2·7174	2·3538
6	4·8131	4·4810	4·1758	3·8950	3·3980	2·9747
7	5·7016	5·3358	4·9985	4·6871	4·1330	3·6577
8	6·6167	6·2248	5·8623	5·5267	4·9268	4·4090
9	7·5593	7·1494	6·7694	6·4167	5·7842	5·2355
10	8·5302	8·1109	7·7217	7·3601	6·7101	6·1446

For Renewing any number of years lapsed in a Lease for TWENTY years.

Years.	3 per Cent.	4 per Cent.	5 per Cent.	6 per Cent.	8 per Cent	10 per Cent.
1	·5537	·4564	·3769	·3118	·2145	·1486
2	1·1240	·9310	·7726	·6423	·4463	·3122
3	1·7114	1·4247	1·1881	·9927	·6965	·4920
4	2·3164	1·9380	1·6244	1·3640	·9668	·6899
5	2·9395	2·4719	2·0826	1·7577	1·2587	·9075
6	3·5814	3·0272	2·5636	2·1749	1·5739	1·1469
7	4·2425	3·6047	3·0686	2·6172	1·9144	1·4102
8	4·9235	4·2053	3·5990	3·0861	2·2821	1·6999
9	5·6249	4·8298	4·1558	3·5830	2·6792	2·0185
10	6·3473	5·4794	4·7405	4·1098	3·1081	2·3690
11	7·0914	6·1550	5·3544	4·6682	3·5713	2·7545
12	7·8578	6·8576	5·9990	5·2601	4·0715	3·1786
13	8·6472	7·5883	6·6758	5·8875	4·6118	3·6451
14	9·4603	8·3482	7·3865	6·5526	5·1953	4·1583
15	10·2978	9·1385	8·1327	7·2576	5·8254	4·7228
16	11·1604	9·9604	8·9163	8·0048	6·5060	5·3437
17	12·0489	10·8152	9·7390	8·7969	7·2410	6·0267
18	12·9640	11·7042	10·6028	9·6365	8·0349	6·7780
19	13·9066	12·6288	11·5098	10·5265	8·8922	7·6045
20	14·8775	13·5903	12·4622	11·4699	9·8181	8·5136

For Renewing any number of years lapsed in a Lease for TWENTY-ONE years.

Years.	3 per Cent.	4 per Cent.	5 per Cent.	6 per Cent.	8 per Cent.	10 per Cent.
1	·5375	·4388	·3589	·2942	·1987	·1351
2	1·0912	·8952	·7358	·6060	·4132	·2838
3	1·6615	1·3699	1·1316	·9365	·6449	·4473
4	2·2489	1·8635	1·5471	1·2868	·8952	·6271
5	2·8539	2·3769	1·9834	1·6582	1·1654	·8250
6	3·4771	2·9108	2·4415	2·0518	1·4573	1·0426
7	4·1190	3·4660	2·9225	2·4691	1·7726	1·2820
8	4·7801	4·0435	3·4276	2·9114	2·1130	1·5453
9	5·4610	4·6441	3·9579	3·3802	2·4807	1·8350
10	6·1624	5·2687	4·5147	3·8772	2·8778	2·1536
11	6·8848	5·9183	5·0994	4·4040	3·3067	2·5041
12	7·6289	6·5938	5·7133	4·9624	3·7699	2·8897
13	8·3953	7·2964	6·3579	5·5543	4·2702	3·3138
14	9·1847	8·0271	7·0348	6·1817	4·8104	3·7803
15	9·9978	8·7870	7·7455	6·8468	5·3939	4·2934
16	10·8353	9·5773	8·4917	7·5587	6·0241	4·8579
17	11·6979	10·3993	9·2752	8·2990	6·7047	5·4788
18	12·5864	11·2541	10·0979	9·0911	7·4397	6·1618
19	13·5016	12·1431	10·9617	9·9307	8·2335	6·9132
20	14·4442	13·0676	11·8688	10·8207	9·0909	7·7396
21	15·4150	14·0292	12·8211	11·7641	10·0168	8·6487

For Renewing any number of years lapsed in a Lease for FORTY years.

Years.	3 per Cent.	4 per Cent.	5 per Cent.	6 per Cent.	8 per Cent.	10 per Cent.
1	·3066	·2083	·1420	·0972	·0460	·0221
2	·6223	·4249	·2922	·2003	·0957	·0464
3	·9475	·6502	·4478	·3095	·1494	·0731
4	1·2825	·8845	·6122	·4253	·2074	·1025
5	1·6276	1·1282	·7849	·5481	·2700	·1349
6	1·9829	1·3816	·9662	·6782	·3377	·1705
7	2·3490	1·6451	1·1565	·8161	·4107	·2096
8	2·7260	1·9192	1·3564	·9622	·4896	·2527
9	3·1143	2·2043	1·5663	1·1172	·5748	·3000
10	3·5143	2·5007	1·7866	1·2815	·6668	·3521
11	3·9263	2·8091	2·0180	1·4556	·7662	·4094
12	4·3507	3·1297	2·2610	1·6401	·8735	·4725
13	4·7877	3·4632	2·5161	1·8358	·9894	·5418
14	5·2379	3·8100	2·7839	2·0431	1·1146	·6181
15	5·7016	4·1707	3·0651	2·2629	1·2498	·7020
16	6·1792	4·5458	3·3604	2·4959	1·3959	·7943
17	6·6712	4·9359	3·6705	2·7429	1·5536	·8958
18	7·1779	5·3417	3·9961	3·0047	1·7239	1·0075
19	7·6997	5·7636	4·3379	3·2822	1·9078	1·1304
20	8·2373	6·2024	4·6969	3·5764	2·1065	1·2655
21	8·7910	6·6588	5·0738	3·8882	2·3210	1·4141
22	9·3613	7·1335	5·4695	4·2187	2·5527	1·5776
23	9·9487	7·6271	5·8850	4·5690	2·8030	1·7575
24	10·5537	8·1405	6·3213	4·9404	3·0732	1·9553
25	11·1768	8·6744	6·7794	5·3340	3·3651	2·1730
26	11·8187	9·2297	7·2604	5·7513	3·6804	2·4124
27	12·4798	9·8071	7·7655	6·1936	4·0208	2·6757
28	13·1608	10·4077	8·2958	6·6625	4·3885	2·9654
29	13·8621	11·0323	8·8527	7·1594	4·7856	3·2840
30	14·5846	11·6819	9·4374	7·6862	5·2145	3·6345
31	15·3287	12·3574	10·0513	8·2446	5·6777	4·0200
32	16·0951	13·0600	10·6959	8·8365	6·1780	4·4441
33	16·8845	13·7907	11·3727	9·4639	6·7182	4·9106
34	17·6976	14·5506	12·0834	10·1290	7·3017	5·4238
35	18·5351	15·3410	12·8296	10·8339	7·9319	5·9883
36	19·3977	16·1629	13·6131	11·5812	8·6125	6·6092
37	20·2862	17·0177	14·4358	12·3733	9·3475	7·2922
38	21·2013	17·9067	15·2997	13·2129	10·1413	8·0435
39	22·1439	18·8312	16·2067	14·1029	10·9987	8·8700
40	23·1148	19·7928	17·1591	15·0463	11·9246	9·7791

§ 101. When the estate is improving in value, the fines which are occasionally levied at any equal intervals will be continually increasing; the yearly rent, or quit-rent, continuing the same: but where it is at its maximum of improvement, such fines may be considered in the light of a fixed periodical income. And since it may be sometimes required to determine the *amount* or *present value* of all the fines certain that may be payable on an estate during any given number of years ($=n$), we shall find that the method of doing this is precisely the same as that which we should pursue in determining the amount or present value of a similar periodical annuity in the same time. For, let ν denote the interval, or number of years, between the payment of each fine; and let ρ denote, as before, the annual rate of interest: then since £1 will amount to $(1+\rho)^{\nu}$ in one interval, to $(1+\rho)^{2\nu}$ in two intervals, to $(1+\rho)^{3\nu}$ in three intervals, and so on: it follows (from the principles already laid down in § 58) that the series $1+(1+\rho)^{\nu}+(1+\rho)^{2\nu}+(1+\rho)^{3\nu}+\ldots(1+\rho)^{n-\nu}$, will express the *amount* of all the fines during n years; a series which in that time will consist of $\frac{n}{\nu}$ terms, and the sum of which, for that period, will be equal to $\frac{(1+\rho)^{n}-1}{(1+\rho)^{\nu}-1}$:* consequently this formula will accurately express the *amount* of all the fines certain of £1 every ν years, during any other period of n years.

§ 102. And, preserving the same characters, since the present value of £1 due at the end of the first interval is $(1+\rho)^{-\nu}$; at the end of the second interval is $(1+\rho)^{-2\nu}$; at the end of the third interval is $(1+\rho)^{-3\nu}$; and so on; it follows (from the principles already laid down in § 72) that the series $(1+\rho)^{-\nu}+(1+\rho)^{-2\nu}+(1+\rho)^{-3\nu}+\ldots(1+\rho)^{-(n-\nu)}$, will express the *present value* of all the fines during n years; a series which in that time will consist of $\frac{n}{\nu}$ terms, and the sum of which, for that period, will be equal to $\frac{1-(1+\rho)^{-n}}{(1+\rho)^{\nu}-1}$: consequently this formula will express the *present value* of all the fines certain of £1 every ν years, during any other period of n years.

§ 103. When n is infinite, this last formula becomes equal to the quantity $\frac{1}{(1+\rho)^{\nu}-1}$ (see § 85), which quantity denotes the present value of the *perpetuity* of all such fines. And either of these expressions being multiplied by any other fine, will give the amount or present value of all such other fines accordingly. For example, the lease of an estate is granted for 7 years, at a pepper-corn rent, with this condition, that the tenant at the expiration thereof may renew the same on paying a fine of £100: what is the value of the landlord's interest therein, at the time of any such renewal,[*] exclusive of the fine then received; allowing compound interest at the rate of 5 per cent per annum? Here we make use of the last formula, which becomes $\frac{1}{(1\cdot05)^7-1}=2\cdot4564$, or near $2\frac{1}{2}$ years' purchase; and this being multiplied by 100, will give 245·64, or £245, 12s. 10d. for the value of the landlord's interest required.

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* See the authors quoted in the note to § 58.

§ 104. By proceeding in the manner here mentioned, I have formed the following table, which shows the number of years' purchase that ought to be given for the *perpetuity* of all the fines which may be paid for renewing an estate granted for 2, 3, 4, 5, 6, 7, 10, 14, 20, 21, and 40 years respectively, and allowing compound interest at the several rates of 3, 4, 5, 6, 8 and 10 per cent per annum: which table we might enlarge or extend at pleasure:—

Table showing the present value of the perpetuity of £1 *payable at every interval therein mentioned.*

Interval of Years.	3 per Cent.	4 per Cent.	5 per Cent.	6 per Cent.	8 per Cent.	10 per Cent.
2	16·4204	12·2549	9·7561	8·0906	6·0096	4·7619
3	10·7843	8·0087	6·3442	5·2352	3·8504	3·0211
4	7·9676	5·8872	4·6402	3·8099	2·7740	2·1547
5	6·2785	4·6157	3·6195	2·9566	2·1307	1·6380
6	5·1532	3·7690	2·9403	2·3894	1·7039	1·2961
7	4·3502	3·1652	2·4564	1·9856	1·4009	1·0541
10	2·9077	2·0823	1·5901	1·2645	·8629	·6275
14	1·9509	1·3667	1·0205	·7931	·5162	·3575
20	1·2405	·8395	·6049	·4531	·2732	·1746
21	1·1624	·7820	·5599	·4167	·2479	·1562
40	·4421	·2631	·1656	·1077	·0483	·0226

§ 105. In the three formulæ above given (§§ 101, 102, 103), I have supposed that ρ expresses the *true* rate of annual interest: but they may be easily converted into others which contain the *nominal* rate only, by referring to what has been said in § 61 and 77. Whence it will be found that, by making the substitutions there alluded to, the first formula (in § 101) will become $\dfrac{(1+\rho)^n-1}{(1+\rho)^\nu-1}=\dfrac{(1+\frac{r}{m})^{mn}-1}{(1+\frac{r}{m})^{m\nu}-1}$; that the second formula (in § 102) will become $\dfrac{1-(1+\rho)^{-n}}{(1+\rho)^\nu-1}=\dfrac{1-(1+\frac{r}{m})^{-mn}}{(1+\frac{r}{m})^{m\nu}-1}$: and that the third formula (in § 103) will become $\dfrac{1}{(1+\rho)^\nu-1}=\dfrac{1}{(1+\frac{r}{m})^{m\nu}-1}$.

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[*] That is, immediately *after* the fine paid at any renewal: but at any *intermediate* time between such renewals, we must multiply the given formula by the amount of £1 at compound interest in the time elapsed since the payment of the *last* fine: thus if the landlord's interest were required at the interval of 3 years from the payment of the last fine, then the value 2·4564 (as found by the formula) being multiplied by 1·1576 (= the amount of £1 at 5 per cent in 3 years) will produce 2·8435 for the number of years purchase, in this case required. The demonstration of this rule will be given in § 106.

§ 106. It should be observed also, that the three formulæ above given denote the true values *immediately after* the fine paid at any one renewal; or on the supposition that the full period of ν years will elapse before the payment of the next fine: but at any *intermediate* time between such renewals, it is evident that the formulæ will not express the true value. In such cases we must adopt the correction alluded to, in the note to § 103; the investigation of which, for that particular formula, is as follows:—Let m denote any period less than ν, between the present time and the payment of the next fine: then will the present value of the first fine be $(1+\rho)^{-m}$; and the present value of the second fine $(1+\rho)^{-(m+\nu)}$; and the present value of the third fine $(1+\rho)^{-(m+2\nu)}$: and so on *ad infinitum:* consequently the present value of all the fines will be equal to $(1+\rho)^{-m}$ multiplied into the series $1+(1+\rho)^{-\nu}+(1+\rho)^{-2\nu}+(1+\rho)^{-3\nu}+\ldots$ *ad infinitum*. But such series being equal to $\dfrac{(1+\rho)^\nu}{(1+\rho)^\nu-1}$, it follows that the present value required will be equal to $(1+\rho)^{-m}\times\dfrac{(1+\rho)^\nu}{(1+\rho)^\nu-1}=(1+\rho)^{\nu-m}\times\dfrac{1}{(1+\rho)^\nu-1}$; where $\nu-m$ denotes the period elapsed since the payment of the last fine: whence the rule is manifest. And it is easy to apply the same method of investigation to the other two formulæ. Indeed this observation is general to all kinds of annuities, whether payable yearly, half-yearly, or at any other intervals similar to those just alluded to. For, the present value of any yearly annuity, the first payment of which is to be received *immediately*, is worth one year's purchase more than a similar annuity the first payment of which is not to be made till the *end* of the year. In *all* cases, therefore, which we have been considering in the present work, it is supposed that the first payment of the annuity does not take place till the *end* of the first of those periods in which it becomes due.

CHAPTER XIV.

ON THE AMOUNT OF SUMS AT COMPOUND INTEREST INCREASED OR DIMINISHED ANNUALLY BY A CONSTANT QUANTITY.

§ 107. If, in calculating the amounts of any given sum at compound interest agreeably to the method laid down in § 14, we not only add the interest to the principal at every time of its payment, but also *increase* it at every such time by a new sum a; the original principal, which I shall still call p, will increase at every time of payment in the following manner: viz. at the end of

1 year, to $p\ (1+\rho)+a$
2 years, to $p\ (1+\rho)^2+a(1+\rho)+a$
3 years, to $p\ (1+\rho)^3+a(1+\rho)^2+a(1+\rho)+a$

and universally at the end of n years, to

$$p\ (1+\rho)^n+a(1+\rho)^{n-1}+a(1+\rho)^{n-2}+\ldots a(1+\rho)+a.$$

§ 108. This last sum consists of two parts, of which the first is $p(1+\rho)^n$; and the other taken inversely forms the series $a+a(1+\rho)+a(1+\rho)^2+a(1+\rho)^3+\ldots a(1+\rho)^{n-1}$; the sum of which is (by § 58) equal to $a\times\frac{(1+\rho)^n-1}{\rho}$. Adding these two, therefore, together, we shall find that the total amount of the principal, of the sums added, and of the interest on the same, will at the end of n years be equal to $p(1+\rho)^n+a\times\frac{(1+\rho)^n-1}{\rho}=s$; whence we deduce the following theorems:—

$$s=p(1+\rho)^n+a.\frac{(1+\rho)^n-1}{\rho} = \text{the Amount,}$$

$$p=s(1+\rho)^{-n}-a.\frac{1-(1+\rho)^{-n}}{\rho} = \text{the Principal,}$$

$$a=\rho.\frac{s-p(1+\rho)^n}{(1+\rho)^n-1} = \text{the Sum Added,}$$

$$n=\frac{\lambda.(a+s\rho)-\lambda.(a+p\rho)}{\lambda.(1+\rho)} = \text{the Number of Years,}$$

$$\rho= \quad = \text{the } \textit{Annual} \text{ Rate of Interest.}^*$$

§ 109. In order to show the use and application of these theorems, let us take the following questions:—

Example 1. If £1000 be put out to compound interest after the rate of 4 per cent per annum, and at the end of every year £20 be added thereto,

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[*] I know of no formula for finding ρ whereby its value may be so readily determined, as by the method of trial and error alluded to in the several notes of the Appendix.

which is also improved at the same rate of interest; what will be the total amount of principal and interest at the end of 30 years? Here we make use of the first theorem, where $p=1000$, $a=20$, $n=30$, and $\rho=\cdot 04$; consequently $s=1000\times(1\cdot 05)^{30}+20\times\frac{(1\cdot 04)^{30}-1}{\cdot 04}=1000\times 3\cdot 243398+20\times 56\cdot 084938=3243\cdot 398+1121\cdot 698=4365\cdot 096$, or £4365, 1s. 11d. By the same process it may be seen that a principal of £1,000,000 put out to compound interest, added to an annual sum of £20,000, would amount in the same time to £4,365,096, which would buy off at the end of that period above six millions of 3 per cents, valued at 70.

Without the help of this formula, however, it is evident, from the statement of the question, that the sum will in this case be equal to the amount of the principal at compound interest added to the amount of the annuity, for the given time and at the given rate; consequently, it might have been solved by referring to Tables III. and V.

Example 2. Suppose a merchant sets up in business with a certain capital, and that by his profits, which are at the rate of 10 per cent compound interest, he adds thereto £500 annually; at the end of 12 years he finds that he is worth £20,000; what was the amount of the capital with which he commenced business? Here we make use of the second theorem, where $s=20{,}000$, $a=500$, $n=12$, and $\rho=\cdot 10$; consequently, $p=20{,}000\times(1\cdot 10)^{-12}-500\times\frac{1-(1\cdot 10)^{-12}}{\cdot 10}=20000\times\cdot 3186308-500\times 6\cdot 8136918=6372\cdot 616-3406\cdot 846=2965\cdot 770$, or £2965, 15s. 5d.

Example 3. A person puts out £1000 at compound interest, after the rate of 5 per cent per annum, as a marriage portion for his daughter; what sum must he annually add thereto in order that, at the end of 14 years, the same may amount to £15,000? Here we make use of the third theorem, where $s=15{,}000$, $p=1000$, $n=14$, and $\rho=\cdot 05$; consequently, $a=\cdot 05\times\frac{15000-1000\times(1\cdot 05)^{14}}{(1\cdot 05)^{14}-1}=\cdot 05\times\frac{13020\cdot 068}{\cdot 9799}=\cdot 05\times 13286\cdot 73=664\cdot 336$, or £664, 6s. 9d.

Example 4. In how many years will a principal of £500, bearing interest at the rate of 6 per cent per annum, and having £80 added to it annually, amount to £2000? Here we make use of the fourth theorem, where $s=2000$, $p=500$, $a=80$, and $\rho=\cdot 06$; consequently, $n=\frac{\lambda(80+2000\times\cdot 06)-\lambda(80+500\times\cdot 06)}{\lambda(1\cdot 06)}=\frac{2\cdot 3010300-2\cdot 0413927}{\cdot 0253059}=10\cdot 260$, or about $10\frac{1}{4}$ years.

§ 110. If, instead of increasing the principal and interest, at the end of every year, by a certain sum, we *diminish* it, by subtracting from it a given sum, a, at the end of every year, we shall have the following values for the amount of principal and interest at the end of each respective year: viz., at the end of

1 year, $p\,(1+\rho)-a$,
2 years, $p\,(1+\rho)^2-a(1+\rho)-a$,
3 years, $p(1+\rho)^3-a(1+\rho)^2-a(1+\rho)-a$,

and universally at the end of

n years, $p(1+\rho)^n-a(1+\rho)^{n-1}-a(1+\rho)^{n-2}-\ldots a(1+\rho)-a$.

§ 111. This latter sum consists of two parts, one of which is $p(1+\rho)^n$, and the other, which is to be *subtracted*, forms the same series as in § 108, the sum of which has been there found equal to $a\times\frac{(1+\rho)^n-1}{\rho}$. Therefore the total amount of principal and interest at the end of n years will be $p(1+\rho)^n-a\times\frac{(1+\rho)^n-1}{\rho}=s$. Whence we deduce the following theorems:—

$$s=p(1+\rho)^n\;-a.\frac{(1+\rho)^n-1}{\rho}\quad=\text{the Amount,}$$

$$p=s(1+\rho)^{-n}+a.\frac{1-(1+\rho)^{-n}}{\rho}=\text{the Principal,}$$

$$a=\rho.\frac{p(1+\rho)^n-s}{(1+\rho)^n-1}\quad=\text{the Sum Subtracted,}$$

$$n=\frac{\lambda.(a-s\rho)-\lambda.(a-p\rho)}{\lambda.(1+\rho)}\quad=\text{the Number of Years,}$$

$$\rho=\qquad\qquad=\text{the }\textit{Annual}\text{ Rate of Interest.}$$

§ 112. It may be remarked here, that these theorems are the same as those which are given in § 108, except that here we have $-a$, instead of $+a$. It may be also further observed, that if a is greater than $p\rho$, the value of s will become less and less, as n increases; till at last it becomes wholly negative. This is readily seen; for, if at each payment we take more from the principal than is added to it by the interest, it is evident that such principal must be continually decreasing, and will at length be absolutely reduced to nothing.

§ 113. I shall now give a few examples to show the use and application of the above theorems.

Example 1. If £1000 be put out to compound interest after the rate of 4 per cent per annum, and at the end of every year £20 be subtracted therefrom; what will be the amount of such principal at the end of 30 years? Here we have $p=1000$, $a=20$, $n=30$, and $\rho=\cdot04$; consequently $s=1000\times(1\cdot04)^{30}-20\times\frac{(1\cdot04)^{30}-1}{\cdot04}=3243\cdot4-1121\cdot7=$

2121·7, or £2121, 14s. By comparing this example with the one given in § 109, we may observe the difference, at the end of any given period, in the property of two persons, one of whom spends *more* than his income, and the other of whom *lays by* a part of the same, and improves it in like manner with the rest of his property.

Example 2. A person has a legacy left him, which he puts out to compound interest at the rate of 5 per cent per annum; out of which he has to pay an annuity of £50 per annum for 18 years; at the end of that term he finds that the property remaining is £5000, what was the amount of the legacy? Here we have $s=5000$, $a=50$, $n=18$, and $\rho=\cdot05$; consequently $p=5000\times(1\cdot05)^{-18}+50\times\frac{1-(1\cdot05)^{-18}}{\cdot05}=2077\cdot603+584\cdot479$ $=2662\cdot082$, £2662, 1s. 8d.

Example 3. A landlord who is worth £10,000, of which he makes 6 per cent compound interest, expects some leases to fall in at the end of 14 years; and as he is desirous of retaining £5000 of his present fortune to that period, he wishes to know what sum he may annually spend so that it may exactly be reduced to that sum at the end of the 14 years. Here we have $s=5000$, $p=10{,}000$, $n=14$, and $\rho=\cdot06$; consequently $a=\cdot06\times\frac{10{,}000\times(1\cdot06)^{14}-5000}{(1\cdot06)^{14}-1}=\cdot06\times15102\cdot4=906\cdot144$, or £906, 2s. 10d.

Example 4. It is required to find in how many years a principal of £100, bearing interest at the rate of 6 per cent per annum, and having £8 subtracted from it every year, will be reduced to nothing or vanish altogether? Here we have $s=0$, $p=100$, $a=8$, and $\rho=\cdot06$; consequently $n=\frac{\lambda8-\lambda2}{\lambda1\cdot06}=\frac{\cdot90309000-\cdot3010300}{\cdot0253059}=23\cdot791$, or 23 years and 289 days. At the close of the American war, when the UNITED STATES funded their domestic debt, they allowed on the stock an interest of 6 per cent, and enacted that such stock should be "subject to redemption by payments not exceeding in one year, on account both of principal and interest," the proportion of 8 per cent:* by which method they established a sinking fund of 2 per cent on the sum loaned; which would extinguish the debt (as appears by the above example) in somewhat less than 24 years.

§ 114. In all these cases I have supposed that the annuity is payable only once a year; but where the annuity, together with the interest, is payable m times a year, the general formula for the amount in such cases will be $s = p(1 + \frac{r}{m})^{mn} \pm a \times \frac{(1+\frac{r}{m})^{mn} - 1}{r}$: whence any of the other quantities concerned may be deduced.

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* Providing, however, at the same time "that the UNITED STATES shall not be bound or obliged to redeem in the proportion aforesaid; but that it shall be understood only that they have a right so to do."

§ 115. It is true that we may also suppose the annuity or sum a to be paid at other intervals than those when the interest becomes due; for the annuity or sum may be payable half-yearly, quarterly, &c.; and the interest payable only yearly. This is the principle adopted by M. de Moivre, who never loses sight of the true annual rate of interest;* and according to him, we have $s = p.(1+\rho)^n \pm \frac{a}{\mu} . \frac{(1+\rho)^n - 1}{(1+\rho)^{\frac{1}{\mu}} - 1}$; where μ varies according to the number of payments of the annuity. But in questions of *this kind* I think it most proper to make the payments of interest correspond with the payments of the annuity; as it is more agreeable to the method of deduction already explained (see §§ 107 and 110); since the sum a is supposed to be added *every time the interest becomes due* and is payable; and together with such interest forms a new sum to accumulate at the same rate for the next period.

§ 116. In this chapter I have shown how to determine the *amount* only of sums increased or diminished by the constant quantity a: but it is easy to apply the same method of deduction in order to find the *present value* of such sums. Indeed the present value here alluded to will evidently be the value of p in the second formula in each case; agreeably to what has been already said in § 36: it would therefore be needless to insist further on this subject.

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CHAPTER XV.

ON THE AMOUNT OF ANNUITIES INCREASING IN A CONSTANT RATIO.

§ 117. In the preceding part of this work I have shown how to determine the amount of a fixed and *constant* annuity for any number of years. I shall now proceed to determine the Amount of annuities *increasing* according to a given scale or ratio: that is, of annuities increasing according to the *order* of the natural numbers, viz., £1 for the first year, £2 for the second year, £3 for the third year, &c.: and of annuities increasing according to the *squares* of the natural numbers, viz., £1 for the first year, £4 for the second year, £9 for the third year, &c.: and of annuities increasing according to the *cubes* of the natural numbers, viz., £1 for the first year, £8 for the second year, £27 for the third year, &c. All which theorems may be of use in the solution of various problems, particularly those relative to life annuities upon the principle of M. de Moivre's hypothesis.

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* See his *Doctrine of Chances*, p. 321.

§ 118. In order to simplify this subject, let us make the amount of an annuity of £1 per annum, remaining constant during the given term, equal to A; the amount of the same annuity increasing according to the order of the natural numbers, equal to B; of the same annuity increasing according to the squares of the natural numbers, equal to C; of the same annuity increasing according to the cubes of the natural numbers, equal to D. Then will these quantities be equal to the different series expressed below, viz.:—

$$
\begin{aligned}
A&=1+\ (1+\rho)+\ (1+\rho)^2+\ (1+\rho)^3+\ (1+\rho)^4+\ldots\ (1+\rho)^{n-1}\\
B&=1+2\ (1+\rho)+3\ (1+\rho)^2+4\ (1+\rho)^3+5\ (1+\rho)^4+\ldots n\ (1+\rho)^{n-1}\\
C&=1+2^2(1+\rho)+3^2(1+\rho)^2+4^2(1+\rho)^3+5^2(1+\rho)^4+\ldots n^2(1+\rho)^{n-1}\\
D&=1+2^3(1+\rho)+3^3(1+\rho)^2+4^3(1+\rho)^3+5^3(1+\rho)^4+\ldots n^3(1+\rho)^{n-1}
\end{aligned}
$$

§ 119. But the process for finding the value of these different series will be rendered still more simple if we make $(1+\rho)=x$: which being done, we shall have the following equations, viz.:—

$$
\begin{aligned}
A&=1+\ x+\ \ x^2+\ \ x^3+\ \ \ x^4+\ldots\ x^{n-1}\\
B&=1+2x+\ 3x^2+\ 4x^3+\ \ 5x^4+\ldots\ nx^{n-1}\\
C&=1+4x+\ 9x^2+16x^3+\ 25x^4+\ldots n^2x^{n-1}\\
D&=1+8x+27x^2+64x^3+125x^4+\ldots n^3x^{n-1}
\end{aligned}
$$

§ 120. The sum of the first series is $A=\frac{x^n-1}{x-1}$; as already found by § 58. In order to find the sum of the second series, divide the whole equation by x, which produces $\frac{B}{x}=\frac{1}{x}+2+3x+4x^2+5x^3+\ldots.nx^{n-2}$. Adding $-\frac{1}{x}+(n+1)x^{n-1}$ to both sides, we have $\frac{B}{x}-\frac{1}{x}+(n+1)x^{n-1}=$ $2+3x+4x^2+5x^3+\ldots nx^{n-2}+(n+1)x^{n-1}=X$. Let us now subtract the series A from this latter series by arranging all the like powers of x under each other as below,

$$
\begin{aligned}
X&=2+3x+4x^2+5x^3+\ldots\ldots\ldots nx^{n-2}+(n+1)x^{n-1}\\
A&=1+\ x+\ x^2+\ x^3+\ldots\ldots\ldots\ x^{n-2}+\qquad\ x^{n-1}\\
\hline
X-A&=1+2x+3x^2+4x^3+\ldots(n-1)x^{n-2}+\qquad nx^{n-1}=B
\end{aligned}
$$

which brings out X−A equal to the original equation B; whence, by substituting the value of X (or $\frac{B}{x}-\frac{1}{x}+[n+1]x^{n-1}$), we shall have $\frac{B}{x}-\frac{1}{x}+(n+1)x^{n-1}-A=B$: consequently, $(x-1)B=(n+1)x^{n}-Ax-1$; and by substituting $(1+\rho)$ for its equal x, we have $B=\frac{n(1+\rho)^{n}-A}{\rho}$.

§ 121. In order to find the sum of the third series, divide the whole equation by x as before, which produces $\frac{C}{x}=\frac{1}{x}+4+9x+16x^{2}+25x^{3}+\ldots\ldots n^{2}x^{n-2}$; adding $-\frac{1}{x}+(n+1)^{2}x^{n-1}$ to both sides, we have $\frac{C}{x}-\frac{1}{x}+(n+1)^{2}x^{n-1}=4+9x+16x^{2}+25x^{3}+\ldots\ldots n^{2}x^{n-2}+(n+1)^{2}x^{n-1}=Y$. Now if we subtract X+B from Y, by arranging all the like powers of x under each other, as below,

$$\begin{array}{r@{\,=\,}l}
Y & 4+9x+16x^{2}+25x^{3}+\ldots\qquad n^{2}x^{n-2}+(n+1)^{2}x^{n-1}\\
X+B & 3+5x+\ \,7x^{2}+\ \,9x^{3}+\ldots(2n-1)x^{n-2}+(2n+1)x^{n-1}\\
\hline
Y-X-B & 1+4x+\ \,9x^{2}+16x^{3}+\ldots(n-1)^{2}x^{n-2}+\qquad n^{2}x^{n-1}=C
\end{array}$$

we shall consequently have $Y-X-B=C$; whence by substituting the value of X and Y we shall have $\frac{C}{x}+(n+1)^{2}x^{n-1}-\frac{B}{x}-(n+1)x^{n-1}-B=C$; that is, $(x-1)C=n(n+1)x^{n}-Bx-B$; and by substituting $(1+\rho)$ for x, we have $C=\frac{n^{2}(1+\rho)^{n}+A-2B}{\rho}$.

§ 122. A similar process is used for finding the sum of the fourth series; for if we divide the whole equation by x, we shall have $\frac{D}{x}=\frac{1}{x}+8+27x+64x^{2}+125x^{3}+\ldots n^{3}x^{n-2}$: adding $-\frac{1}{x}+(n+1)^{3}x^{n-1}$ to both sides, we have $\frac{D}{x}-\frac{1}{x}+(n+1)^{3}x^{n-1}=8+27x+64x^{2}+125x^{3}+\ldots\ldots n^{3}x^{n-2}+(n+1)^{3}x^{n-1}=Z$. Now if we subtract Y+B+2C from Z, by arranging all the like powers of x under each other, as below,

$$\begin{array}{r@{\,=\,}l}
Z & 8+27x+64x^{2}+125x^{3}+\ldots\ldots\ldots\ldots\ldots\ldots n^{3}x^{n-2}+(n+1)^{3}x^{n-1}\\
Y+B+2C & 7+19x+37x^{2}+\ \,61x^{3}+\ldots(3n^{2}-3n+1)x^{n-2}+(3n^{2}+3n+1)x^{n-1}\\
\hline
Z-Y-B-2C & 1+\ \,8x+27x^{2}+\ \,64x^{3}+\ldots\ldots\ldots(n-1)^{3}x^{n-2}+n^{3}x^{n-1}=D
\end{array}$$

we shall have $Z-Y-B-2C=D$; whence by substituting the values of Y and Z, we shall have $\frac{D}{x}+(n+1)^{3}x^{n-1}-\frac{C}{x}-(n+1)^{2}x^{n-1}-B-2C=D$; consequently $(x-1)D=n(n+1)^{2}x^{n}-Bx-2Cx-C$: and by substituting $(1+\rho)$ for x we have $D=\frac{n^{3}(1+\rho)^{n}-A+3B-3C}{\rho}$.

§ 123. In order to render these formulæ more easily to be referred to, I shall bring them all under one view. Thus,

$$A=\frac{(1+\rho)^n-1}{\rho}$$

$$B=\frac{n(1+\rho)^n-A}{\rho}$$

$$C=\frac{n^2(1+\rho)^n+A-2B}{\rho}$$

$$D=\frac{n^3(1+\rho)^n-A+3B-3C}{\rho}.$$

And as an example of each of them, we shall have the following values, when $n=5$, and $\rho=\cdot05$; viz.,

$A = 5\cdot52563125$
$B = 17\cdot11553125$
$C = 64\cdot03215625$
$D = 265\cdot19378125$

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CHAPTER XVI.

ON THE PRESENT VALUE OF ANNUITIES INCREASING IN A CONSTANT RATIO.

§ 124. In order to determine the present value of annuities increasing in a similar manner to those just mentioned; that is, according to the order of the natural numbers, and according to the squares and cubes of the same, let us make A, B, C, D equal to the respective series as before; then shall we have in this case,

$$A=(1+\rho)^{-1}+(1+\rho)^{-2}+(1+\rho)^{-3}+(1+\rho)^{-4}+(1+\rho)^{-5}+\ldots(1+\rho)^{-n}$$
$$B=(1+\rho)^{-1}+2(1+\rho)^{-2}+3(1+\rho)^{-3}+4(1+\rho)^{-4}+5(1+\rho)^{-5}+\ldots n(1+\rho)^{-n}$$
$$C=(1+\rho)^{-1}+2^2(1+\rho)^{-2}+3^2(1+\rho)^{-3}+4^2(1+\rho)^{-4}+5^2(1+\rho)^{-5}+\ldots n^2(1+\rho)^{-n}$$
$$D=(1+\rho)^{-1}+2^3(1+\rho)^{-2}+3^3(1+\rho)^{-3}+4^3(1+\rho)^{-4}+5^3(1+\rho)^{-5}+\ldots n^3(1+\rho)^{-n}$$

§ 125. But if we substitute x for $(1+\rho)$, for the reasons already explained, we shall have

$$A=x^{-1}+x^{-2}+x^{-3}+x^{-4}+x^{-5}+\ldots x^{-n}$$
$$B=x^{-1}+2x^{-2}+3x^{-3}+4x^{-4}+5x^{-5}+\ldots n\,x^{-n}$$
$$C=x^{-1}+4x^{-2}+9x^{-3}+16x^{-4}+25x^{-5}+\ldots n^2x^{-n}$$
$$D=x^{-1}+8x^{-2}+27x^{-3}+64x^{-4}+125x^{-5}+\ldots n^3x^{-n}$$

§ 126. The sum of the first series is $A=\frac{1-x^{-n}}{x-1}$, as already found by § 72. In order to find the sum of the second series, multiply the whole equation by x, which produces $Bx=1+2x^{-1}+3x^{-2}+4x^{-3}+5x^{-4}+\ldots nx^{-(n-1)}$; and adding $-1+(n+1)x^{-n}$ to both sides, we have $Bx-1+(n+1)x^{-n}=2x^{-1}+3x^{-2}+4x^{-3}+5x^{-4}+\ldots nx^{-(n-1)}+(n+1)x^{-n}=X$. Now if we subtract A from X, by arranging all the like powers under each other as below, there will result the original series B, whose sum may be thence deduced, as in the following manner :—

$$\begin{array}{rl} X= & 2x^{-1}+3x^{-2}+4x^{-3}+5x^{-4}+\ldots\ldots\ldots nx^{-(n-1)}+(n+1)x^{-n} \\ A= & x^{-1}+\ x^{-2}+\ x^{-3}+\ x^{-4}+\ldots\ldots\ldots\ x^{-(n-1)}+\qquad x^{-n} \\ \hline X-A= & x^{-1}+2x^{-2}+3x^{-3}+4x^{-4}+\ldots(n-1)x^{-(n-1)}+\qquad nx^{-n}=B. \end{array}$$

Consequently $X-A=B$; whence, by substituting the value of X, we have $Bx-1+(n+1)x^{-n}-A=B$; that is $(x-1)\,B=1-(n+1)x^{-n}+A$; and by restoring $(1+\rho)$ for x, we have $B=\frac{1+A-(n+1).(1+\rho)^{-n}}{\rho}$.

§ 127. The sum of the third series is found in a similar way, by multiplying the whole equation by x, and then adding $-1+(n+1)^2x^{-n}$ to both sides; which produces $Cx-1+(n+1)^2x^{-n}=4x^{-1}+9x^{-2}+16x^{-3}+25x^{-4}+\ldots n^2x^{-(n-1)}+(n+1)^2x^{-n}=Y$. Now if $X+B$ be subtracted from Y, by arranging all the like powers of x under each other, there will result the original series C: consequently we shall have $Y-X-B=C$; and by substituting the value of X and Y, we produce $Cx+(n+1)^2x^{-n}-Bx-(n+1)x^{-n}-B=C$. Whence $(x-1)\ C=(x+1)\ B-n(n+1)x^{-n}$; and by substituting $(1+\rho)$ for x, we have $C=\frac{1+A+2B-(n+1)^2.(1+\rho)^{-n}}{\rho}$.

§ 128. The sum of the fourth series is found by multiplying the whole equation, as before, by x, and then adding $-1+(n+1)^3x^{-n}$ to both sides; which produces $Dx-1+(n+1)^3x^{-n}=8x^{-1}+27x^{-2}+64x^{-3}+125x^{-4}+\ldots n^3x^{-(n-1)}+(n+1)x^{-n}=Z$. Now if $Y+B+2C$ be subtracted from Z, in the manner just mentioned, there will result the original series D: consequently we shall have $Z-Y-B-2C=D$. Therefore by substituting the value of Y and Z we shall have $Dx+(n+1)^3x^{-n}-Cx-(n+1)^2x^{-n}-B-2C=D$; whence $(x-1)\ D=B+(2+x)\ C-n(n+1)^2x^{-n}$; and by substituting $(1+\rho)$ for x, we have $D=\frac{1+A+3B+3C-(n+1)^3.(1+\rho)^{-n}}{\rho}$.

§ 129. In order to render these formulæ more easily to be referred to, I shall bring them all under one view. Thus,

$$A=\frac{1-(1+\rho)^{-n}}{\rho}$$

$$B=\frac{1+A-(n+1)\cdot(1+\rho)^{-n}}{\rho}$$

$$C=\frac{1+A+2B-(n+1)^2(1+\rho)^{-n}}{\rho}$$

$$D=\frac{1+A+3B+3C-(n+1)^3(1+\rho)^{-n}}{\rho}.$$

And, as an example of each of them, I have subjoined the following values, when $n=5$ and $\rho=\cdot05$:—

$A=\ \ 4\cdot32947667$
$B=\ 12\cdot56639339$
$C=\ 45\cdot10643079$
$D=182\cdot12596007$

§ 130. When the annuity is a perpetuity, that is, when n is infinite, these several series will be expressed by the following equations: viz.,

$$A=\frac{1}{\rho}$$

$$B=\frac{1+\rho}{\rho^2}$$

$$C=\frac{(1+\rho)^2+(1+\rho)}{\rho^3}$$

$$D=\frac{(1+\rho)^3+4(1+\rho)^2+(1+\rho)}{\rho^4}.$$

Which equations are easily deduced from the principles already laid down; indeed the formulæ just given for finding the present values of such annuities for a given term will lead us to the true value in the case of perpetuities; since all the quantities affected by n become infinitely small; consequently if we make such quantities equal to nothing, we shall produce the equations here given. We may, however, obtain a general formula for the present value of the perpetuity of an annuity of £1 increasing according to any power ($=m$) of the natural numbers, by attending to the following remarks:—1°. That the nnmber of terms in the numerator of the proposed formula will be always equal to m. 2°. That such terms will consist wholly of the powers of $(1+\rho)$, beginning with the mth power and decreasing by unity. 3°. That the co-efficient of the first term will be $=1$; the co-efficient of the second term $=2^m-\frac{m+1}{1}$; the co-efficient of the third term $=3^m-2^m\times\frac{m+1}{1}+\frac{m(m+1)}{1.2}$; the co-efficient of the fourth term $=4^m-3^m\times\frac{m+1}{1}+2^m\times\frac{m(m+1)}{1.2}-\frac{(m-1).m(m+1)}{1.2.3}$. 4°. That after the numeral co-efficients of half these terms are found, the following co-efficients will be the same with the former, but in a reversed order, similar to what takes place in the expansion of a binomial. 5°. That the denominator of the fraction composing the formula will be always ρ raised to the $(m+1)^{\text{th}}$ power.*

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CHAPTER XVII.

ON THE EQUATION OF PAYMENTS.

§ 131. In a work like the present, it may perhaps be expected that I should say something of the common rule for the equation of payments; a subject which has generally been treated on by preceding writers, and which may be explained in the following manner:—When several sums of money due at different times are owing from one person to another, it is sometimes required to find the time when the whole of such sums may be discharged at one payment without injury to either party: this is called *equating the payments;* and the principle of the rule consists in finding the time when the interest of the sums which are deferred till after they are due, is equal to the discount of those which are paid before they are due. Let us take the case of *two* sums, calling p the first that becomes due, and π the second; and let it be required to find the time x when the interest on the sum p shall be equal to the discount on the sum π; in order to determine this, let the interval of time when the sum p becomes due be denoted by n, and the time when the sum π becomes due, by ν; and let r denote the rate of interest. Then will the interest on the sum p for the time $(x-n)$ be $pr(x-n)$ (see § 7); and the discount on the sum π for the time $(\nu-x)$ will be $\frac{\pi r(\nu-x)}{1+r(\nu-x)}$ (see § 12); and since by the hypothesis these two values are equal, we shall have $pr(x-n)=\frac{\pi r(\nu-x)}{1+r(\nu-x)}$; whence we obtain $x^2-\frac{pr(\nu+n)+\pi+p}{pr}x=-\frac{pr\nu n+\pi\nu+pn}{pr}$. Therefore if we make $\frac{pr(\nu+n)+\pi+p}{pr}=a$, and $\frac{pr\nu n+\pi\nu+pn}{pr}=b$, we shall have $x=\frac{a\pm\sqrt{a^2-4b}}{2}$ for the true equated time. For example: suppose A were indebted to B in the sum of £1000, of which £600 was to be paid in 6 months and the remainder in 12 months; at what time, reckoning interest at 5 per cent, ought the whole to be discharged in one payment without injury to either party? Here we have $p=600$, $\pi=400$, $n=\frac{1}{2}$, $\nu=1$, and $r=\cdot05$; consequently $a=\frac{1045}{30}=34\cdot8333$, and $b=\frac{715}{30}=23\cdot8333$, whence

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* See Dodson's *Mathematical Repository* (vol. ii. p. 74), a work of great merit and utility.

$$x=\frac{34\cdot8333\pm\sqrt{1213\cdot3610-95\cdot3333}}{2}=\frac{34\cdot8333-33\cdot4369}{2}=\cdot6982, \text{or } 255$$

days. As a proof of which it will be found, on trial, that the discount on £400 for 110 days, will be equal to the interest on £600 for 72 days.

§ 132. The above formula brings out the true value for *two* payments; but where the debts are more numerous this method of finding the equated time becomes too complicated and intricate; and there is no rule, fit for general use, by which we can obtain the *exact* values. The common rule which is given in most books on the subject is to "Multiply each sum by the time when it is due, then divide the sum of these products by the total debt: the quotient is the time at which all the money ought to be paid." This rule is founded on the supposition that we are to find the time when the interest of the sums which are kept till after they are due is equal to the *interest* (and not to the *discount*) of those which are paid before they are due: this, however, strictly speaking, is not the correct method; but since the erroneous practice universally prevails of taking the interest instead of the discount from sums which are due at a future period (see § 8), we may adopt this rule as founded on existing circumstances, as affording a near approximation in most cases to the true value, and as being very convenient for practice. The investigation of it is as follows:—

§ 133. Let p, p_1, p_2, &c., denote the several sums due *before* the equated time of payment, x; and n, n_1, n_2, &c., the times at which they respectively become due: further, let π, π_1, π_2, &c., denote the several sums due *after* the time x; and ν, ν_1, ν_2, &c., the times at which they respectively become due. Then, at the time when the interest on the former sums is equal to the interest on the latter, we shall have the equation $pr(x-n)+p_1r(x-n_1)+p_2r(x-n_2)+$ &c., $=\pi r(\nu-x)+\pi_1r(\nu_1-x)+\pi_2r(\nu_2-x)+$ &c.: from which we deduce $x=\frac{pn+p_1n_1+p_2n_2+\&c.+\pi\nu+\pi_1\nu_1+\pi_2\nu_2+\&c.}{p+p_1+p_2+\&c.+\pi+\pi_1+\pi_2+\&c.}$. Whence the rule is manifest.

§ 134. When *compound* interest is concerned, the true equated time may in all cases be determined with the greatest accuracy: for, retaining the same characters as above, we shall find that when the interest on all the former sums is equal to the discount on all the latter, we shall have the equation $p(1+\rho)^{x-n}-p+p_1(1+\rho)^{x-n_1}-p_1+p_2(1+\rho)^{x-n_2}-p_2+$ &c., $=\pi-\pi(1+\rho)^{-(\nu-x)}+\pi_1-\pi_1(1+\rho)^{-(\nu_1-x)}+\pi_2-\pi_2(1+\rho)^{-(\nu_2-x)}+$ &c.; which equation, by multiplying both sides by $(1+\rho)^{-x}$, becomes $p(1+\rho)^{-n}-p(1+\rho)^{-x}+p_1(1+\rho)^{-n_1}-p_1(1+\rho)^{-x}+p_2(1+\rho)^{-n_2}-p_2(1+\rho)^{-x}+\&c.=\pi(1+\rho)^{-x}-\pi(1+\rho)^{-\nu}+\pi_1(1+\rho)^{-x}-\pi_1(1+\rho)^{-\nu_1}+\pi_2(1+\rho)^{-x}-\pi_2(1+\rho)^{-\nu_2}+$ &c.; whence we obtain $(1+\rho)^x=$

$$\frac{p+p_1+p_2+\&c.+\pi+\pi_1+\pi_2+\&c.}{p(1+\rho)^{-n}+p_1(1+\rho)^{-n_1}+p_2(1+\rho)^{-n_2}+\&c.+\pi(1+\rho)^{-\nu}+\pi_1(1+\rho)^{-\nu_1}+\pi_2(1+\rho)^{-\nu_2}+\&c.}$$

Now, in order to abridge the subsequent process, let us make the numerator of the fraction, forming the right side of this equation, equal to a, and the denominator equal to b; then will $(1+\rho)^x=\frac{a}{b}$; whence we obtain $x=\frac{\lambda a-\lambda b}{\lambda(1+\rho)}$. But a is the sum-total of the several debts, and b is the sum of the present values of all those debts; therefore the true equated time, at compound interest, will be accurately expressed by the following rule:—"From the logarithm of the sum of all the debts subtract the logarithm of the sum of the present values of such debts, and divide the remainder by the logarithm of the amount of £1 in a year at the given rate of interest: the quotient will be the equated time required." For example, suppose A were indebted to B in the sum of £750 which was to be paid in three several payments, viz., £250 at the end of 1½ year, £100 at the end of 2 years, and the remaining £400 at the end of 4 years; at what time, reckoning compound interest at 6 per cent, ought the whole to be discharged in one payment? Here we have a, or the sum of all the debts, equal to 750; and b, or the present value of all such debts, equal to 634·913963; consequently $x=\frac{\lambda(750)-\lambda(634{\cdot}913963)}{\lambda(1{\cdot}06)}=\frac{{\cdot}0723463}{{\cdot}0253059}=$ 2·85887, or 2 years and 313 days.

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CHAPTER XVIII.

MISCELLANEOUS QUESTIONS AND REMARKS.

I.

§ 135. In what time will any principal double itself, at any given rate of simple or compound interest? Here we shall have $s=2p$, which in the case of simple interest will bring out $n=\frac{1}{r}$, and in the case of compound interest $n=\frac{\lambda 2}{\lambda(1+\rho)}$. From which theorems I have deduced the following table, which shows the several times at which any principal doubles itself, at the rates therein mentioned:—

Rate of Interest.	Years at	
	Simple Interest.	Compound Interest.
2	50·	35·00278878
2½	40·	28·07103453
3	33·33333333	23·44977225
3½	28·57142857	20·14879168
4	25·	17·67298769
4½	22·22222222	15·74730184
5	20·	14·20669908
6	16·66666667	11·89566105
7	14·28571429	10·24476835
8	12·5	9·00646834
9	11·11111111	8·04323173
10	10·	7·27254090

II.

§ 136. If a *penny* had been put out at 5 per cent *compound* interest at the birth of Christ; to what sum would it amount at the end of the year 1810?

By the first theorem in § 15, it will be seen that its amount in that time would be $\cdot 004166666\times(1\cdot 05)^{1810}=$ 938469 000000 000000 000000 000000 000000 pounds. Now the diameter of the earth is about 8000 miles; consequently its solid contents* will be 68 188963 498145 531559 936000 cubic inches: and if it were made of standard gold, each cubic inch being worth £38, 10s.,† the total value of such a globe would be 2625 275094 678602 965057 536000 pounds. But the amount of a penny in 1810 years as above stated, is more than 357 474 600 times the value of such a globe; consequently if one penny had been put out at 5 per cent compound interest at the time above mentioned, it would at this period have amounted to more money than could be expressed by THREE HUNDRED AND FIFTY-SEVEN MILLIONS of globes, each equal to the earth in magnitude, and all solid gold!!! Whereas if it had been put out at the same rate of *simple* interest, the amount in the same time would have been only *seven shillings and seven pence half-penny.*

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* To find the solid contents of a sphere, multiply the decimal ·785398 163397 448309 615, &c., by the cube of the diameter, and take two-thirds of the product.

† Since a cubic inch of distilled water weighs about 254 grains, and the specific gravities of standard gold and water are to each other as 18·888 to one, it follows that a cubic inch of gold, at the mint price of £3, 17s. 10½d. per ounce, will amount to £38, 10s. 1¼d.

III.

§ 137. If a debt bearing interest be discharged in a certain time by given annual instalments; in how much less time will the same debt be discharged by making such instalments[*] half-yearly, quarterly, &c.? the nominal rate of interest being the same in both cases.

By a deduction from the formula in § 114, it will be found that in all cases we shall have $n=\frac{\lambda(a-sr)-\lambda(a-pr)}{\lambda(1+\frac{r}{m})^{m}}$, according as the instalments are made m times in a year. But in the case of annual instalments this formula becomes $\frac{\lambda(a-sr)-\lambda(a-pr)}{\lambda(1+r)}$; consequently the time in which any debt is discharged by any given annual instalments, will be invariably decreased in the constant ratio $\frac{\lambda(1+r)}{\lambda(1+\frac{r}{m})^{m}}$, according as such instalments are made m times in a year.

For example: Let us suppose a debt of £2000, an annuity of £100, and the rate of interest 4 per cent; in which case such debt would be discharged, by annual payments, in 41·0355 years; and by multiplying this by ·99029 and ·98541, we shall have 40·6370 and 40·4368 respectively, for the number of years in which the same sum would be extinguished by the same annuity payable half-yearly or quarterly; the nominal rate[†] of interest being the same in each case.

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[*] That is, the half-yearly and quarterly instalment, and the half-yearly and quarterly interest being respectively one half or one quarter of the given nominal payment.

[†] M. De Moivre has given a different solution to the above case (*Doctrine of Chances*, p. 321), which arises from his calculation being founded on the supposition of a given *annual* rate of interest; and, under such circumstances, we should have the value of $n=\frac{\lambda(a-s\mathrm{R})-\lambda(a-p\mathrm{R})}{\lambda(1+\rho)}$, where R denotes $\mu[(1+\rho)^{\frac{1}{\mu}}-1]$; μ expressing the number of payments of the annuity in the year. But I think, that in questions of *this kind* we should make the payments of interest and the payments of the annuity correspond with each other; agreeably to the principles already laid down in §§ 114 and 115. I may here remark, however, that M. D'Alembert is of a contrary opinion, and agrees with M. De Moivre in the method of solving questions of this kind. See his *Opusc. Mathem.*, vol. viii. p. 46.

IV.

§ 138. If a loan is borrowed for the service of Government, at a given rate of interest in a 3, 4, or 5 per cent stock; and a sinking fund, equal to ϕ per cent on the capital *stock* created by such loan, is established for the purpose of redeeming the same; the time in which such loan will be redeemed in this way is expressed by the equation $n = \frac{\lambda(1+\frac{\beta}{\phi})}{\lambda(1+\rho)}$; where β is equal to 3, 4, 5, &c., according as the fund, in which the loan is raised, is a 3, 4, or 5 per cent stock.

§ 139. This method of putting all future loans in a course of redemption was adopted in the year 1792 (32 George III. c. 55, § 3); when it was ordained that a fund should be raised of *one* per cent ($\phi=1$) on the *stock* created by all future loans, for the purpose of redeeming the same. The following table will show the number of years in which any principal will be redeemed in this way, according to the assumed price of such stock at the time of its being funded:—

3 per Cents.		4 per Cents.		5 per Cents.	
Price of Stock.	Number of Years.	Price of Stock.	Number of Years.	Price of Stock.	Number of Years.
50	23·791	65	26·950	80	29·555
55	26·102	70	28·962	85	31·347
60	28·413	75	30·975	90	33·139
65	30·724	80	32·987	95	34·931
70	33·035	85	34·999	100	36·724
75	35·345	90	37·011	105	38·516
80	37·656	95	39·023	110	40·308
85	39·967	100	41·035	115	42·100
90	42·278	105	43·048	120	43·892
95	44·589	110	45·060	125	45·684
100	46·900	115	47·072	130	47·476

§ 140. If the loan is borrowed at a given rate of interest as above mentioned; and the sinking fund established for its redemption, instead of being equal to ϕ per cent on the capital stock created by such loan, is equal to ϕ per cent on the *sum* borrowed; then the time, in which such loan will be redeemed in this way, is expressed by the equation $n = \dfrac{\lambda(1+\frac{i}{\phi})}{\lambda(1+\rho)}$; where i denotes the rate of interest per *cent* per annum allowed to the contractors.

§ 141. This method of redeeming the public debts was first suggested by Dr. Price, who proposed to raise a fund of *one* per cent ($\phi = 1$) on the *sum* borrowed in all future loans, for the purpose of redeeming the same; but it was superseded by the plan just mentioned. The following table, however, will show the number of years in which any principal would be redeemed in this way, according to the rate of interest allowed to the contractors:—

Rate of Interest.	Number of Years.
6	33·395
5½	34·960
5	36·724
4½	38·730
4	41·035
3½	43·721
3	46·900

§ 142. If a loan is borrowed at a given rate of interest, as already mentioned; and a certain sum, equal to a per cent on the *sum* borrowed, is applied annually towards paying the interest of such loan, and towards the discharge of the principal (in which case the proper sinking fund, per cent, will be equal to the difference of the given sum a, and the interest per cent per annum); then will the equation become $n = \dfrac{\lambda(\frac{a}{a-i})}{\lambda(1+\rho)}$.

§ 143. This mode of redeeming some of the current loans of Government was first adopted in the year 1807 (47 George III. c. 55), when it was resolved to set apart *ten* per cent per annum on the *sum* borrowed ($a = 10$) as a fund for the payment of the annual interest of the same, and for the gradual redemption of the capital. The following table will show the number of years in which any loan will be redeemed by such annuity of 10 per cent per annum on the sum borrowed, applied annually towards the discharge of the same, on account both of principal and interest, and according to the rate of interest allowed to the contractors:—

Rate of Interest.	Number of Years.
6	15·725
5½	14·914
5	14·207
4½	13·582
4	13·024
3½	12·522
3	12·067

§ 144. These different formulæ are deduced from the principles laid down in § 110; and in the tables here given, the payments are supposed to be made *yearly*;* the value of ρ, or the rate of interest allowed, being computed from the price of the stock at the time of contracting for the loan. But it should be particularly observed that questions of this kind can be solved only with this restriction, that the price of the stock funded continues the same throughout the *whole* term,—a circumstance which is not likely to happen; consequently these formulæ will point out certain limits only as a guide to our inquiries on these subjects.

§ 145. It is possible that considerable variation may also take place in the value of n, owing to the fluctuation in the price of the funds: but, having found its value, at the time the loan was contracted, according to any *one* rate of interest $(=\rho)$, we may easily determine how far that value would be effected by any alteration in the price of the funds, producing any *other* rate of interest $(=\text{R})$; for we have only in such case to multiply either of the above formulæ by the quantity $\frac{\lambda(1+\rho)}{\lambda(1+\text{R})}$. For example, suppose a loan effected in the 3 per cents valued at 75, with a sinking fund of one per cent on the stock created: in this case the rate of interest would be 4 per cent, and the time in which the capital would be redeemed by such sinking fund (provided the stock remained at the same price during the whole term) would be 35·345 years (see § 139). But if, immediately after making this loan, the 3 per cents were to rise to *par*, and to continue at that price during the whole term, then the rate of interest would be only 3 per cent, and the value of n would be $35{\cdot}345 \times \frac{\lambda(1{\cdot}04)}{\lambda(1{\cdot}03)} =$ 46·900. Or, if they were to fall, and continue at 60 during the whole term, the rate of interest would then be 5 per cent, and the value of n only $35{\cdot}345 \times \frac{\lambda(1{\cdot}04)}{\lambda(1{\cdot}05)} = 28{\cdot}413$ years.

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* The sums appropriated for the redemption of the National Debt are issued *quarterly* from the Exchequer, which will consequently make some *small* difference in the values. But the results are, at the best, but *approximations* to the true period of redemption, owing to the fluctuation in the price of the funds. The real differences which would arise from such multiplied payments will be shown in § 146.

In like manner, if a loan were borrowed in a 3 per cent stock valued at 60, and a fund, equal to 10 per cent on the sum borrowed, were established for the payment of the interest and the gradual redemption of the capital; the time in which the whole debt would be redeemed by such fund (provided the stock continued at the same price during the whole term) would be 14·207 years (see § 143). But if, immediately after making this loan, the 3 per cents were to rise to 75, and to continue so during the whole term, then the rate of interest would be 4 per cent, and the value of n would be $14{\cdot}207 \times \frac{\lambda(1{\cdot}05)}{\lambda(1{\cdot}04)} = 17{\cdot}673$. Or, if they were to rise to *par*, the rate of interest would be 3 per cent, and the value of n equal to $14{\cdot}207 \times \frac{\lambda(1{\cdot}05)}{\lambda(1{\cdot}03)} = 23{\cdot}450$ years.

§ 146. From the formula given in § 114, we may easily determine the ratio between the times according as the sums set apart for the redemption of the loan are payable half-yearly, quarterly, &c.; and on the supposition that the interest also is payable at the same intervals. For, if the tables are formed on the supposition of a given rate of interest $(=r)$, which, together with such sums, is payable annually, then will the tabular values thus found be invariably increased by the quantity $\frac{\lambda(1+r)}{\lambda(1+\frac{r}{m})^m}$ according as such interest, &c., is payable m times in a year. Thus, by the table in § 141, it appears that a sinking fund of one per cent on the sum borrowed would annihilate the principal in 41·035 years, reckoning interest at 4 per cent payable yearly: but if such interest, &c., were payable half-yearly, the time would then be $41{\cdot}035 \times \frac{\lambda(1{\cdot}04)}{\lambda(1{\cdot}02)^2} = 40{\cdot}637$ years: or, if the interest, &c., were payable quarterly, the time would then be $41{\cdot}035 \times \frac{\lambda(1{\cdot}04)}{\lambda(1{\cdot}01)^4} =$ 40·436 years: and the same observation will hold good with respect to the other tables.

V.

§ 147. By the Act 32 Geo. III. c. 55, it is ordained that in all future loans there shall be annually set apart, out of the Consolidated Fund, for the purpose of gradually extinguishing such debt, a sum equal to onehundredth part of the capital stock created by such loan. If, therefore, a loan be effected by granting a capital of £175 stock in the 3 per cents for every £100 sterling advanced (such was the loan of 1799), in what time will the whole debt (of 15½ millions then raised) be extinguished by the operation of the fund above mentioned? Here the interest allowed is 5¼ per cent, and the sinking fund of one per cent on the stock is equal to 1·75 on every £100 sterling; the time, therefore, in which this annuity of £1, 15s. will amount to £100, or, in other words, the time, in which the whole debt would be extinguished is (by the third formula in § 58, by the fourth formula in § 111, or by the formula in § 138) equal to 27·093, or rather more than 27 years.

§ 148. This is the exact time in which the whole debt would be extinguished, provided the price of the funds continued the same during the whole period, or provided Government was not obliged to redeem *more* than the *original sum* borrowed. But it is well known, that by borrowing in a 3 or 4 per cent stock, Government is often obliged to redeem such stock at a higher price than that at which it was originally granted; and this will always be the case, since loans are raised during a state of war, when the funds are low; and the operation of the sinking fund above mentioned does not produce much effect till after some years, when probably a peace has taken place and the debt has nearly doubled itself in value.

§ 149. This shows us how vague and uncertain those assurances are which are founded on such calculations; and till Government resorts to the method of borrowing money in use among individuals (where, on the original sum being returned, the debt is cancelled) it will be impossible to determine, with any degree of accuracy, the precise time of the extinction of our present enormous debt.

§ 150. This view of the question will also show us with what limitation we ought to receive Dr. Price's assertion, "That on this plan of redeeming the debt, it will be of less importance to a state what interest it is obliged to give for money; for the higher the interest the sooner will such a fund pay off the principal." * Were it the principal only, or the *sum lent*, which is concerned, his remarks would be just and accurate; but, on the present system, the higher the interest the lower will be the price of the stocks, and their chance of rising afterwards (which involves a loss to the State) will be proportionally greater.

§ 151. It likewise shows us the advantage and propriety of borrowing money on *terminable* annuities rather than by perpetuities. For, we may fairly conclude, that the capital of £27,125,000 3 per cent stock created by this loan of 15½ millions sterling, will not be redeemed at a price less than £75 per cent, if we take the average price of the stocks one year with another, whether of war or peace. Consequently, the Government will have to repay to the public creditor the sum of £20,343,750 sterling, instead of the 15½ millions borrowed as above, a capital which will not be redeemed by the sinking fund above mentioned in less than 35·345 years, reckoning interest at 4 per cent.

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* *Observ. on Rev. Pay.*, vol. i. p. 278.

§ 152. But let us take another view of the subject more agreeable to the actual state of things; let us suppose that the operation of the sinking fund above mentioned continues the same for the first 10 years; that is, that owing to a state of war, the Government can buy up the stock upon the same terms that it was originally granted, and that at the end of that period a peace takes place which continues to the time of the final extinction of the debt. Upon this supposition, we shall find that only £6,040,737 of stock will be redeemed at the end of the first 10 years, and that there will remain £21,084,263 unprovided for, which being valued at 80 (a price perhaps not too extravagant) will make the unredeemed part of the debt equal to £16,867,410 sterling, or £1,367,410 *more* than the original sum borrowed, although near 3½ millions sterling have been paid off!!! And this remaining debt (on the above supposition) will not be finally extinguished till after 23·758 years more; making, in the whole, near 34 years. Or if the 3 per cents were to rise to *par*, this remaining debt would not be finally extinguished till after 29·589 years; making, in this case, the whole number of years nearly equal to 40. How much better, therefore, would it have been, in the first instance, to have granted to the subscribers a terminable annuity of £7 per cent for 27 years, than a perpetuity of 5¼ per cent on the sum advanced, with the contingency of redemption as above-mentioned! Or, since terminable annuities are generally undervalued by the public, to have granted such annuities of £7 per cent for *thirty* years, which would have been an additional bonus to the subscribers of three-fourths of a year's purchase;* and would have secured the extinction of the debt within the time limited, a sacrifice but trifling when we consider the enormous expense which is otherwise incurred. For, in either of the cases first stated, it will appear that the debt is increased nearly one-third of its original amount, or that near 5 millions of money are given for the loan of 15½ millions, over and above the regular rate of interest.

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* I consider this as an *additional* bonus to the subscribers, because I have taken the value of the annuity in this calculation at a price corresponding to the terms on which the loan was contracted, which, of course, includes any advantage gained thereby.

§ 153. Had this loan been borrowed in the 5 per cents, valued at 90, with a *two* per cent sinking fund on the stock created, the whole loan would have been redeemed at that price in 23·170 years; or at par in 25·677 years; and the annual charge to the country on account of this loan would not have been greater than it now is.

VI.

§ 154. In April 1805, those proprietors of the 5 per cents 1797, or Loyalty loan, who had given six months' previous notice, were entitled to have their stock paid off at par; agreeably to the terms of the Act (37 Geo. III. c. 10) by which such stock was created. The Government, however, thought proper to give them their option, in the month of October preceding, of taking, instead of the money, £100 stock in the Navy 5 per cents, and £20, 9s. 8d. stock in the Reduced 3 per cents, for every £100 stock held by them in the 5 per cents 1797, or Loyalty loan. Had the Government been willing to exchange such stock for £7 per cent terminable annuities, for what period ought such annuities to have been granted? Or, had it been willing to exchange them for Long Annuities which expire in January 1860, what sum ought to have been granted therein as an equivalent for every £100 stock in the said 5 per cents 1797; allowing 5½ per cent interest?*

§ 155. By the third example in § 73, it appears that an annuity of £7 ought to be granted for 28¾ years; if, therefore, such annuity had been granted for 30 years, the subscribers would have had full a quarter of a year's purchase for a bonus, or 1¾ per cent on their capital,—a sufficient inducement for them to accept these terms. Now the whole of the charge of the above new stock (including the interest and the 1 per cent sinking fund on the capital) is £6, 16s. 4d. for every £100 sterling: therefore, by the mere addition of 3s. 8d. per cent, the whole debt would have been *certainly* annihilated at the end of the 30 years; whereas by giving the several sums in stock as above mentioned, the whole capital will not probably be redeemed these 40 years.

§ 156. Again, by the second example in § 73, it appears that £5, 16s. per annum in the Long Annuities ought to be given for every £100 5 per cent stock, or, since these terminable annuities are generally undervalued in the market, if £5, 17s. 3d. per annum had been given in exchange, the Long Annuities would have been valued to the subscribers at little more than 17 years' purchase; which was one-fourth of a year's purchase below their current price at that period; and would consequently have been a sufficient inducement in this case also for them to have accepted the terms.

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* From the price of the funds at that period (the 5 per cents being below 90) a person might make 5½ per cent interest of his money; and consequently this rate is taken as a fair ground of calculation.

VII.

§ 157. When the funding system was first introduced, the money required for the service of Government was borrowed for a short period of time (for 5, 6, and 7 years); but as the exigencies of the State increased, this term was lengthened, and in some cases extended to a period of 99 years—a practice which finally gave place to the borrowing of money on *perpetuities;* each succeeding step being more ruinous than the former. Of late years, most of the loans have been raised in a 3 per cent stock valued at about 60; and provision is made by Government for the payment of the same at *par:* a more ruinous system than which could scarcely be devised, had not some late ministers completed the climax by raising money, in the same manner, on the credit of the *revenues of posterity.* This was first done by Mr. Addington (now Lord Sidmouth) in 1802, who raised part of the loan for that year by mortgaging a portion of the Short Annuities. But it has been since carried to a greater extent by Lord Henry Petty in 1807, who has contrived, in a very ingenious way, to shift the burden from the present generation, and to *mortgage a portion of the sinking fund* for the payment of the interest of our current loans.*

§ 158. It will be needless for me to enter into any digression, in this place, on the evils and danger of our present National Debt, or on the various modes proposed of putting it in a state of liquidation, since the subject has been so fully treated on and so ably discussed by the late Dr. Price in his celebrated treatise on *Reversionary Payments;*† and since we find that the best contrived schemes are so liable to be perverted according to the exigencies of the State. There is a plan, however, which, though noticed and condemned by that author, I think might be adopted with advantage to the country, and which would render the operation of the sinking fund of *certain* effect, without the possibility of alienation by any prodigal minister, or by a corrupt and compliant Parliament.

§ 159. The plan which I should propose would be to exchange the *perpetuities* for *terminable* annuities. There are many persons, whose incomes are small, and who have no family, that would be glad to accept of an annuity for 20, 40, or 60 years, according to their views and expectations in life, in exchange for the stock held by them in the public funds: particularly if a small premium were offered to them by way of inducement.* In order to illustrate this plan, let it be proposed that Government should create two new species of funds, or *terminable annuities*, the one for *thirty* and the other for *fifty* years;† and that every half-year, on or before the shutting of the books for the dividends, any proprietor, who might give notice to the Governor and Company of the Bank of England of his wish to embrace the benefits of this plan,‡ should be allowed, for every £100 stock held by him, an annuity for 30 or 50 years (at his option expressed in the notice) equal in amount to its value as calculated according to such rate of interest per cent per annum as the fund, from its current price, may then pay:§ *together with* such additional annuity as shall be equal to 10 per cent ‖ on the dividend which he would receive on his original stock. Thus, suppose the average price of the 3 per cents to be £60, and consequently the rate of interest 5 per cent: the annuity which this sum would purchase for 30 years will be 3·903, or £3, 18s. 1d. per annum; and for 50 years it will be 3·287, or £8, 5s. 9d. per annum:¶ but, since three shillings (being the amount of 10 per cent on the dividend of every £100 stock) must be added to these sums, the annuity which would be received in exchange for every £100 stock is £4, 1s. 1d. and £3, 8s. 9d. respectively.

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* See some observations on this plan in § 166.

† See the chapter entitled *Of Public Credit and the National Debt.*

§ 160. The advantages arising from this method will be, that the National Debt will be put in a state of gradual and *certain* redemption; that many of the inconveniences and losses arising from the present mode of applying the Sinking Fund will be avoided; and that it will be out of the power of any minister to divert it from its intended object. For, on this plan (which I propose, for reasons to be hereafter mentioned, to be adopted during a state of war only), it is probable that Government will not be obliged to redeem any loan at a much higher rate than that at which it was contracted; whereby the loss, arising from an advance in the price of stock, would be obviated,* especially if as much stock were exchanged for a corresponding annuity as might be raised by the loan.

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* If Government were disposed to exchange stock for *life* annuities, I am persuaded that a still greater number of persons would adopt the plan; but as this subject does not form part of the present work, I must defer the consideration of it to another opportunity.

† As the Long Annuities expire in 1860, the latter stock here mentioned might be added to that fund and form one capital; in which case only one new species of annuity would be wanting. Or the terms might be varied still more, and be formed for 10, 20, 30, and 40 years: the present assumption of 30 and 50 years being taken merely by way of example and for the sake of illustration.

‡ Similar to the notice given when the 5 per cents 1797 were exchanged for other stock in October 1804, and April and October 1805.

§ Such price to be estimated from an average of a few preceding days, as was done in the exchange alluded to in the last note.

‖ Any other rate might be taken, according to the wishes of Government, as a bonus to induce the proprietors to accept the exchange. I have taken this sum with a view of explanation only.

¶ In calculating these values, I have considered the annuities as payable *yearly:* they are, however, in all cases payable *half-yearly;* which will make some small difference in their value if calculated on that principle. The table at the end of this section is deduced from the former assumption.

§ 161. But it may perhaps be asked whether the same advantage would not accrue in *raising* the loans by this method, as in *redeeming* them; to which I should reply, that since terminable annuities, particularly for short periods, are generally undervalued in the market, the minister might find it difficult to negotiate a loan for any large amount in this way; whereas many individuals might be induced, for the sake of the bonus, to accept of the terms offered, and this silent and gradual operation would not be so likely to affect the current price of the annuities. The experiment is at least worth a trial, if only on a small scale, as indeed every attempt would be that is likely to put a stop to the present ruinous mode of contracting our debt.

§ 162. I have already pointed out some of the ill effects arising from the present method of applying the Sinking Fund towards the discharge of the National Debt, which arises from the Government being obliged to redeem more than it borrowed. The plan here proposed would in some measure obviate this, as it might be adopted during a state of war only, when the funds are low; by which means such a portion of the debt, as might be subscribed to be exchanged, would be discharged at that low price; since the calculations are made on the supposition that the funds continue at that price during the whole period of 30 or 50 years. Therefore when the 3 per cents are above 65 or 70, perhaps it might not be advisable to pursue this method, because the principal object in view is to secure the low price of the funds.

§ 163. It will be evident that a greater interest will be paid by the public, during the 30 or 50 years, for this terminable annuity, than for the present perpetuities; but this will not cause any additional tax on the country, since the difference may be paid out of the Sinking Fund; it being, in fact, applied to the very purpose for which that fund was established; and, in many cases, the amount of such additional charge will not be equal to the present one per cent Sinking Fund raised on the capital stock created.*

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* See the evil arising from this advance pointed out in § 152, where the probable loss on the loan of 1799 is stated at *five millions sterling*. Had the present plan been adopted in that instance, it would certainly have been redeemed at the end of 30 years by an annuity of £6, 19s. 1d., which is somewhat *less* than its present interest and sinking fund.

§ 164. I may here take the opportunity of answering an objection of Dr. Price's against borrowing money on short annuities. He says, "Some have thought that a good method might be found out of discharging the National Debt by short annuities and life annuities. The following observations will show how groundless an imagination this is. Short annuities and life annuities have been always undervalued by the public, and were they offered to sale to such an amount as would be necessary to make any considerable reduction in the National Debt, they would probably fall to a very low price. Let the 3 per cent stocks be supposed at 86, £100 of this stock would not be given up for a life annuity less than £6, or a short annuity for a less term than 30 years; for this is valuing them at 14⅓ years' purchase, and life annuities, in particular, have never yet been disposed of at so high a price. In order, therefore, to discharge, in this way, a capital in the 3 per cent annuities of 33⅓ millions (the interest of which is a million per annum), it would be necessary to provide a surplus producing another million per annum for 30 years. But this surplus, employed during the same time as a sinking fund, would redeem 47½ millions at par, and 51½ millions at £86 in money for £100 stock. It would therefore be great folly to employ such a surplus in the former way rather than the latter."† Now I by no means agree with him, that £100 of that stock would not be given up for an annuity of £6 for a less term than *thirty* years. The term for which it ought to be given up is but little more than *twenty* years; and it is assuming too much to suppose that the proprietors would insist on its being prolonged half as long again as it ought to be. This being granted, the whole of his subsequent reasoning falls to the ground; for, if the annuity were accepted for *twenty-two* years (which would be a bonus of 5 per cent to the proprietors), the surplus of a million, as above mentioned, employed during that period would redeem only 30½ millions at par, and little more than 32 millions valued at 86. I agree with him, however, that annuities for *very* short terms are too much undervalued by the public to be brought in aid of the present plan;‡ but the same objection will not apply to annuities for 40, 50, or 60 years. I would also observe, that the *price* at which he has taken the stock will not apply to the present method, since I acknowledge that the same advantages will not accrue when the funds are high as when they are low. Indeed, he has stated an *extreme* case *with a view of enforcing his argument*, and to which it would not be politic in any minister to accede; for it will be seen that neither the period for which the annuity is granted, nor the price of the stock, will come within the plan which I have here suggested.

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* That is, in all those cases where the 3 per cents are below 58½.

† *Observ. on Rev. Pay.*, vol. i. p. 287.

‡ They have been more undervalued since the dividends in the funds have been made liable to the Income-tax, owing to the unjust mode of taxing them to the full amount, like perpetuities.

§ 165. The following table will show the annuity for 30 or 50 years respectively that ought to be given in exchange for every £100 3 per cent stock, agreeably to the plan above mentioned, and according to the current price of such stock at the time of the exchange:—

30 Years.		50 Years.	
Price of Stock.	Annuity.	Price of Stock.	Annuity.
50	3·782	50	3·322
60	4·053	60	3·437
70	4·340	70	3·569

VIII.

§ 166. After the observations which I have made in a former part of this work (see § 157), it may perhaps be expected that I should enter a little more at large on the *ruinous* plan of finance introduced by Lord Henry Petty in the course of the last year. This plan, in which there is much *studied* ambiguity and *fallacious* reasoning, is briefly as follows:*— A large loan is contracted for the service of the year, but the minister, either unable to pay the interest of the same, or unwilling to risk his popularity by the imposition of new taxes, ingeniously contrives to avoid both these extremes, for the present, by appropriating from the produce of the war taxes 10 per cent per annum on the amount of such loan, as a fund not only for the payment of the annual interest, but also for the gradual discharge of the principal. This would redeem the loan in about 14 years, the 3 per cents being at 60 *during the whole term* (see § 143): and thus far the plan is unobjectionable, since it is expedient and proper to provide for the redemption of every loan in as speedy a way as possible. But the merits of this part of the project are completely destroyed by the delusive and ruinous scheme which it involves; for, in order to supply the deficiency which is thus made annually in the war taxes,† it is proposed to raise in every succeeding year, till the whole is redeemed, another loan,

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[*] See the *Statement of a Plan of Finance proposed to Parliament in the year* 1807, said to be published from authority.

[†] It is difficult to perceive why this *shifting* of the fund from one subject to another was introduced, unless it was *purposely* to confuse the system.

called a *supplementary* loan, equal in amount to the 10 per cent deducted from the war taxes. These supplementary loans are to be raised with a *one* per cent sinking fund as formerly, and taxes are, from time to time, to be laid on the public to pay the interest, &c., of these supplementary loans only.

§ 167. It must be evident to the most careless observer, that this is not extinguishing the debt, but merely *shifting the mode* and *deferring the time* of its redemption, at an *additional expense* to the nation. The taxes which are to be raised to pay the interest, &c., of these supplementary loans are, during the first years of the plan, so light as not to be an object of much consideration; and after a few years (as these supplementary loans increase in magnitude) *a portion of the sinking fund is mortgaged* for the payment of the interest, &c., of all the future supplementary loans.

This is the first encroachment that has been made on the *present* Sinking Fund, and events will show whether this "sacred blessing and nation's only hope" will not share the fate of all preceding plans which have been adopted for the extinction of our enormous and increasing debt.

§ 168. It is lamentable to see the country pursuing those ruinous measures to which an individual has recourse on the eve of a bankruptcy. Divested of all the extraneous matter with which this plan is encumbered (and it requires no little time and attention to produce anything like simplicity out of it), it can be considered in no other light than as borrowing money at compound interest; and so far from the debt being annihilated in a more speedy manner according to this scheme, I will clearly show that it is *impossible* that any debt can decrease in this way till the original loan has been wholly redeemed, unless we have recourse to such measures as declare to the world the exhausted state of our resources, and are alike fatal to the true interests of the country.*

§ 169. Let the original loan be equal to p, the annual supplementary loan equal to a, and the interest of such supplementary loan, together with its one per cent sinking fund, equal to b. Then will the amount of the unredeemed part of the original loan at the end of n years be represented by $p(1+\rho)^n - a.\frac{(1+\rho)^n-1}{\rho}$; and the amount of the unredeemed part of the supplementary loans, by $(n+1).\frac{b}{\rho} - (\frac{b}{\rho} - a).\frac{(1+\rho)^{n+1}-1}{\rho} - a$. The sum of these two values, or $(p+a)\,(1+\rho)^n + (n+1)\frac{b}{\rho} - \frac{b}{\rho}\,\frac{(1+\rho)^{n+1}-1}{\rho} - a$, will show the total amount of the unredeemed debt at the end of any number of years during the existence of the original loan. But this sum will, for the most part, be continually *increasing* till the time of the final extinction of the original loan,* at which period the unredeemed part of the supplementary loans will *far exceed* the original sum borrowed; and thus the country, instead of finding itself relieved of its burdens, as it was fondly led to imagine, will be encumbered with an *additional* debt without any alleviation of the charge for its redemption.

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* I allude to the encroachments on the Sinking Fund, apparently the ultimate object of the present plan.

§ 170. For example, let us suppose a loan of 12 millions with a sinking fund of 10 per cent on the capital for its redemption, also a supplementary loan of 1,200,000 with a sinking fund of one per cent, on the stock created, for *its* redemption; and let the loan be borrowed in a 3 per cent stock valued at 60. Here we shall have $p=12{,}000{,}000$, $a=1{,}200{,}000$, $b=12{,}000\times(5+1{\cdot}666)=80{,}000$, and $\rho={\cdot}05$. Whence we shall find that the amount† of the unredeemed part of the original loan, of the unredeemed part of the supplementary loans, and of the unredeemed part of the whole accumulated debt, at the end of each year during the existence of the original loan, will be as represented in the following table :‡—

At the end of	Amount of Original Loan.	Amount of Supplemen. Loans.	Total Amount of both Loans united.
1	11 400 000	1 180 000	12 580 000
2	10 770 000	2 338 000	13 108 000
3	10 108 500	3 475 950	13 584 450
4	9 413 925	4 589 747	14 003 672
5	8 684 621	5 679 235	14 363 856
6	7 918 852	6 743 197	14 662 049
7	7 114 795	7 780 356	14 895 151
8	6 270 535	8 789 374	15 059 909
9	5 384 061	9 768 843	15 152 904
10	4 453 264	10 717 285	15 170 549
11	3 475 928	11 633 149	15 109 077
12	2 449 724	12 514 807	14 964 531
13	1 372 210	13 360 547	14 732 757
14	240 821	14 168 575	14 409 396

§ 171. I have here taken an example of *one* year's loan only; but if we suppose that in each *successive* year a loan is borrowed upon similar principles, it must be evident that the loss will be considerably multiplied; and, if persisted in for any length of time, it will be impossible ever to redeem any loan without a very material encroachment on the sinking fund.

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* The time in which any original loan will be redeemed by a series of supplementary loans of this kind may be found by the formula in § 142.

† The amounts here alluded to mean the *money* capital, and not the capital *stock*.

‡ The original loan would be finally extinguished in 14·206699 years, at which period the amount of the unredeemed part of the supplementary loans will be £14,330,718, or £2,330,718 *more* than the sum first borrowed, and which this series of supplementary loans was vainly intended to annihilate.

I have purposely omitted any allusion to the advantages which this system may afford to commercial speculations and pursuits, the energies it may give to our manufactures and agriculture, or the *inconvenience* which may arise at a future period from a too *rapid* reduction of the debt!!! as these broad assertions and commonplace remarks may (like texts of Scripture) be brought to suit any specious proposal. Neither do I know upon what principle it can be presumed that *posterity* will be capable of supporting that load of taxation from which *we* so timorously recoil. My observations are intended merely to show that no plan of finance will be effectual towards discharging our present enormous debt which has not for its object one or both of these two things: either the exchange of the perpetuities for terminable annuities, or the establishment of a PERMANENT sinking fund, which shall be *sacredly* and *inviolably* appropriated for the redemption of its original debt, and "to no other use, intent, or purpose whatsoever." All other schemes are (in the words of a celebrated writer) merely "palliatives which give present relief by increasing future danger, or expedients which postpone a public bankruptcy by rendering it a calamity more unavoidable and dreadful."

APPENDIX

(NOTE A.)

§ 172. IN § 58 I have given a formula for finding the value of ρ, or rate of interest, in the *amount* of annuities; and have referred the reader to the Appendix for an investigation not only of this formula, but of others that have been given by different writers on this subject for determining the same quantity.

§ 173. The primary equation, from which all these formulæ are deduced, is (by § 58) $s = a \times \frac{(1+\rho)^n - 1}{\rho}$; which, in order to avoid circumlocution, I shall call the equation A. If we multiply both sides of this equation by $\frac{\rho}{a}$, and afterwards add unity to each side, we shall have the equation B, $(1+\rho)^n = 1 + \frac{s}{a} \cdot \rho$. Now the quantity ρ is so involved in this equation that its true value in finite terms cannot be expressed by any of the known rules of algebra:* nevertheless, this does not prevent us from approximating to its true value, and to any degree of exactness. For if we expand $(1+\rho)^n$ by means of the binomial theorem into an infinite series ascending by the powers of ρ, and substitute this series in the above equation instead of $(1+\rho)^n$, we shall have the equation C, $1 + n\rho + \frac{n}{1} \cdot \frac{n-1}{2} \cdot \rho^2 + \frac{n}{1} \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \rho^3 + \frac{n}{1} \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \rho^4 + \frac{n}{1} \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \frac{n-4}{5} \cdot \rho^5 + \&c. = 1 + \frac{s}{a} \cdot \rho$; which, by subtracting unity from both sides, and then dividing by $n\rho$, forms the equation D, $1 + \frac{n-1}{2} \cdot \rho + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \rho^2 + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \rho^3 + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \frac{n-4}{5} \cdot \rho^4 + \&c. = \frac{s}{an}$. If we again subtract unity from both sides, and then divide by $\frac{n-1}{2}$, we shall have the equation E, $\rho+\frac{n-2}{3}\cdot\rho^2+\frac{n-2}{3}\cdot\frac{n-3}{4}\cdot\rho^3+\frac{n-2}{3}\cdot\frac{n-3}{4}\cdot\frac{n-4}{5}\rho^4+\&c.=\frac{2(s-an)}{an(n-1)}$. The use and application of these several equations I shall now proceed to show.

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* Such an expression is, in the language of modern algebraists, called an *implicit function* of ρ.

§ 174. There are several methods given, by writers on this subject, for approximating towards the roots of the higher equations,* of which the principal is that by trial and error; and which consists in taking a near value for the root, and correcting the difference between this and the true root, till at last we arrive at the root required. In the doctrine of annuities, however, this process may be much abbreviated, because we know that, in general, a near value of the root of the exponential in the given equation is unity, since the rate of interest, or ρ, is in most cases some small quantity below unity, such as ·02, ·03, ·04, ·05, &c.; and therefore it is manifest that the series which arises from the expansion of such exponential quantity will be a swiftly converging one, particularly when n, or the index of such quantity, is not very great; so that a few of the leading terms will be sufficient for determining, to a tolerable degree of accuracy, the value of the quantity p. It is true that this value may be found by inverting the series agreeably to the method laid down by Sir Isaac Newton; but as other rules more expeditious and easy in practice have been invented, I shall endeavour to explain them as concisely and clearly as possible.

§ 175. I shall first investigate the formula given by Mr. Thomas Simpson,† which is this:—"Assume $Q=\frac{an(n-1)}{2(s-an)}$, then will $\frac{3000Q+(2n-1)400}{6Q(5Q+3n-4)+\frac{1}{6}(n-2)(11n-13)}$ be the rate per cent required." This formula is deduced from the equation E, agreeably to the method laid down by Mr. Simpson himself for the resolution of equations by approximation, in pages 162-165 of his *Treatise of Algebra*. For, he has there shown that in a general equation of the form $z+bz^2+cz^3+dz^4+\&c.=q$ if we make $\sigma=b-\frac{c}{b}$, and $\omega=2b+\frac{d-bc}{b^2-c}$, the value of z will be nearly equal to $\frac{(1+\omega q)q}{1+(b+\omega)q+(\omega-\sigma)bq^2}$.* Now, if we compare this general equation with the equation E, $\rho+\frac{n-2}{3}\cdot\rho^2+\frac{n-2}{3}\cdot\frac{n-3}{4}\cdot\rho^3+\frac{n-2}{3}\cdot\frac{n-3}{4}\cdot\frac{n-4}{5}\cdot\rho^4+\&c.=\frac{2(s-an)}{an(n-1)}$ we shall have $b=\frac{n-2}{3}$, $c=\frac{n-2}{3}\cdot\frac{n-3}{4}$, $d=\frac{n-2}{3}\cdot\frac{n-3}{4}\cdot\frac{n-4}{5}$, $\sigma=\frac{n+1}{12}$, $\omega=\frac{2(2n-1)}{15}$, and

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* On this subject the reader may consult Sir I. Newton's *Geom. Anal.*, cap. 2; Dr. Halley in the *Phil. Trans.* for 1694, No. 210; Raphson's *Anal. Æquat. Univ.*; Maclaurin's *Algebra*, c. 9; Thomas Simpson, in his *Algebra*, p. 158; in his *Select Exercises*, p. 215; in his *Essays*, p. 82; and in his *Dissertations*, p. 102.

† In his *Treatise of Algebra*, p. 239; and in the Supplement to his *Doctrine of Annuities*, p. 87.

$q=\frac{1}{Q}=\frac{2(s-an)}{an(n-1)}=\frac{2\left(\frac{s}{a}-n\right)}{n(n-1)}$; consequently the general formula becomes

$$\frac{\left(1+\frac{2(2n-1)}{15}\cdot\frac{1}{Q}\right)\frac{1}{Q}}{1+\left(\frac{n-2}{3}+\frac{2(2n-1)}{15}\right)\frac{1}{Q}+\left(\frac{2(2n-1)}{15}-\frac{n+1}{12}\right)\frac{n-2}{3}\cdot\frac{1}{Q}};$$

and this, by multiplying both the numerator and the denominator by $30\,Q^2$, becomes $\frac{30Q+(2n-1)4}{6Q(5Q+3n-4)+\frac{1}{6}(n-2)(11n-13)}$, which is very near the true value of ρ when, n is not very great. It will be here seen that Mr. Simpson has multiplied *his* formula by 100, so as to give the rate *per cent*, which, however, is quite useless, as we have only to remove the decimal point two places to the right when the rate *per cent* is required. As an example of the use and application of this formula, let us take the following question, viz. :—If an annuity of £100 per annum for 25 years amount to £4772·710, what rate of interest has been allowed? Here we have $n=25$, $a=100$, $s=4772{\cdot}710$, and $\frac{s}{a}=\frac{4772{\cdot}710}{100}=47{\cdot}7271$; whence $Q=\frac{25\times24}{2(47{\cdot}7271-25)}=\frac{600}{45{\cdot}4542}=13{\cdot}2001$, and

$$\rho=\frac{30\times13{\cdot}2001+49\times4}{6\times13{\cdot}2001\,(5\times13{\cdot}2001+75-4)+\frac{1}{6}(275-13)\times23}=\frac{396{\cdot}003+196}{10850{\cdot}522+1004{\cdot}333}=\frac{592{\cdot}003}{11854{\cdot}855}={\cdot}049938;$$

making a difference of ·000062 from the *true* rate, which is exactly ·05.

§ 176. The next formula that I shall investigate is that given by Dr. Halley,† and which is as follows, viz. :—"Assume $\beta=\left(\frac{s}{an}\right)^{\frac{2}{n-1}}-1$, and $\gamma = \frac{6}{n+1}$; then will $(\gamma^2 + 2\gamma\beta)^{\frac{1}{2}} - \gamma$ be very near the rate required." This formula is deduced from the equation D, and is obtained by raising both sides of that equation to the power indicated by $\frac{2}{n-1}$. Now, in order to raise the series $1 + \frac{n-1}{2} \cdot \rho + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \rho^2 + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \rho^3 + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \frac{n-4}{5} \cdot \rho^4 + \&c.$, to such power, we must have recourse to M. De Moivre's multinomial theorem; and the reason we are directed to raise the series to this index in preference to any other is, that, being equal to the reciprocal of the co-efficient of the first power of ρ, it renders the co-efficient of that power in the resulting series equal to unity; and consequently the approximation towards the value of ρ is rendered so much the more simple and easy. The multinomial theorem here alluded to is as follows:*—Let the original series be $1 + bz + cz^2 + dz^3 + ez^4 + \&c.$, then will $(1 + bz + cz^2 + dz^3 + ez^4 + \&c.)^m$ be expressed by the following series, ascending still by the powers of z, namely:—

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* I have altered Mr. Simpson's notation, lest the characters which he makes use of might be confounded with the letters expressing *other* quantities in the present work. Further, it will be readily seen that the quantity which he denotes by a is here always equal to unity, and therefore may be omitted in the process.

† This formula generally goes by the name of Dr. Halley's formula, because it was first made public in Dr. Halley's *Discourse on Compound Interest*, inserted in the introduction to Sherwin's *Mathematical Tables*, which were published in 1705.

$$1 + mb.z + \left.\begin{array}{l} \frac{m}{1} \cdot \frac{m-1}{2} . b^2 \\ + mc \end{array}\right\} . z^2 + \left.\begin{array}{l} \frac{m}{1} \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} . b^3 \\ + m.(m-1).bc \\ + mb \end{array}\right\} . z^3 + \left.\begin{array}{l} \frac{m}{1} \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} \cdot \frac{m-3}{4} . b^4 \\ + \frac{m}{1} \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} . b^2 c \\ + m.(m-1).bd \\ + \frac{m}{1} \cdot \frac{m-1}{2} . c^2 \\ + me \end{array}\right\} . z^4 + \&c.$$

Therefore if we compare this with our present series, we shall have $b = \frac{n-1}{2}$, $c = \frac{n-1}{2} \cdot \frac{n-2}{3}$, $d = \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4}$, $e = \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \frac{n-4}{5}$ &c., $m = \frac{2}{n-1}$ and $z = \rho$; consequently the $\frac{2}{n-1}$th power of the series $1 + \frac{n-1}{2} \cdot \rho + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \rho^2 + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \rho^3 + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \frac{n-4}{5} \cdot \rho^4 + \&c.$, will be $1 + \rho + \frac{n+1}{12} \cdot \rho^2 - \frac{n^3 - 4n^2 + n + 6}{1440} \cdot \rho^4 + \&c.$ That is, the equation $(1 + \frac{n-1}{2} \cdot \rho + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \rho^2 + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \rho^3 + \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \frac{n-4}{5} \cdot \rho^4 + \&c.)^{\frac{2}{n-1}} = \left(\frac{s}{an}\right)^{\frac{2}{n-1}}$ will become (after reducing the several co-efficients of the powers of ρ in the resulting series, to their lowest terms) the equation F, $1+\rho+\frac{n+1}{12}\cdot\rho^2=\left(\frac{s}{an}\right)^{\frac{2}{n-1}}$. For, it fortunately happens that the co-efficients of the third power of ρ in this equation destroy one another, consequently the fourth term is equal to nothing; and since the fifth and all the subsequent terms are of very little value, particularly when n is not very great, we shall find that the first three terms will nearly express the value of the whole series, and we shall also find that this equation (reduced as it now is, by this operation, to so few terms) will be more easily resolved than any of the other preceding ones. Having, therefore, by subtracting unity from both sides of the equation F, obtained $\rho+\frac{n+1}{12}\cdot\rho^2=\left(\frac{s}{an}\right)^{\frac{2}{n-1}}-1$, let us make $\left(\frac{s}{an}\right)^{\frac{2}{n-1}}-1=\beta$; and then we shall form the equation G, $\rho+\frac{n+1}{12}\cdot\rho^2=\beta$. If this be multiplied by $\frac{12}{n+1}$, it will become $\rho^2+\frac{12}{n+1}\cdot\rho=\frac{12}{n+1}\cdot\beta$; and lastly, if we make $\frac{6}{n+1}=\gamma$, this equation will become $\rho^2+2\gamma\rho=2\gamma\beta$, which is the one from which Dr. Halley's formula is immediately derived; for, if we extract the root of this quadratic equation we shall have $\rho=(\gamma^2+2\gamma\beta)^{\frac{1}{2}}-\gamma$, which is the very formula alluded to. But this formula may be rendered more convenient for practice without the help of the new character γ; for by retaining its equal, $\frac{6}{n+1}$, we have $\rho=\left[\frac{36}{(n+1)^2}+\frac{12}{n+1}\cdot\beta\right]^{\frac{1}{2}}-\frac{6}{n+1}=$ $\frac{1}{n+1}\times[36+12.(n+1).\beta]^{\frac{1}{2}}-\frac{6}{n+1}=\frac{\sqrt{36+12.(n+1).\beta}-6}{n+1}$.

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* See De Moivre's *Miscellanea Analytica,* p. 87; or Lacroix *Calcul. Diff. et Integ.*, vol. i. p. 32.

As an example of the use and application of this formula, let us take the following question: viz., If an annuity of £100 per annum for 50 years amount to £20934·800, what is the rate of interest that has been allowed? Here we have $n=50$, $a=100$, $s=20934{\cdot}800$, and $\frac{s}{a}=$ $\frac{20934{\cdot}800}{100}=209{\cdot}348$; whence $\left(\frac{s}{an}\right)^{\frac{2}{n-1}}-1=\beta=\left(\frac{209{\cdot}348}{50}\right)^{\frac{2}{49}}-1=$ $\cdot06019$, and $\rho=\frac{\sqrt{36+12\times51\times\cdot06019}-6}{51}=\frac{\sqrt{72{\cdot}83628}-6}{51}=\frac{2{\cdot}53441}{51}$ $=\cdot049694$, which is within ·000306 of the true rate, or 5 per cent.

§ 177. Of these two formulæ for finding the value of ρ, that of Mr. Simpson is obtained in the most simple manner, though it does not give the value so exact as that of Dr. Halley; this latter, however, is troublesome in practice, since it involves the square root of a binomial, the extraction of which is not always performed with ease or expedition;

and, indeed, Mr. Simpson's formula is very laborious and intricate for common purposes, as any one will easily discover in working out a few examples. These circumstances induced me to re-consider the equations, and endeavour to find some other formula which might not only be more readily solved, but express the value with an equal degree of accuracy. The result of my inquiry is the formula given in the present work, page 38, which, at the same time that it is more easily resolved in numbers, expresses the value of ρ to a much greater degree of exactness than any I have yet seen. The manner in which it is obtained is by a union of the two methods pursued by Mr. Simpson and Dr. Halley in deducing their formulæ, and the investigation is as follows:—

§ 178. Let us pursue the steps just laid down in deducing Dr. Halley's formula till we come to the equation G, and then instead of resolving that quadratic equation by extracting the root, let us approximate towards such root agreeably to the rule laid down by Mr. Simpson for approximating towards the roots of equations, in page 164 of his *Treatise of Algebra;* where he has shown that in a general equation of the form $z+bz^2+cz^3+$ &c.,$=q$, if we make $\sigma=b-\frac{c}{b}$, the value of z will be nearly equal to $\frac{(1+\sigma q)q}{1+(b+\sigma)q}$. Therefore, if we compare the equation G, $\rho+\frac{n+1}{12}.\rho^2=\beta$, with this general equation, we shall have $b=\frac{n+1}{12}$, $c=0$, $\sigma=b$, $q=\beta$, and $z=\rho$; consequently the general formula becomes $\rho=$ $\frac{(1+\frac{n+1}{12}.\beta).\beta}{1+\frac{2(n+1)}{12}.\beta}=\frac{[12+(n+1)\beta].\beta}{12+2.(n+1)\beta}$, which is the formula alluded to.

§ 179. I shall now subjoin a few examples, in numbers, of each of these three rules for finding the value of ρ, in order that the reader may compare them readily together, and be thereby enabled to judge of the degree of accuracy which each of them possesses. It may be necessary to premise, that the true rate in all these examples is exactly ·05, or 5 per cent, which I have chosen as a proper medium rate for showing the relative advantages of the three formulæ. The first column denotes the value of n or the number of years during which the annuity is to continue, and the value of $\frac{s}{a}$ in the general equation is taken from Table V., under 5 per cent, and opposite to the given number of years.

Years.	Simpson's Formula.	Halley's Formula.	Baily's Formula.
10	·049999	·049997	·050002
25	·049938	·049956	·050007
50	·048943	·049694	·050003
75	·045524	·049158	·049975
100	·040015	·048401	·049925

From this table it will be seen that, even when the term is 100 years, and the rate of interest not more than 5 per cent, my formula will express such rate almost to a ten-thousandth part of its true value; whereas Mr. Simpson's formula, in such case, would not express it within *one per cent* of the true value.

§ 180. It will be readily perceived that the lower the rate of interest, and the smaller the number of years, the nearer will either of these formulæ express the value of ρ; and, on the contrary, that the higher the rate of interest and the greater the number of years, the farther will they diverge from the true value: because not only the powers of ρ (in the equations D and E, from which these formulæ are deduced), but also the co-efficients of such powers increase in proportion to the value of n and ρ; and, indeed, the value of n and ρ may in some instances be so great that neither of the formulæ will bring out the value sufficiently exact, which arises from our rejecting all the terms affected with the higher powers of ρ as being of little or no value, whereas they ought (if it could conveniently be done) to be admitted into the calculation;* but as there is no mode of doing this without at the same time rendering the formulæ much more complex than they are at present, we must rest satisfied with them as they now stand. When the rate of interest is under 10 per cent, and the number of years not greater than 100, the formula which I have given in page 40 will always give the value of ρ true to the *fourth* place, and in most cases even to the *sixth* place of decimals, which is as great a degree of accuracy as need be desired.

§ 181. For common purposes, where great exactness is not required, we may in general discover the rate of interest sufficiently near by inspection of Table V. at the end of this work, where, by comparing the value of $\frac{s}{a}$ with the values in the table against the given number of years, we may find the value of ρ tolerably exact, or at least sufficiently so as to serve as a foundation for a further approximation, which I shall shortly mention. For example; suppose an annuity of £100 per annum amounts to £3800 in 20 years, then will $\frac{s}{a}$ be equal to $\frac{3800}{100}=38$. Now the value of $\frac{s}{a}$ in the same time at 6 per cent interest is 36·785, and at 7 per cent interest it is 40·995; we therefore know that the true rate must lie somewhere between ·06 and ·07; and as 38 exceeds 36·785 by something more than ¼ of the difference between these two values, we may conclude that the rate is rather more than 6¼ per cent; which we find on trial to be the case, as the true rate is ·063029, or about £6, 6s. per cent. As this method is attended with very little trouble, we may often resort to it with advantage, particularly when we wish to find a first near value of ρ to serve as a ground for a further approximation. For when we have obtained a near value, either by conjecture, or by any of the methods above laid down, we may correct the difference between this and the true value till we arrive at *any degree* of accuracy required. The formula for correcting this value I shall now endeavour to explain.

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* When n, or the number of years, is 25, and the rate of interest 5 per cent, the fifth term in the equation F (were it continued) would be $\frac{n^3-4n^2+n+6}{1440}.\rho^4 = \frac{15625-2500+25+6}{1440} \times \cdot00000625 = 9\cdot1361 \times 00000625 = \cdot0000571$, a quantity which we may very properly reject as of little or no value; but if the number of years were 100 instead of 25, this term would become $\frac{1000000-40000+100+6}{1440} \times \cdot00000625 = 666\cdot7361 \times \cdot00000625 = \cdot0041671$.

§ 182. Let R be assumed as somewhat near the true value of ρ, and let x be the unknown difference between these two values, then will $\text{R}+x=\rho$, and by substituting $\text{R}+x$ for ρ in the general equation B, $(1+\rho)^n=1+\frac{s}{a}\cdot\rho$, such equation will become $(1+\text{R}+x)^n=1+\frac{s}{a}\cdot\text{R}+\frac{s}{a}\cdot x$. If we expand $[(1+\text{R})+x]^n$ by means of the binomial theorem, into a series ascending by the powers of x, it will become $(1+\text{R})^n+n(1+\text{R})^{n-1}x+\frac{n(n-1)}{2}.(1+\text{R})^{n-2}x^2+$ &c. Now, since x is generally some *very* small quantity, it is evident that this series must converge very swiftly; we may therefore reject all the terms affected with the second, third, and higher powers of x, as inconsiderable in respect to the rest, and take $(1+\text{R})^n+n(1+\text{R})^{n-1}x$ for the value of the whole series; consequently the equation becomes $(1+\text{R})^n+n(1+\text{R})^{n-1}x=1+\frac{s}{a}\cdot\text{R}+\frac{s}{a}\cdot x$, whence $x=\dfrac{1+\frac{s}{a}\cdot\text{R}-(1+\text{R}^n)}{n(1+\text{R})^{n-1}-\frac{s}{a}}$.* If R has been assumed pretty near the true value of ρ, then will $R+x$ express the value of ρ to double the number of places that were true in R; and by substituting this value instead of R, and repeating the process, we shall have the value of ρ twice corrected; and thus we may proceed till we have arrived at any degree of accuracy required.

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* It is evident on inspection that the numerator of this fraction consists of the same

As an example of this method of correcting the value of ρ, let us suppose the amount of an annuity of £20 per annum for 30 years to be £1600; consequently $n=30$, and $\frac{s}{a}=\frac{1600}{20}=80$; and let R, or the assumed near value of ρ, be ·06, taken either by conjecture or from any of the methods of approximation above mentioned;* then will $x=\frac{1+80\times{\cdot}06-(1{\cdot}06)^{30}}{30(1.06)^{29}-80}=\frac{5{\cdot}8-5{\cdot}74349117}{30\times5{\cdot}41838790-80}=\frac{{\cdot}05650883}{82{\cdot}55163700}={\cdot}00068452$; whence $\rho=R+x={\cdot}06068452$. If this value of ρ should not be conceived to be sufficiently exact, we may make a second correction of it, by substituting it instead of ·06, or R, in the equation above given, and repeating the process. Thus, by making $R={\cdot}06068452$, we shall have $x=\frac{1+80\times{\cdot}06068452-(1{\cdot}06068452)^{30}}{30(1{\cdot}06068452)^{29}-80}=\frac{5{\cdot}8547616-5{\cdot}85580927}{30\times5{\cdot}5207834-80}=\frac{-{\cdot}00104767}{85{\cdot}6235020}=-{\cdot}000012236$; which being negative shows that R has been in this case taken too great, and that the value of x must be subtracted; consequently we have for the second correction $\rho=R+x={\cdot}060684520-{\cdot}000012236={\cdot}060672284$, which is true to the last figure.

§ 183. In this manner we may find the value of ρ in the amount of annuities to any degree of accuracy; and though the operations may appear tedious and intricate, there is no mode by which they can be simplified or abridged. By always operating, however, agreeably to one formula, the steps of the process will soon be rendered easy and familiar, and the results obtained without much trouble or difficulty.

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terms and quantities as the equation B, except that the assumed value R is substituted for the true value ρ; and it will readily occur to the experienced analyst that the denominator is equal to the *differential co-efficient* of such assumed equation, supposing R variable. Consequently the rule for correcting the value of ρ may be concisely stated thus: Let the assumed value of ρ be substituted in the general equation, and let the error thence arising be divided by the differential co-efficient of such equation, the quotient, added to the assumed value, will give the value of ρ corrected.—See on this subject, Simpson's *Mathematical Essays*, p. 81, and Lacroix, *Calcul. Differ. et Integ.*, vol. i. p. 546.

* This first near value of ρ is found by inspection of the table, agreeably to the method which I have just laid down; if, however, the formula which I have given in page 38 be made use of, the value of ρ will come out ·06069, which is sufficiently near without any correction.

§ 184. I shall close this note by bringing into one view the three formulæ above mentioned, in order that the reader may more readily refer to them, or compare them together. I shall likewise subjoin the formula for correcting the value of ρ thus obtained, if on trial it should not be found sufficiently exact.

$$\rho=\frac{30Q+(2n-1)4}{6Q(5Q+3n-4)+\frac{1}{6}(11n-13)(n-2)} \quad \text{where } Q=\frac{n(n-1)}{2\left(\frac{s}{a}-n\right)}$$

$$\left.\begin{array}{l} \rho=\dfrac{\sqrt{36+12(n+1)\beta}-6}{n+1} \\ \rho=\dfrac{[12+(n+1)\beta].\beta}{12+2(n+1)\beta} \end{array}\right\} \text{where } \beta=\left(\frac{s}{an}\right)^{\frac{2}{n-1}}-1$$

$$\rho=\text{R}+\frac{1+\frac{s}{a}\cdot\text{R}-(1+\text{R})^{n}}{n(1+\text{R})^{n-1}-\frac{s}{a}}$$

where R denotes any near value of ρ taken either by trial or conjecture.

◆

(NOTE B.)

§ 185. In the preceding note I have investigated the several formulæ which have been given for finding the rate of interest in the amount of annuities: my object in the present note is to pursue the same course with respect to those formulæ which have been given for finding such rate in the *present value* of annuities. And it will readily be seen that there is a great similarity in the method of deduction.

§ 186. The primary equation from which they are all deduced is (by § 72) $p=a\times\frac{1-(1+\rho)^{-n}}{\rho}$; which I shall call the equation A, and which, by multiplying both sides by $-\frac{\rho}{a}$, and afterwards adding unity thereto, becomes the equation B, $(1+\rho)^{-n}=1-\frac{p}{a}\cdot\rho$. If we expand $(1+\rho)^{-n}$ by means of the binomial theorem into an infinite series ascending by the powers of ρ, and substitute such series in this equation, we shall have the equation C, $1-n\rho+\frac{n}{1}\cdot\frac{n+1}{2}\cdot\rho^{2}-\frac{n}{1}\cdot\frac{n+1}{2}\cdot\frac{n+2}{3}\cdot\rho^{3}+\frac{n}{1}\cdot\frac{n+1}{2}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}\cdot\rho^{4}-\frac{n}{1}\cdot\frac{n+1}{2}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}\cdot\frac{n+4}{5}\cdot\rho^{5}+\&c.=1-\frac{p}{a}\cdot\rho$; which, by subtracting unity from both sides, and dividing by $-n\rho$, forms the equation D, $1-\frac{n+1}{2}\cdot\rho+\frac{n+1}{2}\cdot\frac{n+2}{3}\cdot\rho^2-\frac{n+1}{2}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}\cdot\rho^3+\frac{n+1}{2}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}\cdot\frac{n+4}{5}\cdot\rho^4-\&c.=\frac{p}{an}$. If we again subtract unity from both sides, and divide by $-\frac{n+1}{2}$, we shall have the equation E, $\rho-\frac{n+2}{3}\cdot\rho^2+\frac{n+2}{3}\cdot\frac{n+3}{4}\cdot\rho^3-\frac{n+2}{3}\cdot\frac{n+3}{4}\cdot\frac{n+4}{5}\cdot\rho^4+\&c.=\frac{2(an-p)}{an(n+1)}$: the use and application of which equations will be shown hereafter.

§ 187. I shall first of all investigate the formula given by Mr. Thomas Simpson,* which is this: "Assume $Q=\frac{an(n+1)}{2(an-p)}$ so shall $\frac{3000Q-(2n+1)400}{6Q(5Q-3n-4)+\frac{1}{6}(n+2).(11n+13)}$ express the rate per cent very nearly." This formula is deduced from the equation E, agreeably to the method laid down by Mr. Simpson himself for the resolution of equations by approximation, already quoted in the preceding note, § 175. For if we compare the equation E, $\rho-\frac{n+2}{3}\cdot\rho^2+\frac{n+2}{3}\cdot\frac{n+3}{4}\cdot\rho^3-\frac{n+2}{3}\cdot\frac{n+3}{4}\cdot\frac{n+4}{5}\cdot\rho^4+\&c.=\frac{2(an-p)}{an(n+1)}$ with the general equation there given (viz., $z+bz^2+cz^3+dz^4+\&c.=q$), we shall have $b=-\frac{n+2}{3}$, $c=\frac{n+2}{3}\cdot\frac{n+3}{4}$, $d=-\frac{n+2}{2}\cdot\frac{n+3}{4}\cdot\frac{n+4}{5}$, $\sigma=b-\frac{c}{b}=-\frac{n-1}{12}$, $\omega=2b+\frac{d-bc}{b^2-c}=-\frac{2(2n+1)}{15}$, and $q=\frac{1}{Q}=\frac{an(n+1)}{2(an-p)}=\frac{n(n+1)}{2(n-\frac{p}{a})}$; consequently, the general formula

$$\frac{(1+\omega q)q}{1+(b+\omega)q+(\omega-\sigma)bq^2}$$ becomes

$$\frac{(1-\frac{2(2n+1)}{15}\cdot\frac{1}{Q})\times\frac{1}{Q}}{1-\left(\frac{n+2}{3}+\frac{2(2n+1)}{15}\right)\frac{1}{Q}+\left(\frac{2(2n+1)}{15}-\frac{n-1}{12}\right)\cdot\frac{n+2}{3}\cdot\frac{1}{Q^2}};$$ and this, by multiplying both the numerator and the denominator by $30Q^2$, will become $\frac{30Q-(2n+1)4}{6Q(5Q-3n-4)+\frac{1}{6}(11n+13)\cdot(n+2)}$, which expresses a near value of ρ, when n or the number of years is not very great; and this is the same as Mr. Simpson's formula, except that he has multiplied his by 100, in order to express the rate *per cent*, which is needless, as we have only to remove the decimal point, in such case, two places to the right.

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* In his *Treatise of Algebra*, p. 240; and in his *Doctrine of Annuities, Sup.* p. 88.

As an example of the use and application of this formula, let us take the following question, viz.:—If an annuity of £100 per annum, for 25 years, be purchased for £1409·394, what is the rate of interest allowed? Here we have $n = 25$, $a = 100$, $p = 1409{\cdot}394$, and $\frac{p}{a} = \frac{1409{\cdot}394}{100} = 14{\cdot}09394$; whence $\text{Q} = \frac{25 \times 26}{2 \times 10{\cdot}90606} = \frac{650}{21{\cdot}81212} = 29{\cdot}80$; and $\rho = \frac{894 - 204}{178{\cdot}8 \times (149 - 79) + \frac{1}{6}(288 \times 27)} = \frac{690}{12516 + 1296} = \frac{690}{13812} = {\cdot}049957$. Now the true rate in this example is exactly ·05, so that the formula gives the value of ρ in this case within ·000043 of its real value.

§ 188. I shall next investigate the formula given by Dr. Halley, which is as follows:—"Assume $\beta = \left(\frac{an}{p}\right)^{\frac{2}{n+1}} - 1$, and $\gamma = \frac{6}{n-1}$; then will $\gamma - (\gamma^2 - 2\gamma\beta)^{\frac{1}{2}}$ be very near the rate required." This formula is deduced from the equation D, and is obtained by raising both sides of that equation to the power indicated by $-\frac{2}{n+1}$; which index is made choice of, because it is equal to the reciprocal of the co-efficient of the first power of ρ, and (by the method adopted for raising the series to the required power) renders that co-efficient in the resulting series equal to unity; a circumstance which simplifies very much the formula thence deduced for the purpose of a subsequent approximation. Now, in order to raise the series, which forms the left side of this equation, to the given power, we must have recourse to M. De Moivre's multinomial theorem, as already explained in the preceding note, § 176. By a comparison of that theorem with our present series, we shall have $b = -\frac{n+1}{2}$, $c = \frac{n+1}{2} \cdot \frac{n+2}{3}$, $d = -\frac{n+1}{2} \cdot \frac{n+2}{3} \cdot \frac{n+3}{4}$, $e = \frac{n+1}{2} \cdot \frac{n+2}{3} \cdot \frac{n+3}{4} \cdot \frac{n+4}{5}$, and $m = -\frac{2}{n+1}$: consequently the $-\frac{2}{n+1}$th power of the series $1 - \frac{n+1}{2} \cdot \rho + \frac{n+1}{2} \cdot \frac{n+2}{3} \cdot \rho^2 - \frac{n+1}{2} \cdot \frac{n+2}{3} \cdot \frac{n+3}{4} \cdot \rho^3 + \frac{n+1}{2} \cdot \frac{n+2}{3} \cdot \frac{n+3}{4} \cdot \frac{n+4}{5} \cdot \rho^4 - \&\text{c.}$, will be $1 + \rho - \frac{n-1}{12} \cdot \rho^2 + \frac{n^3 + 4n^2 + n - 6}{1440} \cdot \rho^4 + \&\text{c.}$; that is, the equation $\left(1 - \frac{n+1}{2} \cdot \rho + \frac{n+1}{2} \cdot \frac{n+2}{3} \cdot \rho^2 - \frac{n+1}{2} \cdot \frac{n+2}{3} \cdot \frac{n+3}{4} \cdot \rho^3 + \&\text{c.}\right)^{-\frac{2}{n+1}} = \left(\frac{p}{an}\right)^{-\frac{2}{n+1}} = \left(\frac{an}{p}\right)^{\frac{2}{n+1}}$*

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* The *negative* power of any quantity is equal to the same *positive* power of the reciprocal of that quantity; for $\left(\frac{p}{a}\right)^{-m} = \frac{1}{\left(\frac{p}{a}\right)^m} = \frac{1}{\frac{p^m}{a^m}} = \frac{a^m}{p^m} = \left(\frac{a}{p}\right)^m$.

will become (after reducing the several co-efficients of the powers of ρ in the resultant series to their lowest terms) the equation F, $1+\rho-\frac{n-1}{12}\cdot\rho^2=\left(\frac{an}{p}\right)^{\frac{2}{n+1}}$: for it fortunately happens that in this formula (as in the one in page 118) the co-efficient of the third power of ρ vanishes; consequently the fourth term is equal to nothing; and since the fifth and all the subsequent terms are but small in comparison to the rest when n is not very great, we may take the first three terms to denote the value of the whole series. Whence we have $\rho-\frac{n-1}{12}\cdot\rho^2=\left(\frac{an}{p}\right)^{\frac{2}{n+1}}-1$; and by making $\left(\frac{an}{p}\right)^{\frac{2}{n+1}}-1=\beta$, we shall have the equation G, $\rho-\frac{n-1}{12}\cdot\rho^2=\beta$: if we multiply both sides by $-\frac{12}{n-1}$, we shall have $\rho^2-\frac{12}{n-1}\cdot\rho=-\frac{12}{n-1}\cdot\beta$; and lastly, by making $\gamma=\frac{6}{n-1}$, we shall have $\rho^2-2\gamma\rho=-2\gamma\beta$, which is the very equation from which Dr. Halley's formula is immediately derived; for, by extracting the root of this quadratic, we have $\rho=\gamma-(\gamma^2-2\gamma\beta)^{\frac{1}{2}}$, which is the formula alluded to. But this formula may be rendered more convenient for practice without the help of the new character γ; for, by retaining its equal, $\frac{6}{n-1}$, we have $\rho=\frac{6}{n-1}-\left[\frac{36}{(n-1)^2}-\frac{12\beta}{n-1}\right]^{\frac{1}{2}}=\frac{6}{n-1}-\frac{1}{n-1}\times[36-12(n-1)\beta]^{\frac{1}{2}}=\frac{6-\sqrt{36-12.(n-1)\beta}}{n-1}$.

As an example of the use and application of this formula, let us take the following question, viz.:—If an annuity of £100 per annum for 50 years be purchased for £1825·592, what is the rate of interest allowed? Here we have $n=50$, $a=100$, and $p=1825{\cdot}592$; whence $\beta=\left(\frac{100\times50}{1825{\cdot}592}\right)^{\frac{2}{51}}-1=\left(\frac{5000}{1825{\cdot}592}\right)^{\frac{2}{51}}-1={\cdot}040302$, and $\rho=\frac{6-\sqrt{36-12\times49\times{\cdot}040302}}{49}=\frac{6-\sqrt{12{\cdot}302424}}{49}=\frac{2{\cdot}49252}{49}={\cdot}050868$.

§ 189. Of the respective merits of these two formulæ I have already given my opinion in the preceding note, § 177; and for the same reasons which I have there mentioned, I endeavoured to deduce some other formula which might prove more expeditious and accurate; the result of my inquiries has been the formula which I have already given in the body of the work (§ 72), and the investigation of which I shall now proceed to explain. The steps of the process are the same as in deducing Dr. Halley's formula, till we come to the equation G, where, instead of resolving that quadratic equation by extracting the root, we must approximate towards such root according to Mr. Simpson's formula, as laid down in page 164 of his *Treatise of Algebra*, and already alluded to in the last note (see § 178). Whence, by comparing that formula with the equation G, $\rho - \frac{n-1}{12} . \rho^2 = \beta$, we shall have $b = -\frac{n-1}{12}$, $c=0$, $\sigma = b$, $q = \beta$, and $z = \rho$; consequently the general formula becomes $\rho = \frac{(1 - \frac{n-1}{12} . \beta) . \beta}{1 - \frac{2.(n-1)}{12} . \beta} = \frac{[12 - (n-1)\beta] . \beta}{12 - 2(n-1)\beta}$ which is the formula given in the body of this work.

§ 190. I shall now subjoin a few examples of each of these methods in numbers, that the reader may be enabled to judge of the degree of accuracy which they possess in comparison to each other. It must be observed that the true rate in all these examples is exactly ·05; that the first column denotes the value of n, or the number of years during which the annuity is to continue; and that the value of $\frac{p}{a}$ in the general equation is taken from Table VI., under 5 per cent, and opposite the given term.

Years.	Simpson's Formula.	Halley's Formula.	Baily's Formula.
10	·049991	·050007	·050003
25	·049957	·050091	·050030
50	·049552	·050868	·050188
75	·048538	·054265	·050599
100	·047042	*fails.*	·051401

From this table it will be seen that, even when the term is 100 years, and the rate of interest not more than 5 per cent, my formula will express such rate to the 700th part of its true value, whereas Mr. Simpson's formula will not then express it within a 400th part of such value; and Dr. Halley's formula in such case wholly fails.*

§ 191. It will be readily perceived that the lower the rate of interest, and the smaller the number of years, the nearer will either of these formulæ express the value of ρ; on the contrary, the higher the rate of interest, and the greater the number of years, the further will it diverge from the true value, because not only the powers of ρ, but the co-efficients of such powers will not in this case decrease so rapidly in any of the equations;* and, indeed, it will be seen that when n and ρ are very great, the present formulæ will not express the rate so accurately as the former ones in page 113, where the *amounts* of annuities are concerned. We can always, however, in such cases, determine the value of ρ to a great degree of exactness by another formula which I shall explain hereafter.

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* It fails because $12(n-1)\beta = 1188\beta$ is greater than 36, consequently the square root of $36 - 1188\beta$ cannot be extracted.

§ 192. For common purposes, where great exactness is not required, we may in general discover the rate of interest sufficiently near by inspection of Table VI. at the end of this work, where, by comparing the value of $\frac{p}{a}$ with the values there set down against the given number of years, we may find the value of ρ tolerably exact, or at least sufficiently so as to serve as a foundation as a further approximation by the method which I shall presently explain. For example, suppose an annuity of £50 per annum, for 26 years, cost £685, then will $\frac{p}{a}=\frac{685}{50}=13{\cdot}7$. Now the value of $\frac{p}{a}$ for the same time at 5 per cent interest is 14·375, and at 6 per cent interest it is 13·003; we therefore know that the true rate must lie somewhere between ·05 and ·06; and as 13·7 is nearly an arithmetical mean between these two values, we may presume that the rate is about 5½ per cent; which we find on trial to be the case, as the true rate is ·05473, or about £5, 9s. 6d. per cent. When greater accuracy, however, is required, we must have reccurse to the formulæ above given; and if upon trial they do not express the rate of interest sufficiently exact, we may in all cases correct the error to any degree of accuracy required by the following method:—

§ 193. Let R be assumed as somewhat near the true value of ρ, and let x be the unknown difference between them, then will $\text{R}+x=\rho$, and by substituting $\text{R}+x$ for ρ, the general equation B, $(1+\rho)^{-n}=1-\frac{p}{a}\cdot\rho$, will become $(1+\text{R}+x)^{-n}=1-\frac{p}{a}\cdot\text{R}-\frac{p}{a}\cdot x$. If we expand the quantity

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* Thus, in the equation F, let us suppose $n=25$, and the rate of interest 5 per cent, then will the fifth term of the series, if it were continued, be $\frac{n^3+4n^2+n-6}{1440}\cdot\rho^4=$ $\frac{15625+2500+25-6}{1440}\times{\cdot}00000625=12{\cdot}6\times{\cdot}00000625={\cdot}0007875$, a quantity which is not of any great value: But if the number of years be 100, then will this term become $\frac{1000000+40000+100-6}{1440}\times{\cdot}00000625=722{\cdot}2875\times{\cdot}00000625={\cdot}0045143$.

$[(1+\text{R})+x]^{-n}$ by means of the binomial theorem, into a series ascending by the powers of x, it will become $(1+\text{R})^{-n}-n(1+\text{R})^{-(n+1)}x+\frac{n(n+1)}{2}.(1+\text{R})^{-(n+2)}x^2-$ &c. Now, since x is generally some quantity much smaller than ·02, ·03, ·04, ·05, &c., it is evident that this series will converge very swiftly; we may therefore reject all the terms affected with the second, third, and higher powers of x, as very small in respect to the first, and retain the two first terms as expressing the value of the whole series; which, though not strictly true, will approach very near to it and answer the present purpose. Our equation consequently becomes $(1+\text{R})^{-n}-n(1+\text{R})^{-(n+1)}x=1-\frac{p}{a}\cdot\text{R}-\frac{p}{a}\cdot x$, whence $x=\dfrac{1-\frac{p}{a}\cdot\text{R}-(1+\text{R})^{-n}}{\frac{p}{a}-n(1+\text{R})^{-(n+1)}}$.* If R has been assumed pretty near the true value of ρ, then will $\text{R}+x$ express the value of ρ to double the number of figures that are true in R; and by substituting this value again instead of R, and repeating the process, we shall have the value of ρ twice corrected. For example, in the equation B, $(1+\rho)^{-n}=1-\frac{p}{a}.\rho$, let the present value of an annuity of £30 per annum for 50 years be £540; consequently $n=50$, and $\frac{p}{a}=\frac{540}{30}=18$; and let R, or the assumed near value of ρ, be ·05, then will $x=\dfrac{1-18\times{\cdot}05-(1{\cdot}05)^{-50}}{18-50(1.05)^{-51}}=\dfrac{1-{\cdot}9-{\cdot}08720373}{18-50\times{\cdot}08305117}=\dfrac{{\cdot}01279627}{13{\cdot}8474415}={\cdot}00092409$; whence $\rho=\text{R}+x={\cdot}05092409$. If this value of ρ should be supposed not to be sufficiently exact, we may make a second correction of it, by substituting it instead of ·05 in the equation above mentioned, and repeating the process. Thus, by this second substitution, we shall find $x=\dfrac{1-18\times{\cdot}05092409-(1{\cdot}05092409)^{-50}}{18-50(1{\cdot}05092409)^{-51}}=\dfrac{1-{\cdot}91663362-{\cdot}0834512}{18-50\times{\cdot}07940743}=\dfrac{-{\cdot}00008482}{14{\cdot}02962850}=-{\cdot}00000604$; which being negative shows that R has been in this case taken too great, and must therefore be subtracted; consequently $\rho=\text{R}+x={\cdot}05092409-{\cdot}00000604={\cdot}05091805$, which is true to the last figure.

§ 194. I have already observed that when n and ρ are very great, the formulæ above given will not express the rate of interest sufficiently exact,* and that in such cases we must have recourse to other formulæ for this purpose. Thus, let the equation A, $p = a \times \frac{1-(1+\rho)^{-n}}{\rho}$ be multiplied by $\frac{\rho}{p}$, and it will become the equation H, $\rho = \frac{a}{p} - \frac{a}{p} \times (1+\rho)^{-n}$. Now, since $\frac{a}{p}$ is always less than unity, and $(1+\rho)^{-n}$ is very small when n and ρ are very great, it follows that $\rho = \frac{a}{p}$ nearly,† which I call an approximation of the first degree. But if this value be substituted for ρ in $(1+\rho)^{-n}$, the equation H will be converted into the equation I, $\rho = \frac{a}{p} - \frac{a}{p} \times \left(1+\frac{a}{p}\right)^{-n}$, which is an approximation of the second degree, and shows the value of ρ much nearer than the former. We may approach still nearer to the value of ρ by the following method: let $\frac{a}{p}$ be assumed as nearly equal to the value of ρ, and let x be the unknown difference between them, then will $\frac{a}{p} + x = \rho$; and if this value be substituted for ρ in the equation H, we shall have $\frac{a}{p} + x = \frac{a}{p} - \frac{a}{p} \times \left(1+\frac{a}{p}+x\right)^{-n}$: whence by expanding $\left(1+\frac{a}{p}+x\right)^{-n}$ into the series $\left(1+\frac{a}{p}\right)^{-n} - n\left(1+\frac{a}{p}\right)^{-(n+1)} x + \&c.$, and rejecting all the powers of x above the first, we shall have the equation $\frac{a}{p} + x = \frac{a}{p} - \frac{a}{p}\left(1+\frac{a}{p}\right)^{-n} + \frac{a}{p} n \left(1+\frac{a}{p}\right)^{-(n+1)} x$, from which we deduce $x = \frac{-\frac{a}{p}\left(1+\frac{a}{p}\right)^{-n}}{1-\frac{a}{p} n \left(1+\frac{a}{p}\right)^{-(n+1)}}$, and which, by multiplying the numerator and denominator by $\frac{p}{a}\left(1+\frac{a}{p}\right)^{n+1}$, becomes $-\frac{1+\frac{a}{p}}{\frac{p}{a}\left(1+\frac{a}{p}\right)^{n+1} - n}$; consequently $\rho = \frac{a}{p} - \frac{1+\frac{a}{p}}{\frac{p}{a}\left(1+\frac{a}{p}\right)^{n+1} - n} =$

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* The rule for finding the value of x may be concisely stated thus: Let the assumed value of ρ be substituted in the general equation, and let the error thence arising be divided by the *differential co-efficient* of such equation; the quotient, added to the assumed value, will give the value of ρ corrected.—See the note in § 182.

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* This is always the case when $\frac{p}{a}.\text{R}$, in the above formula, is greater than unity; and we shall then find that $\frac{a}{p}$ will denote the rate much nearer than such assumed value R.

† When n is infinite, that is, when the annuity is a perpetuity, the quantity $\frac{a}{p} \times (1+\rho)^{-n}$ vanishes entirely, and ρ is in such case exactly equal to $\frac{a}{p}$.

$$\frac{\left(1+\frac{a}{p}\right)^{n+1}-\frac{p+a(n+1)}{p}}{\frac{p}{a}\left(1+\frac{a}{p}\right)^{n+1}-n}.$$ If the value of ρ thus obtained be not thought sufficiently exact, we may, by substituting it for $\frac{a}{p}$ in this last equation, correct it to any required degree of accuracy.

I shall now give an example of these last two methods of resolving the value of ρ; for this purpose, let the present value of an annuity of £40 per annum for 60 years be £500; consequently $n=60$, $\frac{p}{a}=\frac{500}{40}=12{\cdot}5$, and $\frac{a}{p}=\frac{40}{500}={\cdot}08$; whence the approximation of the second degree above mentioned is $\rho={\cdot}08-{\cdot}08\times(1{\cdot}08)^{-60}={\cdot}08-{\cdot}08\times{\cdot}00987585=$ ${\cdot}079209932$. But if we make use of the third approximation we shall have $\rho=\frac{(1{\cdot}08)^{61}-\frac{500+40\times 61}{500}}{12{\cdot}5\times(1{\cdot}08)^{61}-60}=\frac{109{\cdot}35762876-5{\cdot}88}{1366{\cdot}9703595-60}=\frac{103{\cdot}47762876}{1306{\cdot}9703595}$ $={\cdot}07917367$. These two values differ but ${\cdot}00003626$ from each other; and if either of them be substituted for $\frac{a}{p}$ in the last equation, we shall have the rate come out equal to ${\cdot}0791728$, which is true to the last figure.

§ 195. I shall now close this note by bringing into one view the different formulæ above mentioned, in order that the reader may more readily refer to them, or compare them together. I have also subjoined the formula for correcting the value of ρ, if on trial it should not be found sufficiently exact.

$$\rho=\frac{30Q-(2n+1)4}{6Q(5Q-3n-4)+\frac{1}{6}(11n+13).(n+2)} \quad \text{where } Q=\frac{n(n+1)}{2\left(n-\frac{p}{a}\right)}$$

$$\left.\begin{aligned}\rho&=\frac{6-\sqrt{36-12(n-1)\beta}}{n-1}\\ \rho&=\frac{[12-(n-1)\beta].\beta}{12-2(n-1)\beta}\end{aligned}\right\} \quad \text{where } \beta=\left(\frac{an}{p}\right)^{\frac{2}{n+1}}-1$$

$$\rho=\text{R}+\frac{1-\frac{p}{a}.\text{R}-(1+\text{R})^{-n}}{\frac{p}{a}-n(1+\text{R})^{-(n+1)}}$$

where R denotes any near value of ρ taken either by trial or conjecture.

When the number of years and rate of interest are very great, then will

$$\rho=\frac{a}{p}-\frac{a}{p}\cdot\left(1+\frac{a}{p}\right)^{-n}$$

or more nearly

$$\rho=\frac{\left(1+\frac{a}{p}\right)^{n+1}-\frac{p+a.(n+1)}{p}}{\frac{p}{a}\cdot\left(1+\frac{a}{p}\right)^{n+1}-n}$$

(NOTE C.)

§ 196. THE formula given in page 66, for finding the rate of interest in Reversionary annuities is an improvement on the formula which Dr. Halley has given for this purpose,* since it is not only more easily solved, but expresses the value more accurately. I shall pursue a similar plan, in investigating these methods, to that which I have adopted in the preceding notes.

§ 197. The primary equation from which they are deduced is (by § 81) the equation A, $p=a\times\frac{(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}}{\rho}$; which being multiplied by $\frac{\rho}{a}$ and transposed, forms the equation B, $(1+\rho)^{-\nu}-(1+\rho)^{-(\nu+n)}=\frac{p\rho}{a}$. If we expand $(1+\rho)^{-\nu}$ into a series ascending by the powers of ρ, by means of the binomial theorem, it will become $1-\nu\rho+\frac{\nu}{1}\cdot\frac{\nu+1}{2}\cdot\rho^2-\frac{\nu}{1}\cdot\frac{\nu+1}{2}\cdot\frac{\nu+2}{3}\cdot\rho^3+\frac{\nu}{1}\cdot\frac{\nu+1}{2}\cdot\frac{\nu+2}{3}\cdot\frac{\nu+3}{4}\cdot\rho^4-$ &c.; and if we expand $(1+\rho)^{-(\nu+n)}$ into a similar series, it will become $1-(\nu+n).\rho+\frac{\nu+n}{1}\cdot\frac{\nu+n+1}{2}\cdot\rho^2-\frac{\nu+n}{1}\cdot\frac{\nu+n+1}{2}\cdot\frac{\nu+n+2}{3}\cdot\rho^3+\frac{\nu+n}{1}\cdot\frac{\nu+n+1}{2}\cdot\frac{\nu+n+2}{3}\cdot\frac{\nu+n+3}{4}\cdot\rho^4-$&c.; and by subtracting this latter series from the former, there will remain the series $\Big\}$ $n\rho-\frac{2\nu n+n^2+n}{2}\cdot\rho^2+\frac{3\nu^2n+3\nu n^2+6\nu n+n^3+3n^2+2n}{6}\cdot\rho^3$ $-\frac{4\nu^3n+6\nu^2n^2+18\nu^2n+4\nu n^3+18\nu n^2+22\nu n+4n^4+6n^3+11n^2+6n}{24}\cdot\rho^4+$

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* In Gardiner's Logarithms. See the note to § 176.

&c.; which being substituted instead of its equal in the equation B, forms the equation C, $n\rho - \frac{2\nu n + n^2 + n}{2} \cdot \rho^2 + \frac{3\nu^2 n + 3\nu n^2 + 6\nu n + n^3 + 3n^2 + 2n}{6} \cdot \rho^3 - \&c.$[*] $= \frac{p\rho}{a}$. If we divide by $n\rho$, we shall have the equation D, $1 - \frac{2\nu + n + 1}{2} \cdot \rho + \frac{3\nu^2 + 3\nu n + 6\nu + n^2 + 3n + 2}{6} \cdot \rho^2 - \&c. = \frac{p}{an}$. Let us now raise both sides of this equation to the power indicated by $-\frac{2}{2\nu + n + 1}$;[†] and in order to raise the series which forms the left hand of this equation to such power, we must have recourse to M. de Moivre's multinomial theorem, as already explained in § 176, where, by comparing the several terms of the present series with the general formula there given, it will be found that the equation $\left(1 - \frac{2\nu + n + 1}{2} \cdot \rho + \frac{3\nu^2 + 3\nu n + 6\nu + n^2 + 3n + 2}{6} \cdot \rho^2 - \&c.\right)^{-\frac{2}{2\nu + n + 1}} = \left(\frac{p}{an}\right)^{-\frac{2}{2\nu + n + 1}} = \left(\frac{an}{p}\right)^{\frac{2}{2\nu + n + 1}}$[‡] will become the equation E, $1 + \rho - \frac{n^2 - 1}{12(2\nu + n + 1)} \cdot \rho^2 + \&c. = \left(\frac{an}{p}\right)^{\frac{2}{2\nu + n + 1}}$; whence we have $\rho - \frac{n^2 - 1}{12(2\nu + n + 1)} \cdot \rho^2 = \left(\frac{an}{p}\right)^{\frac{2}{2\nu + n + 1}} - 1$; and by making $\left(\frac{an}{p}\right)^{\frac{2}{2\nu + n + 1}} - 1 = \beta$, we shall have the equation F, $\rho - \frac{n^2 - 1}{12(2\nu + n + 1)} \cdot \rho^2 = \beta$. If we multiply both sides by $-\frac{12(2\nu + n + 1)}{n^2 - 1}$, we shall have $\rho^2 - \frac{12(2\nu + n + 1)}{n^2 - 1} \cdot \rho = -\frac{12(2\nu + n + 1)}{n^2 - 1} \cdot \beta$; and lastly, if we make $\frac{6(2\nu + n + 1)}{n^2 - 1} = \gamma$, then will $\rho^2 - 2\gamma\rho = -2\gamma\beta$; and by extracting the root of this quadratic equation we shall have $\rho = \gamma - (\gamma^2 - 2\gamma\beta)^{\frac{1}{2}}$, which is the very formula given by Dr. Halley: it may, however, be rendered more convenient for practice by excluding the new character γ, and making $2\nu + n + 1 = \delta$; for then will $\rho = \frac{6\delta}{n^2 - 1} - \left(\frac{36\delta^2}{(n^2 - 1)^2} - \frac{12\delta\beta}{n^2 - 1}\right)^{\frac{1}{2}} = \frac{6\delta - \sqrt{36\delta^2 - 12\delta(n^2 - 1)\beta}}{n^2 - 1}$.

As an example of the use and application of this formula, let us take the following question:—If the reversion of an annuity of £60 per annum for 50 years after the next 10 years cost £672·45, what rate of interest was allowed? Here we have $\nu=10$, $n=50$, $a=60$, and $p=672{\cdot}45$; whence $\delta=71$, $\beta=\left(\frac{60\times 50}{672{\cdot}45}\right)^{\frac{2}{71}}-1={\cdot}043025$, and $\rho=\frac{6\times 71-\sqrt{36\times 5041-12\times 71\times 2499\times{\cdot}043025}}{2499}=\frac{426-\sqrt{89869{\cdot}407}}{2499}=\frac{126{\cdot}21776}{2499}={\cdot}050507$. Now the true rate being exactly ·05, this formula gives the value, in the present case, as accurate as could be expected.

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[*] I have rejected the fourth and all the following terms of this series, as they are too complicated to be admitted in the subsequent calculations.

[†] This index is chosen in preference to any other, for the reasons already given in §§ 176 and 188.

[‡] I have already shown, in the note to § 188, that the *negative* power of any quantity is equal to the *positive* power of the reciprocal of that quantity.

§ 198. If, instead of extracting the root of the equation above given, in order to find the value of ρ, we approximate towards such root by Mr. Simpson's method, already explained in § 178, we shall find on a comparison of the equation F, $\rho-\frac{n^2-1}{12(2\nu+n+1)}\cdot\rho^2=\beta$, with the general equation there given, that $b=\frac{n^2-1}{12(2\nu+n+1)}=\frac{n^2-1}{12\delta}$, $c=0$, $\sigma=b$, $q=\beta$, and $z=\rho$; consequently the value of the root will be expressed by this equation, $\rho=\frac{(1-\frac{n^2-1}{12\delta}\cdot\beta)\cdot\beta}{1-2\frac{n^2-1}{12\delta}\cdot\beta}=\frac{[12\delta-(n^2-1)\beta].\beta}{12\delta-2(n^2-1)\beta}$, which is the formula that I have given for general use in page 66, as it shows the value of ρ more correctly than Dr. Halley's formula, in those cases where n is greater than ρ, and, moreover, does not involve the extraction of the square root.

§ 199. I shall now give a few examples of each of these formulæ expressed in numbers, that the reader may be enabled not only to compare them readily together, but to judge of the degree of accuracy which they possess. I should previously observe that the true rate in all these examples is exactly ·05; that the first column denotes the value of ν, or the number of years during which the annuity is deferred; and the second column the value of n, or number of years during which the annuity is to continue after it has commenced; and that the value of $\frac{p}{a}$ is deduced from the values in Table VI., under 5 per cent, and opposite the respective number of years.*

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* In the manner alluded to in the note to § 202.

Value of ν.	Value of n.	Halley's Formula.	Baily's Formula.
10	10	·050001	·050001
90	10	·050000	·050000
10	25	·050046	·050033
75	25	·050001	·050001
10	50	·050507	·050288
50	50	·050188	·050168
10	75	·052380	·050966
25	75	·051439	·050918

§ 200. It should be remarked here, that the lower the rate of interest and the smaller the number of years during which the annuity is to continue, the nearer will either of these formulæ express the value of ρ; and on the contrary, the higher the rate of interest and the greater the number of years, the further will it diverge from the true value: because not only the powers of ρ but the co-efficients of such powers will not in this case decrease so rapidly in the equation from which the formulæ are deduced. It may be further remarked, that, with respect to the accuracy of the results, it signifies little what the value of ν is, provided n is not very great; as may be seen from the table above given. If, however, the value of ρ as found by this formula, or by any other method either of trial or conjecture, should not prove sufficiently exact, we may always correct such value to any required degree of accuracy, by the following method of approximation:—

§ 201. Let R be assumed as somewhat near the true value of ρ, and let x be the unknown difference between them; then will $\mathrm{R}+x=\rho$; and by substituting $\mathrm{R}+x$ for ρ, the general equation B, will become $(1+\mathrm{R}+x)^{-\nu} - (1+\mathrm{R}+x)^{-(\nu+n)} = \frac{p}{a}.\mathrm{R}+\frac{p}{a}.x$. If we expand $[(1+\mathrm{R})+x]^{-\nu}$ by means of the binomial theorem into a series ascending by the powers of x, it will become $(1+\mathrm{R})^{-\nu} - \nu(1+\mathrm{R})^{-(\nu+1)}x + \frac{\nu}{1}.\frac{\nu+1}{2}.(1+\mathrm{R})^{-(\nu+2)}x^2 -$ &c.; and if we expand $[(1+\mathrm{R})+x]^{-(\nu+n)}$ into a similar series, it will become $(1+\mathrm{R})^{-(\nu+n)} - (\nu+n).(1+\mathrm{R})^{-(\nu+n+1)}x + \frac{\nu+n}{1}.\frac{\nu+n+1}{2}$ $(1+\mathrm{R})^{-(\nu+n+1)}x^2 -$ &c.: by subtracting the latter series from the former, and rejecting all the terms affected with the second, third, and higher powers of x, as very small in comparison with the first, the equation will become $(1+\mathrm{R})^{-\nu} - (1+\mathrm{R})^{-(\nu+n)} - \nu(1+\mathrm{R})^{-(\nu+1)}x +$

$(\nu + n) \,.\, (1 + \text{R})^{-(\nu+n+1)}x = \frac{p}{a} \,.\, \text{R} + \frac{p}{a} \,.\, x$; whence we have

$$x = \frac{(1+\text{R})^{-\nu} - (1+\text{R})^{-(\nu+n)} - \frac{p}{a} \,.\, \text{R}}{\frac{p}{a} + \nu(1+\text{R})^{-(\nu+1)} - (\nu+n) \,.\, (1+\text{R})^{-(\nu+n+1)}} .*$$

§ 202. If R has been assumed pretty near the true value of ρ, then will $\text{R}+x$ express the value of ρ to double the number of places that are true in R; and by substituting this value instead of R, and repeating the process, we shall have the rate twice corrected. For example, in the equation B, $(1+\rho)^{-\nu} - (1+\text{R})^{-(\nu+n)} = \frac{p}{a} \,.\, \rho$, let the present value of a reversionary annuity of £40 per annum for 26 years after the next 14 years be £240; consequently, $\nu = 14$, $n = 26$, and $\frac{p}{a} = \frac{240}{40} = 6$: and let R, or the assumed near value of ρ be ·06;† then will $x = \frac{(1{\cdot}06)^{-14} - (1{\cdot}06)^{-40} - 6 \times {\cdot}06}{6 + 14 \times (1{\cdot}06)^{-15} - 40 \times (1{\cdot}06)^{-41}}$ $= \frac{{\cdot}44230096 - {\cdot}09722219 - {\cdot}36}{6 + 14 \times {\cdot}41726506 - 40 \times {\cdot}09171905} = \frac{-{\cdot}01492123}{8{\cdot}17294884} = -{\cdot}0018257$; whence $\rho = \text{R} + x = {\cdot}06 - {\cdot}0018257 = {\cdot}0581743$. If this value should not be thought sufficiently exact, we may make a second correction of it by substituting it instead of ·06 in the equation above given, and repeating the process: thus by this second substitution we shall find $x = \frac{(1{\cdot}0581743)^{-14} - (1{\cdot}0581743)^{-40} - 6 \times {\cdot}0581743}{6 + 14 \times (1{\cdot}0581743)^{-15} - 40 \times (1{\cdot}0581743)^{-41}} =$ $\frac{{\cdot}45310516 - {\cdot}10416255 - {\cdot}34904580}{6 + 14 \times {\cdot}42819525 - 40 \times {\cdot}09843609} = \frac{-{\cdot}00010319}{8{\cdot}05728990} = -{\cdot}0000128$; whence $\rho = \text{R} + x = {\cdot}0581743 - {\cdot}0000128 = {\cdot}0581615$, which is true to the last figure.

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[*] The rule for finding the value of x may be concisely stated thus: Let the assumed value of ρ be substituted in the general equation, and let the error thence arising be divided by the *differential co-efficient* of such equation; the quotient, added to the assumed value, will give the value of ρ corrected. See the note in § 182.

[†] This first near value of ρ may generally be estimated from the Tables, by finding the number of years' purchase that the reversion is worth, according to the several rates of interest therein mentioned: and if any of the resulting values come pretty near the given value of $\frac{p}{a}$, we may assume the rate of interest from which it arises, as the near rate sought for. Thus, in the present case, it appears that at the rate of 6 per cent, the present value of the reversion is worth $15{\cdot}046 - 9{\cdot}295 = 5\frac{3}{4}$ years' purchase; which is nearly the value of $\frac{p}{a}$: I therefore conclude that the true rate is pretty near 6 per cent, and I take this as the first near value accordingly.

§ 203. I shall close this note by bringing into one view, the several formulæ above mentioned, in order that the reader may more readily consult them.

$$\left.\begin{array}{l} \rho = \dfrac{6\delta - \sqrt{36\delta^2 - 12\delta(n^2-1)\beta}}{n^2-1} \\ \rho = \dfrac{[12\delta-(n^2-1)\beta].\beta}{12\delta-2(n^2-1)\beta} \end{array}\right\} \text{ where } \delta = 2\nu+n+1, \text{ and } \beta = \left(\frac{an}{p}\right)^{\frac{2}{\delta}} - 1$$

$$\rho = \mathrm{R} + \frac{(1+\mathrm{R})^{-\nu} - (1+\mathrm{R})^{-(\nu+n)} - \frac{p}{a}\,\mathrm{R}.}{\frac{p}{a} + \nu(1+\mathrm{R})^{-(\nu+1)} - (\nu+n).(1+\mathrm{R})^{-(\nu+n+1)}}$$

where R denotes any near value of ρ taken by trial or conjecture.

◆

(NOTE D.)

§ 204. WHEN the reversion is a perpetuity, the rate of interest is deduced from the equation A, $p = a \times \frac{(1+\rho)^{-\nu}}{\rho}$; see § 85: and this multiplied by $\frac{\rho}{a}$, forms the equation B, $(1+\rho)^{-\nu} = \frac{p}{a}.\rho$. If we multiply this by $\frac{a}{p}.(1+\rho)^{\nu}$, we shall have the equation C, $\rho(1+\rho)^{\nu} = \frac{a}{p}$. Let $(1+\rho)^{\nu}$ be expanded, by means of the binomial theorem, into an infinite series ascending by the powers of ρ, and it will become $1 + \nu\rho + \frac{\nu}{1}\cdot\frac{\nu-1}{2}\cdot\rho^2 + \frac{\nu}{1}\cdot\frac{\nu-1}{2}\cdot\frac{\nu-2}{3}\cdot\rho^3 + \&c.$: and this being multiplied by ρ, the equation C will become the equation D, $\rho + \nu\rho^2 + \frac{\nu}{1}\cdot\frac{\nu-1}{2}\cdot\rho^3 + \frac{\nu}{1}\cdot\frac{\nu-1}{2}\cdot\frac{\nu-2}{3}.\rho^4 + \&c. = \frac{a}{p}$. If we multiply both sides by ν, and afterwards add unity thereto, we shall have the equation E, $1 + \nu\rho + \nu^2\rho^2 + \frac{\nu^2}{1}\cdot\frac{\nu-1}{2}.\rho^3 + \frac{\nu^2}{1}\cdot\frac{\nu-1}{2}\cdot\frac{\nu-2}{3}\cdot\rho^4 + \&c. = \frac{p+a\nu}{p}$. Let both sides of this equation be raised to the power indicated by $\frac{1}{\nu}$; and in order to raise the series which forms the left side of this equation to that power, we must have recourse to M. De Moivre's multinomial theorem, as already explained in § 176:

when by comparing the several terms of the present series with the formula there given, it will be found that, when the co-efficients are reduced to their lowest terms, the equation $(1+\nu\rho+\nu^2\rho^2+\frac{\nu^2}{1}\cdot\frac{\nu-1}{2}\cdot\rho^3+\&c.)^{\frac{1}{\nu}}=\left(\frac{p+a\nu}{p}\right)^{\frac{1}{\nu}}$ is equal to the equation F, $1+\rho+\frac{\nu+1}{2}\cdot\rho^2-\frac{\nu^2-1}{6}\cdot\rho^3+\&c.=\left(\frac{p+a\nu}{p}\right)^{\frac{1}{\nu}}$: whence we have $\rho+\frac{\nu+1}{2}\cdot\rho^2-\frac{\nu^2-1}{6}\cdot\rho^3=\left(\frac{p+a\nu}{p}\right)^{\frac{1}{\nu}}-1$.*

Let us now make $\left(\frac{p+a\nu}{p}\right)^{\frac{1}{\nu}}-1=\beta$; whence we shall have the equation G, $\rho+\frac{\nu+1}{2}\cdot\rho^2-\frac{\nu^2-1}{6}\cdot\rho^3=\beta$: and by comparing this with Mr. Simpson's formula, in § 178, for approximating towards the roots of equations, we shall have $b=\frac{\nu+1}{2}$, $c=-\frac{\nu^2-1}{6}$, $\sigma=\frac{5\nu+1}{6}$, $q=\beta$, and $z=\rho$; consequently the approximated value of the root of this equation will be $\rho=$

$$\frac{(1+\frac{5\nu+1}{6}.\beta).\beta}{1+\left(\frac{\nu+1}{2}+\frac{5\nu+1}{6}\right).\beta}=\frac{[6+(5\nu+1)\beta]\beta}{6+4(2\nu+1)\beta},$$

which is the formula given in page 69.

§ 205. In order that the reader may judge of the degree of accuracy which this formula possesses, I have subjoined the following table of examples:—the first column denotes the value of ν, or the number of years during which the annuity is deferred; and the value of $\frac{p}{a}$ is deduced from Table VI., by taking the present value of an annuity for the time ν from the present value of the perpetuity. The true rate of interest is marked at the head of each respective column, and the values, set under the same, show the approximations thereto.

Value of ν.	True rate ·03.	True rate ·05.	True rate ·08.
5	·03001	·05008	·08040
10	·03005	·05024	·08080
20	·03012	·05031	·07999
30	·03013	·04986	·07793
40	·03002	·04910	·07568

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* By rejecting all the terms affected with the fourth, fifth, and higher powers of ρ as inconsiderable when compared with the others, as in the former cases.

§ 206. Within the limits here given, the present formula will give the value of ρ sufficiently exact for common purposes; or we may have recourse to the Table in page 66, where we may find a near value of ρ, as it were by inspection, agreeably to what I have already laid down respecting the preceding formulæ in §§ 181 and 192. Where we have not this resource, or where the present formula fails, we must adopt the method of Trial and Error; and after having obtained a near value of ρ by this mean, we may approximate to its true value to any degree of accuracy by the following method:—

§ 207. Let R be assumed as somewhat near the true value of ρ, and let x be the unknown difference between them; then will $\mathrm{R}+x=\rho$, and by substituting $\mathrm{R}+x$ for ρ, the general equation B will become $(1+\mathrm{R}+x)^{-\nu}=\frac{p}{a}\cdot\mathrm{R}+\frac{p}{a}\cdot x$. If we expand $[(1+\mathrm{R}+x)]^{-\nu}$, by means of the binomial theorem, into a series ascending by the powers of x, it will become $(1+\mathrm{R})^{-\nu}-\nu(1+\mathrm{R})^{-(\nu+1)}x+\frac{\nu}{1}\cdot\frac{\nu+1}{2}\cdot(1+\mathrm{R})^{-(\nu+2)}x^2-$&c.: therefore by rejecting all the terms affected with the second, third, and higher powers of x, as very small in comparison to the rest, the equation will become $(1+\mathrm{R})^{-\nu}-\nu(1+\mathrm{R})^{-(\nu+1)}x=\frac{p}{a}\cdot\mathrm{R}+\frac{p}{a}\cdot x$; whence $x=\frac{(1+\mathrm{R})^{-\nu}-\frac{p}{a}\cdot\mathrm{R}}{\nu(1+\mathrm{R})^{-(\nu+1)}+\frac{p}{a}}$.* If R has been assumed pretty near the value of ρ, then will $\mathrm{R}+x$ express the value of ρ to double the number of figures that are true in R; and by substituting this value instead of R, and repeating the process, we shall have the value of ρ twice corrected. For example, in the equation B, $(1+\mathrm{R})^{-\nu}=\frac{p}{a}\cdot\mathrm{R}$, let the present value of a reversionary annuity of £1000 for ever, after the next 40 years, be £575·3875; consequently $\nu=40$, and $\frac{p}{a}=\frac{575\cdot3875}{1000}=\cdot575387$; and let R, as found by the formula in page 64, be taken equal to ·07568: then will

$$x=\frac{(1\cdot07568)^{-40}-\cdot575387\times\cdot07568}{40\times(1\cdot07568)^{-41}+\cdot575387}=\frac{\cdot0540351-\cdot0435453}{2\cdot009337+\cdot575387}=\frac{\cdot0104898}{2\cdot584724}$$

$=\cdot00406$, whence $\rho=\mathrm{R}+x=\cdot07568+\cdot00406=\cdot07974$; and this differs but ·00026 from the true value, which is exactly 8 per cent, as may be seen by substituting ·07974 for R, and repeating the process.

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* This rule may be concisely stated in the manner already explained in the note to § 182.

(NOTE E.)

§ 208. IF anywhere in the axis of a *Logarithmic Curve* (Fig. 1.) there be taken any number of equal parts AB, BC, CD, &c., and at the points A, B, C, D, &c., there be raised the perpendiculars AA′, BB′, CC′, DD′, &c.; then from the nature of the curve all these perpendiculars will be continually proportional.* Consequently if any one of these ordinates (AA′ for instance) represent any sum of money put out to compound interest, and the next ordinate be taken so that the increment thereof (βB′ for instance) be equal to the interest of the given sum at the end of the given time represented by the corresponding absciss AB; then will the whole ordinate BB′ denote the amount of principal and interest at the end of that period; also, the next ordinate CC′ will denote the amount of principal and interest at the end of twice that period; and the next following ordinate DD′ will denote the amount of principal and interest at the end of thrice that period; and so on.

§ 209. Now let any one of these ordinates (AA′ for instance) represent unity, and let the rate of interest per annum be denoted by the increment βB′; then will the parts AB, AC, AD, &c., of the axis express the number of years. Also, the *amount* of £1 at the end of any one of those years will be truly expressed by the corresponding ordinate BB′, CC′, DD′ &c.; and the increments βB′, γC′, δD′, &c., will show the quantity by which the sum of £1 is increased in every succeeding year. In like manner the ordinates *bb′*, *cc′*, *dd′*, &c., will denote the *present value* of £1 due at the end of 1, 2, 3, &c., years: all which is evident from the nature of the curve.

§ 210. The equation of the logarithmic curve is $y=b.a^x$; y denoting the ordinate, x the absciss, a a quantity greater than unity, and b any value taken at pleasure. But in order to apply this expression more conveniently to the doctrine of interest and annuities, let us substitute for these symbols the characters made use of in the body of this work (see § 15): whereby the equation $y=b.a^x$ will be converted into $s=p(1+\rho)^n$.

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* For the construction, &c., of the logarithmic curve, the reader may consult Robertson's *Geo. Treat. on the Conic Sec.*, 8vo, 1802; Keill's *Tract on Logarithms*, at the end of his edition of Euclid; Euler's *Introd. in Anal. Inf.*, vol. ii.; La Croix *Calcul. Diff. et Integ.*; Emerson *on Curve Lines*; or Hugenii *Opera Reliqua*, vol. ii., in which latter work the principal properties of this curve are pointed out.

Consequently s will now denote the ordinate, and n the absciss; and if, in order to simplify the subject, we make p equal to the first ordinate or unity, then will this last equation become $s=(1+\rho)^n$.

§ 211. If AB, AC, AD, &c., be equal to 1, 2, 3, &c., years (that is, equal to n), and AA′ be taken to represent unity, then will βB′ be equal to ρ; and BB′, CC′, DD′, &c., will be respectively equal to $(1+\rho)^1$, $(1+\rho)^2$, $(1+\rho)^3$, &c. (that is, equal to s); also bb', cc', dd', &c., will be respectively equal to $(1+\rho)^{-1}$, $(1+\rho)^{-2}$, $(1+\rho)^{-3}$, &c.

§ 212. The most remarkable property of the logarithmic curve is that its subtangent (AE for instance), from whatever part of the curve the tangent may be drawn, is a constant quantity, being always equal to $\frac{1}{N\lambda.(1+\rho)}$. And the different species or forms of these curves, as related to the same points of the axis, are determined from this quantity.

Thus, if in any given logarithmic curve YA′Y′ (Fig. 2) the increment yY be taken equal to r, or (which is the same thing) the ordinate BY be taken equal to $(1+r)$, then will the value of the intermediate ordinate bb', equidistant from AA′ and BY, be denoted by $(1+r)^{\frac{1}{2}}$, because these three ordinates are in continued proportion, or $1:(1+r)^{\frac{1}{2}}=(1+r)^{\frac{1}{2}}:(1+r)$; and the subtangent to this curve will be $\frac{1}{N\lambda(1+r)}$. But if this intermediate ordinate be indefinitely extended, and there be taken hh' equal to half yY, then will the logarithmic curve HA′H′ be generated, whose ordinate at the end of one year will be BH$=(1+\frac{r}{2})^{2}$, because $1:(1+\frac{r}{2})=(1+\frac{r}{2}):(1+\frac{r}{2})^2$; and whose subtangent is $\frac{1}{N\lambda.(1+\frac{r}{2})^2}$. Or, if in the indefinite ordinate βq, equidistant from AA′ and bh', there be taken qq' equal to a quarter of yY, then will the logarithmic curve QA′Q′ be generated, whose ordinate at the end of the year will be BQ, and whose subtangent will be $\frac{1}{N\lambda.(1+\frac{r}{4})^4}$. Further, if the absciss AB be divided into an infinite number of points, and if the ordinates corresponding to each of these points increase proportionally by infinitely small portions of r to the end of the year represented by AB, then will the curve MA′M′ be generated, whose ordinate at the end of the year will be denoted by BM, and whose subtangent is $\frac{1}{r}$ (see §§ 20-22).

§ 213. Therefore if a set of logarithmic curves be formed whose ordinates (at the end of every year represented by equal portions of the absciss) denote the amount of £1 in such number of years according to any normal rate of interest ($=r$) payable yearly, half-yearly, quarterly, or momently, then will the subtangents of such curves vary from $\frac{1}{N\lambda.(1+r)}$ to $\frac{1}{r}$; and these will be the limits of the values of such subtangents, and the measure of the curves generated in such cases.

§ 214. The same observations will apply to the present values of sums put out to interest if ordinates be drawn from the curve on the left side of AA′, where the curve approaches the axis; and it would consequently be easy to apply to these cases the observations made in § 42, respecting the decrease of such values.

§ 215. The logarithms or distances from unity of the same number in two logarithmic curves of different species will be proportional to the subtangents of their curves. Thus, let BA′B′, CA′C′ (Fig. 3), be two logarithmic curves formed by taking two different rates of interest, ρ and R respectively: and in either of them let there be taken any point B, from which let there be drawn the line BC parallel to the axis till it meets the other curve in C; then will the ordinates βB, γC be equal to each other, and the absciss Aβ will be to the absciss Aγ as the subtangent (Ab) of the curve BA′B′ is to the subtangent (Ac) of the curve CA′C′. Thus, if ν denote the absciss of the first curve, and n denote the absciss of the second curve, then because the ordinates are equal we shall have $(1+\rho)_\nu = (1+\mathrm{R})^n$, that is $\nu \times N\lambda.(1+\rho) = n \times N\lambda.(1+\mathrm{R})$: whence this proportion $\nu : n = \frac{1}{N\lambda.(1+\rho)} : \frac{1}{N\lambda.(1+\mathrm{R})}$. Consequently, having found the time when a given principal will amount to a given sum at one rate of interest, we may easily determine the time when the same principal would amount to the like sum at any other rate.

§ 216. In order to give a sensible idea of the rapid increase of money when put out at a high rate of interest, and also to illustrate more clearly the proportion which the amounts or present values of sums, put out at *different* rates of interest, bear to each other, I have drawn, from the same scale, three different logarithmic curves (Fig. 4) each related to the common axis XAX′, which is divided into equal parts, each part denoting the period of five years. The ordinate AA′ represents unity; and the other ordinates to the right and left of this denote respectively the *amount* and *present value* of £1 according to the several rates of 2, 5 and 10 per cent interest per annum, and at the end of the several periods marked on the axis. Thus the amount of £1 at the end of 20 years will be represented by the ordinate XD if the rate of interest be *two* per cent per annum; but if it be *five* per cent, the amount will be represented by XC; and if *ten* per cent, by XB. In like manner, the *present value* of £1 at the end of 40 years will be represented by the ordinates XD′, XC′, XB′ respectively, according as the rate of interest is 2, 5 or 10 per cent per annum: and thus for any of the intermediate periods.

§ 217. This last figure will also tend to explain more fully the observations made respecting the maximum of the difference in the present values of two equal sums at two different rates of interest (see § 51). For it will be seen that the ordinate AA′, which is *common* to all the curves, decreases faster in proportion as the rate of interest is higher: consequently, since it vanishes at last in all the curves, or becomes equal to nothing, when n is infinite, it follows that there must be some point in the axis where the difference between the corresponding ordinates of any two curves is a maximum. Thus, the difference between the ordinates of the curve BA′B′, and the curve CA′C′ is greatest about the 14th or 15th year: and it will be seen, from the figure, that the curves after crossing each other at A′ mutually recede and come to their greatest distance at the point just mentioned; after which they approach each other till they meet again at an infinite distance.

§ 218. If instead of making the ordinates of the logarithmic curve equal to $(1+\rho)^n$ we make them equal to $\frac{N\lambda.(1+\rho)}{\rho} \times (1+\rho)^n$, then will the *area* of such curve denote the value of an *annuity* of £1 per annum in the given time. In this case, the ordinate AA′ (taking A as the origin of the abscisses, in Fig. 1) will be equal to $\frac{N\lambda.(1+\rho)}{\rho}$ instead of being equal to unity as in the former cases: and the *space* A′B′BA will now represent unity, if AB denote one year. Also, the space A′C′CA will represent the *amount* of the annuity in two years; and A′D′DA, the *amount* of the annuity in three years, &c., &c. In like manner, the space A′*b*′*ba* will represent the *present value* of the annuity for one year; the space A′*c*′*ca*, the *present value* of the annuity for two years, &c., &c. And the rectangle A′A × AE (equal to twice the triangle A′EA) will represent the present value of the *perpetuity* of £1 per annum: that is, it will be equal to the space comprehended between the indefinite right line AX and the curve A′X′ extended *ad infinitum*.

§ 219. The present value of *reversions* also is correctly shown by means of this curve: for, the present value of the reversion of the annuity for the time denoted by the absciss *cd*, after the time denoted by A*c*, is equal to the area *c′d′dc*. And the value of the reversion of a *perpetuity* of £1 per annum after the time denoted by A*c* is equal to the rectangle of the ordinate *cc′* and the constant subtangent: that is, equal to *cc′* × AE.

§ 220. In this manner most of the leading theorems in the doctrine of interest and annuities may be rendered *obvious to the sight:* but enough has been said to show the connexion of this subject with the nature of the curve here alluded to; and the reader may readily extend his inquiries and researches should he think the matter worthy attention.

PART II.—FORMULÆ

FOR ALL QUESTIONS IN COMPOUND INTEREST AND LIFE CONTINGENCIES RELATING TO ONE LIFE, TWO LIVES, AND THREE LIVES.

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THEOREMS

In Compound Interest and Life Assurance.*

1. Amount to which P will accumulate in $(n+\frac{a}{b})$ years

$$=P(1+r)^n(1+\frac{ar}{b}).$$

2. Amount to which P will accumulate in $n.m$th parts of a year, at interest payable by instalments $\frac{r}{m}$ at the end of each such mth part

$$=P(1+\frac{r}{m})^{mn}.$$

3. Amount to which P accumulates at momentaneous interest $=P\epsilon^{nr}$ where $\epsilon =$ the base of the Naperian logarithms, or $= 2{\cdot}71828$, &c.

4. True annual rate of interest equal to a nominal rate paid in mth portions throughout the year

$$=(1+\frac{r}{m})^m-1.$$

5. True annual rate equal to momentaneous interest $= \epsilon^r - 1$.

6. Nominal rate corresponding to $(\epsilon^r - 1)$, if $\epsilon^r - 1 = k$,

$$r = \text{N}\lambda(1+k).$$

7. The amount of £1 at compound interest in $m + n$ years = product of the amount in m years multiplied by the amount in n years.

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* The substance of the first two sections is extracted from the "Treatise on Associations for Provident Investment." By Arthur Scratchley, Esq., M.A. [Except articles 60, 61, 65, 66, and from 80 to 114 inclusive, which are of different authors, as enumerated in the Preface to this work by the Editor.]

8. The amount of £1 at yearly interest r in n years = amount in $\frac{n}{m}$ years at interest payable m times a year at the nominal rate of mr; for, the amount of a given sum, at the end of any number of periods of time, does not depend on the length of each period, but on the number of them and the quantity of interest payable at the end of each.

9. If A be the amount of £1 in n years at yearly interest, the amount at interest payable m times a year

$$= \mathrm{A}\left[1 + \frac{n(1-\frac{1}{m})}{2} \cdot r^2\right] \text{ nearly.}$$

10. At momentaneous interest, $nr = \mathrm{N}\lambda(\frac{\mathrm{A}}{\mathrm{P}})$.

If P amount to A in N_1 years at r_1 interest, it would amount to A in $\frac{\mathrm{N}_1 r_1}{r_2}$ years at r_2 interest, or in $q\mathrm{N}$ years at $\frac{r}{q}$ interest.

11. At momentaneous interest, the number of years in which £1 will become £f $= \frac{\mathrm{N}\lambda f}{r}$.

If $f = 2$, money will double at r per cent in $\frac{69{\cdot}3147}{r}$ years.

If $f = 3$, money will become trebled in $\frac{109{\cdot}8612}{r}$ years.

12. At yearly interest, payable m times a year, money will become f-fold in $\frac{\mathrm{N}\lambda f}{\mathrm{N}\lambda(1+\frac{r}{m})}$ intervals, or nearly $\frac{\mathrm{N}\lambda f}{(\frac{r}{m})} + \frac{\mathrm{N}\lambda f}{2}$.

If $f = 2$, money will double in nearly $\left(\frac{69{\cdot}3147}{(\frac{r}{m})} + {\cdot}3465\right)$ intervals, or

nearly $= \left(\frac{70}{\text{rate of interest per cent}}\right)$ years.

13. If the specific velocity of doubling be represented by 1, the relative velocities of attaining successive units of capital would be

$$1,\ \frac{\mathrm{N}\lambda 2}{\mathrm{N}\lambda(\frac{3}{2})},\ \frac{\mathrm{N}\lambda 2}{\mathrm{N}\lambda(\frac{4}{3})},\ \ldots\ \frac{\mathrm{N}\lambda 2}{\mathrm{N}\lambda(1+\frac{1}{f})},$$

or, very nearly, by

1, 1·70, 2·40, 3·10, 3·80, &c.

PRESENT VALUE AND DISCOUNT.

14. The discount for present payment of S due at the end of n years

$$=S\{1-(1+r)^{-n}\}.$$

15. The present value of S, if the interest be payable in mth portions throughout the year,

$$=S\{1+\frac{r}{m}\}^{-mn}.$$

16. If it be paid momentaneously,

$$=S\epsilon^{-nr}.$$

17. The present value of £1 due at the end of $(n_1+n_2+n_3+\&c.)$ years, is equal to the product of the separate present values.

18. If P be the value of a sum at yearly interest, the value at interest payable m times a year is nearly

$$=P\left[1-\frac{n(1-\frac{1}{m})}{2}\cdot r^2\right].$$

19. The difference between the present values of the same sum at two different rates of yearly interest, r,

$$=S\{(1+r_1)^{-n}-(1+r_2)^{-n}\}.$$

If the interest be realized m times a year

$$=S\left[(1+\frac{r_1}{m})^{-mn}-(1+\frac{r_2}{m})^{-mn}\right].$$

If momentaneously $=S\{\epsilon^{-nr_1}-\epsilon^{-nr_2}\}.$

These differences increase up to a certain point with the value of n, and then diminish. If the interest be momentaneous, the maximum difference is when

$$n=\frac{N\lambda r_2-N\lambda r_1}{r_2-r_1}.$$

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ANNUITIES.

20. Amount of an annuity of £a

$$=a\left[\frac{(1+r)^n-1}{r}\right].$$

21. Amount at yearly interest r, payable in mth portions,

$$=a\left[\frac{(1+\frac{r}{m})^{mn}-1}{r}\right].$$

22. Amount at momentaneous interest

$$=\frac{a}{r}(\epsilon^{nr}-1).$$

23. The differences of a table of the amounts of annuities gives an annuity table for the amount of a single sum at the end of n years, for, if A be the amount of an annuity,

$$A_{n+1}-A_n=(1+r)^n.$$

24. The present value of an annuity of £a

$$=a\left[\frac{1-(1+r)^{-n}}{r}\right].$$

25. The difference between the reciprocals of the present value and of the amount of an annuity is the rate of interest; for

$$\frac{1}{P_n}-\frac{1}{A_n}=r.$$

The difference of the reciprocals of the present value of annuities for two terms is equal to the difference of the reciprocal of their amounts; for

$$\frac{1}{P_m}-\frac{1}{P_n}=\frac{1}{A_m}-\frac{1}{A_n}.$$

26. The present value of £1 due in n years = the difference of the present values of an annuity of £1 for n and $(n-1)$ years; and the differences in a table of annuity-values will give the values of single sums.

27. The present value of an annuity payable and interest realizable m times a year

$$=\frac{a}{r}\left[1-(1+\frac{r}{m})^{-mn}\right].$$

28. If the annuity and interest are payable momentaneously

$$=\frac{a}{r}(1-\epsilon^{-n}{}_{r}).$$

29. An annuity payable for n years at the beginning of each year

$$=A_{n+1}-1.$$

30. The repayment to be made m times a year for a loan of P for n years

$$=\frac{P(\frac{r}{m})}{1-(1+\frac{r}{m})^{-mn}}.$$

31. If the term of years be such that the money would become f-fold if it were not paid, the periodical repayment would be

$$=\frac{f}{f-1}\mathrm{P}\frac{r}{m},$$

and the annual repayment would be $\frac{f}{f-1}$ times a year's interest on the debt.

32. In the time in which money would double, the annuity to repay the loan would be twice a year's interest.

33. The amount of an annuity of £1 in the time in which a single sum would become f-fold

$$=\frac{100\,(f-1)}{\text{rate of interest per cent}}.$$

In the exact time in which the money would double

$$=\frac{100}{\text{rate of interest per cent}}.$$

34. The balance due on a loan P for a term of years, after x yearly repayments on account of principal and interest have been made, is

$$=\frac{a}{r}-(\frac{a}{r}-\mathrm{P})\,(1+r)^{x}.$$

35. The number of years in which an annuity of c per cent would repay a loan, principal and interest, is

$$=-\frac{\log(1-\frac{100r}{c})}{\log(1+r)}.$$

36. Let $$d=(\frac{\mathrm{A}_n}{an})^{\frac{2}{n-1}}-1,$$

then the rate of interest at which a per annum will amount to A_n in n years is

$$=\frac{(12+\overline{n+1}.d)d}{12+2(n+1)d}.$$

37. If tables be at hand showing A_{r_1}, the amount of £1 a year in n years at r_1 interest, and $\mathrm{A}_{(r_1-\frac{1}{100})}$ the amount of £1 a year in n years at $r_1-\frac{1}{100}$ interest, then to find r, the rate at which a per annum will amount to A in n years,

$$r=r_1+\frac{\frac{\mathrm{A}}{a}-\mathrm{A}_{r_1}}{100\,(\mathrm{A}_{r_1}-\mathrm{A}_{(r_1-\frac{1}{100})})}\text{ nearly.}$$

38. The rate of interest at which P is the present value of a per annum for n years

$$r=\frac{\{12-(n-1)d\}d}{12-2(n-1)d}$$

where
$$d=\left[\frac{an}{\mathrm{P}}\right]^{\frac{2}{n+1}}-1$$

By a table of present values

$$r=r_1+\frac{\mathrm{P}r_1-\frac{\mathrm{P}}{a}}{100[\mathrm{P}_{(r_1-\frac{1}{100})}-\mathrm{P}r_1]}.$$

39. The annual profit made by borrowing a per annum at a low rate r and lending it out at r_1

$$=a\left[1-\frac{r}{r_1}\cdot\frac{1-(1+r_1)^{-n}}{1-(1+r)^{-n}}\right].$$

40. The monthly payment corresponding to a yearly payment a for n years

$$=\frac{a}{12}\cdot\frac{1-(1+r_1)^{-n}}{1-(1+\frac{r_1}{12})^{-12n}}.$$

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LIFE ASSURANCE.

SECT. I.

SYMBOLS USED IN THIS SECTION.

l_x= the number of lives in existence at age x by the table of mortality adopted.

v = the present value of £1 due at the end of one year $=\frac{1}{1+r}$, the rate of interest per £ being r.

d will frequently be employed in lieu of $\frac{r}{1+r}$ or $1-v$, and the character $\hat{a}_{\propto}$ for $1+\frac{1}{r}$ or $\frac{1}{d}$.

a_x= present value of an annuity of £1 payable at the *end* of each year, to continue during the existence of a life aged x.

A_x $\hat{a}_x=1+a_x$= present value of a similar annuity payable at the *beginning* of each year.

Π_x= single premium to assure £1, to be received at the end of the year in which a life now aged x may die = present value of £1 reversionary.

? π_x= annual premium, payable at the *beginning* of each year, during the life of a person aged x, to assure £1 as before, at the end of the year in which he may die.

p_x= Annual Office premium to assure £1 at death on a life aged x.

41. Now π_x may be determined from a table of annuities thus:—A person borrowing £1 at once for the term of his life could repay it with interest by an annual payment at the beginning of each year, consisting of the premium of assurance, to restore the principal at his death, and annual interest r per £, which, if paid yearly in advance, would be reduced to $\frac{r}{1+r}$ or d.

∴ £1 would be repaid by, and is equivalent to, an annuity payment of π_x+d.

But £1 would buy an annuity of $\frac{1}{\hat{a}_x}$ on the same life, and at the same rate of interest, ∴ these two values must therefore be equal, and we have

$$\pi_x+d=\frac{1}{\hat{a}_x}, \text{ or } \pi_x=\frac{1}{\hat{a}_x}-d\text{.........................}(1).$$

This form, by simple reasoning, can be obtained thus:—

Since £1 will buy either an annuity *due* of $\frac{1}{a_x}$ during a life x, or a *perpetuity due* of d, then the difference of the purchasable annuities, or $\frac{1}{\hat{a}_x}-d$, should be sufficient to purchase a perpetuity *due* at death of life x, which would cost £1; or, in other words, an annuity of $\frac{1}{\hat{a}_x}-d$ set aside during a life x would amount to £1 at the *end* of the year in which he may die, and is therefore equal to π_x.

42. Into the value of $\hat{a}_x$, which is made use of in this formula, enter the considerations depending on the table of mortality. The formula itself is identical in form, as it should be, with that by which would be determined the requisite annual payment at the beginning of each year, or the sinking fund, to accumulate to £1 at the end of a term certain of $x+1$ years, in terms of the *present* value π_x of an ordinary annuity of £1 for x years, where

$$\pi_x=\frac{1-(1+r)^{-x}}{r} \text{ (See § 24)},$$

and annual payment, or sinking fund,

$$=\frac{1}{1+\pi_x}-d\text{...............................}(1 \text{ bis.})$$

It may be mentioned here, incidentally, that if for $\hat{a}_x$ were substituted the values of an annuity for the whole duration of two or more lives, the formula (1) would bear the same relation with regard to them as it does to the single life, and the symmetrical form would be preserved.

43. To determine the relation between Π_x and $\hat{a}_x$. Since $\hat{a}_x$ is the

difference between a perpetuity *due* commencing at once, and a perpetuity due not to commence until the *end* of the year in which the life x may die, of which the present value is $\Pi_x . \hat{a}_\infty$, then

$$\begin{aligned}\hat{a}_x &= \hat{a}_\infty - \Pi_x . \hat{a}_\infty \\ &= \hat{a}_\infty . (1 - \Pi_x) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2),\end{aligned}$$

a formula of most convenient use, by which a life-annuity table can be deduced from that of the present value of a reversion.

From (2) we have

$$\Pi_x = 1 - \frac{\hat{a}_x}{\hat{a}_\infty} = 1 - d\hat{a}_x \quad \ldots\ldots\ldots\ldots\ldots\ldots (3),$$

a simple form for the present value of a reversion, or the *single premium* for an assurance of £1 in terms of the life annuity.

Again, since the present value of all the yearly premiums π_x must be the same as the single premium Π_x, we have

$$\pi_x . \hat{a}_\infty = \Pi_x \; \therefore \; \pi_x = \frac{\Pi_x}{\hat{a}_x},$$

or from (2),

$$\pi_x = \frac{\Pi_x . d}{1 - \Pi_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4).$$

44. The formula for the *single premium* can be proved otherwise, thus:—If the possession of £1 reversionary were deferred until the time of the last annuity payment of interest r thereon which could be received by a life aged x, its present value would be simply £1 less ra_x, or $1 - ra_x$.

But the £1 is not receivable until twelve months after, or until the *end* of the year in which the life x dies; hence we must discount ($\hat{a}$) for one year to make the result equal to Π_x,

$$\therefore \Pi_x = \frac{1 - ra_x}{1 + r},$$

which is of the same value as (3).

Two useful results may be obtained immediately from (1) and (4)—the former gives

$$\hat{a}_x = \frac{1}{\pi_x + d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5),$$

and the latter,

$$\Pi_x = \frac{\pi_x}{\pi_x + d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6).$$

45. It is worthy of attention that if formulæ relating to life contingencies be made to depend, when practically, on but *one unknown* quantity, and to involve symbols for annuities due or payable at the *beginning* instead of at the *end* of each year, their forms are not only more elegant, but more simple for practical use and remembrance. Such forms are also preferable, as they are then analogous to formulæ for *premiums*, which are customarily payable at the *beginning* of the year; and they can be as easily adapted to cases of *half-yearly* or *quarterly* annuities, as formulæ involving annuities due at the end of the year. It is an objection to exist-

ing works on life contingencies that this analogy is disregarded; and we would add, that in considering the relative value of formulæ, the Actuary should distinguish between those which are merely valuable for the computation of new tables, and others which enable him with but one or two tables to solve questions submitted to him. For the latter purpose he should prefer formulæ which, as far as practicable, are functions of as few tabular results as possible. An instance of simplicity can be found in the formula for the value of a life assurance policy.

Let p_x be the OFFICE premium charged to assure £1 on life x. Then if a renewal premium be just due but not paid, the value of a policy of £1 at the end of n years

$$\begin{aligned} &= \Pi_{x+n} - p_x . \hat{a}_{x+n} \\ &= \Pi_{x+n} - p_x . \hat{a}_{\propto}(1-\Pi_{x+n}) \\ &= 1-(1+p_x \hat{a}_{\propto})\,(1-\Pi_{x+n}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (7), \end{aligned}$$

which expression for the value of a policy contains only one variable Π_{x+n}.

Again, if p_x coincide with the *nėt* unloaded premium π_x, which by (4) is

$$= \frac{\Pi_x . d}{1-\Pi_x},$$

$$\therefore \text{ value of policy from (7) is } = \frac{\Pi_{x+n}-\Pi_x}{1-\Pi_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8).$$

If the policy be in amount P instead of £1, the results above found would have to be multiplied by P.

46. Let $\Pi_{\overline{xy}}$ = present value of £1 in reversion after two lives x, y are both dead; $\Pi_{x\cdot y}$ = present value of £1 at death of either; then $\Pi_x + \Pi_y$, which is the sum of the present values separately on the two lives, must be equal to

$$\Pi_{x\cdot y} + \Pi_{\overline{xy}}$$

whence

$$\Pi_{\overline{xy}} = \Pi_x + \Pi_y - \Pi_{x\cdot y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9),$$

a form analogous to that of an annuity on the *last* of two lives being $a_{\overline{xy}} = a_x + a_y - a_{x\cdot y}$; and generally it must be noticed that symmetrical forms of functions of Π_x attend those of $\hat{a}_x$, Π_x being in reality but the present value of an annuity for one year only, deferred till the death of x.

Similarly,

$$\hat{a}_{\overline{xy}} = \hat{a}_{\propto}\{1-(\Pi_x + \Pi_y - \Pi_{x\cdot y})\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10).$$

If the annual premium in (4) to assure £1 at death of life x were to be payable during the existence of a life y instead of x, then

$$\pi_y = \frac{\Pi_x d}{(1-\Pi_y)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11).$$

If, as in a *Debtor* and *Creditor* policy, the assurance were on a life x (the *borrower*), but the annual premium to be payable only while *both* are alive, that is to say, to cease when either creditor or borrower die, then

$$\pi_{x\cdot y} = \frac{\Pi_x . d}{.(1-\Pi_{x\cdot y})} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12).$$

47. To pass from the *net* premium $\pi_{x\cdot y}$ to the proper *office* premium $p_{x\cdot y}$,

corresponding in margin to office premium p_x given by the Society's Tables :—

We have $$p_{x\cdot y}.\hat{a}_{x\cdot y}=p_x.\hat{a}_x$$

$$p_{x\cdot y}=p_x\frac{\hat{a}_x}{.\hat{a}_{x\cdot y}} \quad\text{.............................}\ (13),$$

or, $$=p_x\frac{1-\Pi_x}{1-\Pi_{x\cdot y}} \quad\text{.............................}\ (14).$$

The price or present value of an assurance of £1 to begin on a life now aged x, at the expiration of n years, and to secure £1 to his heirs whenever he die *after* that period has expired, is

$$=\frac{l_{x+n}}{l_x}.v^n.\Pi_{x+n} \quad\text{.............................}\ (15).$$

The price of an assurance of £1 on same life, the risk to begin at once, and to last for a term of n years only, is clearly the difference between the cost of an assurance for the whole of life, Π_x, and one deferred n years,

$$=\Pi_x-\frac{l_{x+n}}{l_x}.v^n\Pi_{x+n} \quad\text{.........................}\ (16).$$

ON DEPOSIT LIFE ASSURANCES.

48. The formulæ for the system of Deposit Life Assurance are simple, and easily determined. As it presents another feature of provident investment for the savings of the industrious classes, a few words upon the system will not be out of place. The advantage offered consists of the Savings Banks' facilities of withdrawal of the whole or part of the deposit premium, whilst the benefits of an ordinary life office are secured.

Let P' = the single deposit money to purchase a life assurance policy A_x to be received at death, with the understanding that, after twelve months have elapsed, a sum equal to $\frac{P'}{m}$ may be withdrawn on demand, or at very brief notice; then it would be requisite for the safety of the company to invest the withdrawable portions at a much lower rate of interest, or in immediately convertible securities, if it be desired to be always ready to meet all *withdrawal* demands. The chance of early repayment of the *whole* deposit, that is, of its being deducted through *death*, does not enter into that consideration, as, on an average of lives, supposing the general funds of the company to be invested in the usual manner, the payments and receipts would follow the law of mortality.

49. The calculation of A_x will therefore be analogous to that for determining the *single* premium for the assurance of £1; with this difference, that our plan consists in keeping the rates of interest allowed upon P' or

$\left(\frac{m-1}{m}\mathrm{P}' \text{ and } \frac{\mathrm{P}'}{m}\right)$ independent of the rate of interest and margin originally adopted in the determination of p_x, and in making a table containing the values of p_x the basis for calculating A_x.

Let an ordinary whole life annual premium table be in use by the company.

$r =$ the highest rate of annual interest that the company can afford to credit upon $\frac{m-1}{m}.\mathrm{P}'$.

$r' =$ a lower annual rate allowed upon $\frac{\mathrm{P}'}{m}$, and let $d' = \frac{r'}{1+r'}$.

50. From the moment P′ is deposited, the assurance risk represented by A_x commences, although, according to the usual theoretical hypothesis (contrary to the occurrence of actual practice), A_x is not payable until the *end* of the year in which the life (x) may die. Proceeding, however, on that supposition, the sum assured A_x will be equal to the deposit P′ and a sum $(\mathrm{A}_x - \mathrm{P}')$ arising from the interests upon P′. Now P′, deposited, is credited with annual interest $=\mathrm{P}'.\frac{(m-1)r+r'}{m}$ due at the end of each year, which is equivalent to

$$\frac{\mathrm{P}'}{m}\left\{\frac{(m-1)r}{1+r}+\frac{r'}{1+r'}\right\} \text{ or } \frac{\mathrm{P}'}{m}\left\{(m-1)d+d'\right\} \text{ at the beginning.}$$

Hence, since the annual premium π_x paid at the beginning of every year of life would assure £1 payable at the *end* of that year in which the life may die, the deposit P′ may be considered by the company as producing at the beginning of each year a premium $=\frac{\mathrm{P}'}{m}\left\{(m-1)\,d+d'\right\}$.

$$\therefore \mathrm{A}_x = \mathrm{P}' + \frac{\mathrm{P}'}{m}\left\{(m-1)d+d'\right\}\cdot\frac{1}{p_x}$$

$$= \mathrm{P}'\left[1+\frac{1}{m}\left\{(m-1)d+d'\right\}\frac{1}{p_x}\right] \ldots\ldots\ldots\ldots (17).$$

51. By this equation a table of deposit assurances can be deduced from an ordinary life table on substituting the values of p_x at various ages; and the *Deposit-Policies* will be with or without profits, according to the hypothesis relative to p_x. In practice, it would be generally expedient and reasonable not to allow any withdrawals until after a small number of years m', greater than one, in which case, the equation (17) should still be used, as it is not worth while to complicate it by the consideration that during $(m'-1)$ years the rate of interest r might be credited on the whole of P′. The advantage would be in favour of the company. When m' is

not less than 3, we consider that r' may be taken $= \cdot 025$ and $r = \cdot 035$. This will be understood by the consideration that in all investments of this kind accepted by Insurance Companies, or Benefit Building and other similar Societies, the larger the portion of his deposit over which the depositor has power of withdrawal on demand, the greater will be the capital which must be kept by the company in immediately convertible securities to meet sudden withdrawals; and the lower will be the average interest derived on the aggregate of its funds. If six months' or a year's notice were required of intended withdrawal, the case would be different.

If $p_x = \pi_x$, $r' = r =$ the same rate involved in π_x, and $m=1$, $A=1$, equation (17) is reduced to

$$1 = P' . \left\{ 1 + \frac{r}{1+r} \cdot \frac{1}{\pi_x} \right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18),$$

the ordinary formula expressing the relation between the single premium P' and the annual premium π_x to assure £1.

52. As regards assurers, equation (17) gives the amount of policy which an *occasional* deposit would assure upon his life. To the industrious classes one or two smaller policies, created by single deposits, are more convenient than the general system by which a fixed premium is required at regularly recurring intervals, without their having, in most offices, any protection against the loss of the policy, in the event of the assurer's means not enabling him to keep up his payments. The advantage of an ordinary savings' bank is also presented, since a portion of his deposit-money may be withdrawn on demand. Such policies would thus serve as negotiable commodities in commercial transactions.

Suppose an ordinary *Annual Premium* Policy of Assurance to be taken out by a life aged x years, and, after a certain number of n years, the assurer desire to suspend all future payments, and to obtain, for his acquired interest in the company, a *Deposit-Policy;* then the amount thereof

$$= (\text{office value of old policy}) \times \left[1 + \frac{1}{m} \left\{ (m-1)d + d' \right\} \frac{1}{p_{x+n}} \right] \ldots\ldots (19)$$

(the mth part of the office value of the old policy being withdrawable).

53. If after n years, $\frac{P'}{m}$ be withdrawn on a deposit policy, the *diminution* in the policy, found by reducing equation (17) to the case of the whole of the deposit being withdrawable on demand, and applying it to the effect produced by withdrawing $\frac{P'}{m} = \frac{P'}{m} \left\{ 1 + \frac{d'}{p_{x+n}} \right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$, an endorsement upon the original deposit policy would then be made stating that it is diminished to the extent represented by (20).

The society should always get the benefit of the difference between the *office* and *real* age of the depositor, both at the time of entry and at that of withdrawal, with the distinction that x in (17) should be taken at *next* birthday, but $(x+n)$ in (20) at *last* birthday.

54. By way of further illustration of the analogy between functions of annuities for *Terms certain* and for the *whole* duration of a life or lives, we will refer to Art. 23, and will show how the property $\frac{1}{P'_n}-\frac{1}{A_n}=r\ldots(21)$ will serve, in the case of a life annuity, to determine the *single premium* Π_x *to assure* £1 *at the death of a life aged* x. [In No. 51 of these notes P′ stands for Π_x.]

For the *present value* of an annuity of £1, payable at the end of *every* year that the life x may enter, is $a_x+\Pi_x$, this corresponds to P'_x in equation (21). Again, the *accumulated amount* of such an annuity by the end of the year in which life (x) may die would be $1+\frac{a_x}{\Pi_x}$; since it would consist of the last annuity payment + the improved amount of a_x, at the close of the duration of the life, which by proportion would be $\frac{a_x}{\Pi_x}$; so that $1+\frac{a_x}{\Pi_x}$ corresponds to A_x in (21).

Then, since the analogy should subsist, the equation $\frac{1}{a_x+\Pi_x}-\frac{1}{1+\frac{a_x}{\Pi_x}}=r$ must hold, whence

$$\Pi_x=\frac{1-r.a_x}{1+r},$$

the ordinary formula.

The property in (21) can be remembered by the consideration that it expresses merely that

$$\left\{\begin{matrix}\textit{The annuity for n years,}\\ \textit{which £1 would purchase}\end{matrix}\right\}-\left\{\begin{matrix}\textit{The sinking fund to create}\\ \textit{£1 at the end of n years}\end{matrix}\right\}=\left\{\begin{matrix}\textit{One year's}\\ \textit{interest.}\end{matrix}\right\}$$

Sect. II.

On the Columnar Method of Notation.

55. In order to shorten calculations involving life contingencies, extensive tables of what may be termed artificial numbers have been computed; and as these tables are very generally employed—owing to the facilities they afford in certain instances—it will be proper to give some account of their construction and use. According to this system—called sometimes the *columnar* and sometimes the *commutation* method—if x be any age,

l_x the number of persons living at that age in the table of mortality, v^x the present value of £1 due at the end of x years, and z the last age in the table,

$$D_x = l_x v^x.$$
$$N_x = D_x + D_{x1} + D_{x2} + D_{x3} \ldots + D_z.$$
$$M_x = d_x v^{x1} + d_{x1} v^{x2} + \ldots\ldots\ldots + l_z v^{z1}.$$
$$S_x = N_x + N_{x1} + N_{x2} + \ldots\ldots + N_z = \Sigma N_x.$$
$$R_x = M_x + M_{x1} + M_{x2} + \ldots\ldots + M_z = \Sigma M_x.$$

The values of the quantities D_x, N_x, M_x, &c., are calculated in the tables referred to for every age x, and the manner in which such tables become subservient to actuarial investigations will now be shown. It will be as well, however, first to draw attention to three relations between the capital letters in the above system which are often useful, namely—

$$D_x + N_{x1} = N_x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\alpha)$$
$$M_x + N_{x1} = vN_x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta)$$
$$D_x - M_x = dN_x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\gamma).$$

The first of these is quite obvious, and the second (β) is easily proved; for

$$M_x = l_x v^{x1} - l_{x1} v^{x1} + l_{x1} v^{x2} - l_{x2} v^{x2} + \ldots\ldots$$
$$N_{x1} = l_{x1} v^{x1} + l_{x2} v^{x2} + l_{x3} v^{x3} + \ldots.$$
$$\therefore M_x + N_{x1} = l_x v^{x1} + l_{x1} v^{x2} + l_{x2} v^{x3} + \ldots$$
$$= vN_x.$$

The third relation (γ) is obtained by subtracting (β) from (α).

56. IMMEDIATE ANNUITIES.—The present value of an annuity of £1 for the whole of life at age x being $\frac{l_{x1}v + l_{x2}v^2 + l_{x3}v^3 + \&c.}{l_x}$, it becomes, by multiplying the numerator and denominator by v^x, $\frac{l_{x1}v^{x1} + l_{x2}v^{x2} + l_{x3}v^{x3} + \&c.}{l_x v^x}$, which, under the above system of notation will evidently be denoted by $\frac{N_{x1}}{D_x}$

57. *Temporary Annuity.*—The present value of a life annuity of £1 to continue during n years only being $\frac{l_{x1}v + l_{x2}v^2 \ldots\ldots\ldots + l_{x+n}v^n}{l_x}$ $= \frac{l_{x1}v^{x1} + l_{x2}v^{x2} \ldots\ldots + l_{x+n}v^{x+n}}{l_x v^x}$ will be expressed by $\frac{N_{x1} - N_{x1+n}}{D_x}$.

58. If the annuity is to be paid at the *beginning* of each year, it will consist of the first payment, made immediately, and of an annuity for $n-1$ years paid at the end of each year, and will therefore be

$$= 1 + \frac{N_{x1} - N_{x+n}}{D_x} = \frac{D_x + N_{x1} - N_{x+n}}{D_x} = \frac{N_x - N_{x+n}}{D_x} \text{ by } (\alpha).$$

59. *Deferred Annuity by Single Premium.*—Since the value of a deferred annuity to commence at the end of n years $=\frac{D_{x1+n}+D_{x2+n}+D_{x3+n}+\&c.}{D_x}$, the expression for this will become $\frac{N_{x1+n}}{D_x}$.

60. *Deferred Annuity by Annual Premiums.*—The annual premium payable by x during n years to insure a deferred annuity of £1, to commence at the end of n years, the premiums to be returned in the event of his death before the age $x+n$.

$$\pi_{x(n}=\frac{N_{x+n1}}{N_{x|n}-R_{x|n}+nM_{x+n}}.$$

61. If the premiums are not returnable, strike out $(-R_{x|n}+nM_{x+n})$.

62. *An intercepted Annuity.*—If the annuity is to cease after m payments, that is, to commence at age $x+n$ and continue till age $x+n+m$, its value will evidently be that of a life annuity deferred n years, minus the value of a life annuity deferred $n+m$ years, and is therefore $=\frac{N_{x1+n}-N_{x1+n+m}}{D_x}$.

Lastly, if the annuity is to be payable at the *beginning* instead of at the end of each of the m years, its value would be $\frac{N_{x+n}-N_{x+n+m}}{D_x}$.

63. WHOLE LIFE ASSURANCES.—The single premium to be paid down in order to assure £1 at death, is $\frac{d_xv+d_{x1}v^2+\&c.}{l_x}=\frac{d_xv^{x1}+d_{x1}v^{x2}+\&c.}{l_xv^x}=\frac{M_x}{D_x}$.

Let π_x denote the annual premium for the same, payable at the beginning of each year; then since the present value of these yearly payments must equal the present value of the risk, or the single premium just found, we have

$$(1+\frac{N_{x1}}{D_x})\,\pi_x=\frac{M_x}{D_x}\quad\therefore\ \pi_x=\frac{M_x}{D_x+N_{x1}}=\frac{M_x}{N_x}.$$

If the annual premiums are to be payable for the first n years only, the sum assured being still payable at death, we must have, calling the annual payment π,

$$\left(\frac{N_x-N_{x+n}}{D_x}\right)\pi=\frac{M_x}{D_x},$$

$$\therefore\ \pi=\frac{M_x}{N_x-N_{x+n}}.$$

64. *Temporary Assurances.*—If from the single premium for a whole life assurance we subtract that for a similar assurance deferred n years, the remainder will be the value of a temporary assurance to commence imme-

diately and to continue for n years only. We have, therefore, single premium for temporary assurance to commence at present age x, and to last till age $x+n=\frac{M_x-M_{x+n}}{D_x}$.

Let $\pi_{x)n}$ be the equivalent annual premium, the first payment of which is made immediately, the remaining $n-1$ payments constituting a temporary life annuity for that number of years, then the following relation must obtain,

$$\left(\frac{N_x-N_{x+n}}{D_x}\right)\Pi=\frac{M_x-M_{x+n}}{D_x},$$

$$\therefore \pi_{x)n}=\frac{M_x-M_{x+n}}{N_x-N_{x+n}}=\frac{D_{x|n}-dN_{x|n}}{D_x}.$$

65. *Deferred Assurance.*—Single premium to insure £1, payable at the death of x if it happen after n years.

$$\Pi_{x(n}=\frac{M_{x+n}}{D_x}=\frac{D_{x+n}-dN_{x|n}}{D_x}.$$

66. *Assurance by Limited Payments.*—The annual premium during n years from age x, to insure £1 at death.

$$\pi_{x(n)}=\frac{M_x}{N_{x|n}}=\frac{D_x-dN_x}{N_{x|n}}.$$

67. *Endowments—Premiums non-returnable.*—The single premium for a person aged x to secure the payment of £1 on his completing his $(x+n)$th year, the premium to be forfeited if he die previously, is $\frac{l_{x+n}v^n}{l_x}=\frac{l_{x+n}v^{x+n}}{l_xv^x}$ $=\frac{D_{x+n}}{D_x}$. The annual premium for the risk, found as in the case of a temporary assurance, $=\frac{D_{x+n}}{N_x-N_{x+n}}$.

68. *Endowments—Premiums returnable.*—Suppose an endowment of £1 to be secured to a person aged x on his attaining the age $x+n$, by a single payment Π, which is to be returned if death happen before the specified age is reached. The whole risk will consist of a temporary assurance for the sum Π for n years, added to the value of a simple endowment payable at the end of n years, we must therefore have

$$\Pi=\frac{M_x-M_{x+n}}{D_x}\times\Pi+\frac{D_{x+n}}{D_x}.$$

$$\therefore (D_x-M_x+M_{x+n})\Pi=D_{x+n}$$

$$\therefore \Pi=\frac{D_{x+n}}{D_x-M_x+M_{x+n}}.$$

69. If the endowment be paid for by an annual premium π, returnable in the event of death happening within the n years, the risk will consist, in addition to a simple endowment, of an increasing assurance for n years, commencing at π and increasing π every year; hence the following relation must obtain

$$\pi\left(\frac{N_{x|n}}{D_x}\right)=\frac{\pi(R_{x|n}-nM_{x+n})}{D_x}+\frac{D_{x+n}}{D_x}$$

$$\therefore\ \pi(N_{x|n}-R_x+R_{x+n}+nM_{x+n})=D_{x+n}$$

$$\therefore\ \pi=\frac{D_{x+n}}{N_{x|n}-R_x+R_{x+n}+n\,M_{x+n}},$$

which is therefore the expression for the annual premium.

70. *Endowment Assurances.*—In these the sum assured is paid on the individual attaining a given age, or at death if that happen previously. It is therefore simply an endowment and a temporary assurance combined in one; hence, adding together the single premiums for these two risks, we get the single premium to secure an endowment assurance of £1 on a life aged x, to be received at age $(x+n)$ or at previous death

$$=\frac{M_x+(D_{x+n}-M_{x+n})}{D_x},$$

and by adding together the *yearly* premiums for the two risks referred to above, we get the annual premium for the endowment assurance,

$$=\frac{M_x+(D_{x+n}-M_{x+n})}{N_{x|n}}.$$

71. Endowment assurance by $n-m$ annual premiums.

$$\frac{M_x+(D_{x+n}-M_{x+n})}{N_x-N_{x+n-m}}.$$

72. *Increasing Annuities.*—Let a person aged x be in possession of an annuity of £1, and suppose he is to enter upon a second annuity of the like amount at the end of one year, also a third similar annuity at the end of the second year, and so on; the value of the whole will be

$$\frac{N_{x1}+N_{x2}+N_{x3}+\&c.}{D_x}\ \text{or}\ \frac{S_x}{D_x};$$

this, therefore, must express the value of an annuity of which the successive payments are 1, 2, 3, 4, &c., the increase continuing to the end of life.

73. If the first annuity of £1 is to continue for n years only, the second for $n-1$ years, the third for $n-2$ years, and so on, all payments ceasing at the end of n years, the value of the whole will be

$$=\frac{N_x-N_{x+n}}{D_x}+\frac{N_{x2}-N_{x1+n}}{D_x}+\frac{N_{x3}-N_{x2+n}}{D_x}+\&c.\ \ldots+\frac{N_{x+n}-N_{x1+n}}{D_x}$$

$$=\frac{S_{x1}-S_{x1+n}-nN_{x1+n}}{D_x},$$

which therefore denotes the value of a temporary annuity for n years, of which the successive payments are 1, 2, 3,...n.

74. If, when the annuity reaches the amount n, it is to remain constant and to continue for the remainder of life, its value will evidently be increased by $\frac{n.\mathrm{N}_{x1+n}}{\mathrm{D}_x}$, and will therefore become $=\frac{\mathrm{S}_{x1|n}}{\mathrm{D}_x}$.

75. *Increasing and Decreasing Premiums.*—Let the annual premium to assure £1 at death be π' for the first n years, $\pi'+\delta$ for the second n years, $\pi'+2\delta$ for the third n years, $\pi'+3\delta$ for the fourth n years, and $\pi'+4\delta$ for the rest of life; then we must have

$$\pi'\left(\frac{\mathrm{N}_x-\mathrm{N}_{x+n}}{\mathrm{D}_x}\right)+(\pi'+\delta)\left(\frac{\mathrm{N}_{x+n}-\mathrm{N}_{x+2n}}{\mathrm{D}_x}\right)+(\pi'+2\delta)\left(\frac{\mathrm{N}_{x+2n}-\mathrm{N}_{x+3n}}{\mathrm{D}_x}\right)$$
$$+(\pi'+3\delta)\left(\frac{\mathrm{N}_{x+3n}-\mathrm{N}_{x+4n}}{\mathrm{D}_x}\right)+(\pi'+4\delta)\cdot\frac{\mathrm{N}_{x+4n}}{\mathrm{D}_x}=\frac{\mathrm{M}_x}{\mathrm{D}_x}.$$

$$\therefore \pi'\mathrm{N}_x+\delta(\mathrm{N}_{x+n}+\mathrm{N}_{x+2n}+\mathrm{N}_{x+3n}+\mathrm{N}_{x+4n})=\mathrm{M}_x.$$

$$\therefore \pi'=\frac{1}{\mathrm{N}_x}[\mathrm{M}_x-\delta(\mathrm{N}_{x+n}+\mathrm{N}_{x+2n}+\mathrm{N}_{x+3n}+\mathrm{N}_{x+4n})].$$

76. In a similar manner, if the premium *decrease* δ every five years and remain constant when it becomes $\pi'-4\delta$, the first premium,

$$\pi'=\frac{1}{\mathrm{N}_x}[\mathrm{M}_x+\delta(\mathrm{N}_{x+n}+\mathrm{N}_{x+2n}+\mathrm{N}_{x+3n}+\mathrm{N}_{x+4n})].$$

77. *Increasing and Decreasing Assurances.*—Let it be required to find the value of an assurance for n years, A being the sum assured the first year, 2A the second, 3A the third, and so on, nA being the amount for the last year. The value will be

$$\mathrm{A}\left(\frac{\mathrm{M}_x-\mathrm{M}_{x+1}}{\mathrm{D}_x}\right)+2\mathrm{A}\left(\frac{\mathrm{M}_{x+1}-\mathrm{M}_{x+2}}{\mathrm{D}_x}\right)+3\mathrm{A}\left(\frac{\mathrm{M}_{x+2}-\mathrm{M}_{x+3}}{\mathrm{D}_x}\right)\cdots\cdots\cdots$$
$$\cdots\cdots\cdots+n\mathrm{A}\left(\frac{\mathrm{M}_{1x+n}-\mathrm{M}_{x+n}}{\mathrm{D}_x}\right)$$
$$=\frac{\mathrm{A}}{\mathrm{D}_x}\cdot(\mathrm{M}_x+\mathrm{M}_{x+1}+\mathrm{M}_{x+2}\cdots\cdots\mathrm{M}_{1x+n}-n.\mathrm{M}_{x+n})$$
$$=\frac{\mathrm{A}(\mathrm{R}_{x|n}-n.\mathrm{M}_{x+n})}{\mathrm{D}_x}.$$

78. If the assurance continued to increase to the end of life, the value will evidently be $\mathrm{A}.\frac{\mathrm{R}_x}{\mathrm{D}_x}$.

79. If the assurance be $m+\mathrm{A}$ the first year, $m+2\mathrm{A}$ the second, $m+3\mathrm{A}$ the third, to $m+n\mathrm{A}$, its value will evidently be obtained by adding to the

last result the single premium for assurance of £m for n years, and is therefore

$$=\frac{m(\mathrm{M}_x-\mathrm{M}_{x+n})+\mathrm{A}(\mathrm{R}_x-\mathrm{R}_{x+n}-n\mathrm{M}_{x+n})}{\mathrm{D}_x},$$

and writing $-\mathrm{A}$ for A we obtain the value of a decreasing assurance for n years, commencing with $(m-\mathrm{A})$ and diminishing £A annually to the end of the term,

$$=\frac{m(\mathrm{M}_x-\mathrm{M}_{x+n})-\mathrm{A}(\mathrm{R}_x-\mathrm{R}_{x+n}-n\mathrm{M}_{x+n})}{\mathrm{D}_x}.$$

The last two results, if the increasing or the decreasing assurance be continued to the end of life, become respectively

$$\frac{m\mathrm{M}_x+\mathrm{AR}_x}{\mathrm{D}_x} \text{ and } \frac{m\mathrm{M}_x-\mathrm{AR}_x}{\mathrm{D}_x}.$$

Annuities dependent on Two Lives.

80. The value of an annuity payable during the joint continuance of two lives aged x, y.

$$a_{xy}=\frac{\mathrm{N}_{xy|1}}{\mathrm{D}_{xy}}.$$

81. The value of annuity payable during the joint lines x, y, and n years after y if x live so long.

$$=\frac{\mathrm{N}_{x1|n}}{\mathrm{D}_x}+\frac{\mathrm{N}_{(x1+n)-y1}}{\mathrm{D}_{xy}}.$$

82. An annuity on the life x, to commence at the end of the year in which y may die.

$$a_{x)y}=a_x-a_{xy}.$$

83. A similar annuity on the life y.

$$a_{y)x}=a_y-a_{xy}.$$

84. An annuity on the longest of two lives x, y. This is equivalent to the sum of the preceding three annuities.

$$a_{\overline{xy}}=a_x+a_y-a_{xy}=a_{xy}+a_{x)y}+a_{y)x}.$$

85. A temporary joint annuity on two lives x, y, to cease after n years, or at the death of the first.

$$a_{xy)n}=\frac{\mathrm{N}_{xy|n1}}{\mathrm{D}_{xy}}=\frac{\mathrm{N}_{xy|1}-\mathrm{N}_{xy|n1}}{\mathrm{D}_x}.$$

86. A deferred joint annuity on two lives, x, y, to commence after n years, provided both are then alive, and to cease on the death of the first.

$$a_{xy(n}=\frac{\mathrm{N}_{xy|n1}}{\mathrm{D}_{xy}}.$$

INSURANCES ON TWO LIVES BY SINGLE PREMIUMS.

87. Value of an assurance of £1, payable on the first death of two lives x and y.

$$\Pi_{xy}=1-d\hat{a}_{xy}.$$

88. An assurance of £1, payable at the end of the year of the last death of two lives x and y.

$$\Pi_{\overline{xy}}=1-d\hat{a}_{\overline{xy}}=1-d(\hat{a}_x+\hat{a}_y-\hat{a}_{xy}).$$

89. A temporary assurance of £1, payable on the death of either x or y (or both), if taking place within n years.

$$\Pi_{xy)n}=1-d.\frac{N_{xy|n}}{D_{xy}}.$$

90. A deferred assurance of £1, payable on the death of either x or y (or both), provided it take place after n years.

$$\Pi_{xy(n}=1-d.\frac{N_{xy|n}}{D_{xy}}.$$

91. A temporary assurance of £1, payable on the death of either x or y, provided it take place last, within n years.

$$\Pi_{\overline{xy})n}=1-d\hat{a}_{\overline{xy})n}=1-d\left(\frac{N_{x|n}}{D_x}+\frac{N_{y|n}}{D_y}+\frac{N_{xy|n}}{D_{xy}}\right).$$

92. A similar assurance if death take place after n years.

$$\Pi_{\overline{xy}(n}=1-d\left(\frac{N_{x+n}}{D_x}+\frac{N_{y+n}}{D_y}-\frac{N_{xy(n}}{D_{xy}}\right).$$

INSURANCES ON TWO LIVES BY ANNUAL PREMIUMS.

93. The annual premium during the joint lives of x, y, for an assurance of £1 payable on the first death.

$$\pi_{xy}=\frac{D_{xy}}{N_{xy}}-d.$$

94. The annual premium during the longest life to insure £1 at his death.

$$\pi_{\overline{xy}}=\frac{1}{\hat{a}_{xy}}-d.$$

95. The annual premium for a temporary assurance of £1, payable on the death of the first of two lives, x, y, provided it happen within n years.

$$\pi_{xy)n}=\frac{1}{\hat{a}_{xy)n}}-d.$$

96. To insure £1, payable on the death of the longest of two lives, x, y, provided it take place within n years.

$$\pi_{\overline{xy})n}=\frac{1}{\hat{a}_{xy)n}}-d.$$

97. A similar insurance of £1 *deferred* n years.

$$\pi_{xy(n}=\frac{\mathrm{N}_{xy\|n}}{\mathrm{N}_{xy|n}}.$$

SURVIVORSHIP INSURANCES DEPENDENT ON THE ORDER IN WHICH THE DEATH OF X OR Y HAPPENS.

98. The sum insured is due at the end of the year of age in which the specific death happens; and there are four cases on which this insurance may depend: (1.) the death of x, if it happen first $(x'y)$; (2.) the death of x, if it happen second $(x''y)$; (3.) the death of y, if it happen first (xy'); (4.) the death of y, if it happen second (xy'').

I. Value of £1 at the death of x, if first.

$$\Pi_{x'y}=\frac{v\mathrm{N}_{xy}+v^{\frac{1}{2}}\mathrm{N}_{x\cdot y1}-v^{\frac{1}{2}}\mathrm{N}_{y\cdot x1}-\mathrm{N}_{x1\cdot y1}}{2\mathrm{D}_{xy}}.$$

II. Value of £1 at the death of y, if first.

$$\Pi_{xy'}=\frac{v\mathrm{N}_{xy}+v^{\frac{1}{2}}\mathrm{N}_{y\cdot x1}-v^{\frac{1}{2}}\mathrm{N}_{x\cdot y1}-\mathrm{N}_{x1\cdot y1}}{2\mathrm{D}_{xy}}.$$

III. Value of £1, payable on the death of x, if he die second.

$$\Pi_{x''y}=\Pi_x-\Pi_{x'y}.$$

IV. In case of y dying second.

$$\Pi_{xy''}=\Pi_y-\Pi_{xy'}.$$

99. Value of an annuity on the survivor of X and Y to be divided equally during the joint lives, and to be received wholly by the survivor during the remainder of his life.

$$\text{X's share}=a_x-\frac{a_{xy}}{2}.$$

$$\text{Y's share}=a_y-\frac{a_{xy}}{2}.$$

100. Value of £1, payable on the death of the longest of two lives, x y.

$$\Pi_{\overline{xy}}=\Pi_{x''y}+\Pi_{xy''}=\Pi_x+\Pi_y-\Pi_{xy}.$$

101. In case of equal ages.

$$\Pi_{\overline{xx}}=\Pi_{x''x}+\Pi_{xx''}=2\Pi_x-\Pi_{xx}.$$

102. The annual premium to insure £1 at the death of the first life is usually paid during the joint lives only.

So $$\frac{\Pi_{xy}}{\hat{a}_{xy}}=\frac{\Pi_{x'y}}{\hat{a}_{xy}}+\frac{\Pi_{xy'}}{\hat{a}_{xy}}=\pi_{xy}=\pi_{x'y}+\pi_{xy'}.$$

103. The annual premium to insure £1 at the death of the longest life is usually paid during the longest life.

Hence $$\frac{\Pi_{\overline{xy}}}{\hat{a}_{\overline{xy}}}=\frac{\Pi_{x''y}}{\hat{a}_{xy}}+\frac{\Pi_{xy''}}{\hat{a}_{xy}}=\pi_{\overline{xy}}=\pi_{\overline{x''y}}+\pi_{\overline{xy''}},$$

and $$\frac{\Pi_{\overline{xx}}}{\hat{a}_{\overline{xx}}}=\frac{\Pi_{x''x}}{\hat{a}_{xx}}+\frac{\Pi_{xx''}}{\hat{a}_{xx}}=\pi_{\overline{xx}}=2\pi_{x''x}.$$

104. $\pi_{x''x}=\frac{\pi_{\overline{xx}}}{2}=$ annual premium payable during last of two lives, x_1 and x_2 designated, provided he be the last to die.

105. *Temporary Survivorship.*—Value of an insurance of £1 payable on the death of the survivor of x, y, provided it take place within n years.

$$\Pi=v[1-v^n(c_{x)n}+c_{y)n}-c_{xy)n})-d(a_{x)n}+a_{y)n}-a_{xy)n})$$

$$\pi=\frac{1-v^n(c_{x)n}+c_{y)n}-c_{xy)n}}{1-v^n(c_{x)n}+c_{y)n}-c_{xy)n})+a_{x)n}+a_{y)n}-a_{xy)n}}.$$

106. *Temporary Survivorship on the Survived Life.*—Value of an insurance of £1 payable in the event of y dying within n years and x survive him.

$$\Pi=N_{xy}+N_{x\cdot y1}-(N_{xy|n}+N_{x\cdot y1|n})+N_{xy|n1}+N_{x1\cdot y|n}-(N_{xy|1}+N_{x1\cdot y})$$

$$\pi=\frac{\Pi}{\hat{a}_{xy)n}-v^n c_{xy)n}}.$$

Value of Policies, and the Amounts in Deposit.

107. $\pi_x.\hat{a}_{x+n}=$ The value at age $x+n$ of the future annual premiums payable on or by a life X, at and after age $x+n$.

108. $\pi\frac{N_{x|n}}{D_{x+n}}=$ The amount in deposit at the age $x+n$ properly belonging to X before the payment of the future premiums π_x commences.

109. $\Pi_{x+n}-\pi_x.\hat{a}_{x+n}=$ The sum in deposit immediately before the $(n+1)$th premium is due, to each £1 insured at age x by the annual premium π_x.

110. The value of a policy, be it for an *Ordinary assurance*, an *Endowment assurance*, a *Survivorship assurance*, or any other *Contingent* assurance, except if it be of a temporary or deferred character, can be

ascertained by the following simple formulæ (*depending only on a Table of Reversions*) :—

$$\text{I. } \frac{\Pi_{x+n}-\Pi_x}{1-\Pi_x}. \qquad \text{II. } 1-\frac{\hat{a}_{x+n}}{\hat{a}_x}.$$

111. If in the valuation office premiums (p_x) be used in lieu of net premiums (π_x), the value of the policy can be determined thus :—

$$\text{III. } \frac{\pi_{x+n}-p_x}{\pi_{x+n}-d}. \qquad \text{IV. } \pm[1-(d+p_x)\hat{a}_{x+n}].$$

112. *Alteration of Policies from one system of Assurance to another.*—Cases of this kind not unfrequently occur in practice ; it may therefore be useful to add an example or two in illustration of the principles on which these changes should be made.

Example 1. A person whose present age is x has the sum P assured at death at an annual premium π, which is just due but not paid, and he desires, by increasing his yearly premium from π to p to have the sum assured s payable on his completing his $(x+n)$th year, or at death if that should happen previously. What should be the amount of p ?

The change will require the cancelling of the present assurance from and after the age $x+n$, this will be a gain to the society to the extent of $s\cdot\frac{M_{x+n}}{D_x}$. There will be a further gain on the part of the company of the additional premium, or $p-\pi$, for a period of n years, the value of which is $(p-\pi)\cdot\frac{N_x-N_{x+n}}{D_x}$. On the other hand, the office will have to provide an endowment, s, at the end of n years, the present worth of which is $s\cdot\frac{D_{x+n}}{D_x}$, and to lose the annual premium π after n years, the value of which is $\pi\cdot\frac{N_{x+n}}{D_x}$. Now, in order that the position of the society may be unaffected, the gains and losses which the changes involve should balance ; hence we have the following equation for determining p :—

$$\frac{s.M_{x+n}}{D_x}+(p-\pi.)\frac{N_{x|n}}{D_x}=s.\frac{D_{x+n}}{D_x}+\pi.\frac{N_{x+n}}{D_x}$$

$$\therefore s(D_{x+n}-M_{x+n})+\pi N_x=p(N_x-N_{x+n}),$$

whence

$$p=\frac{s(D_{x+n}-M_{x+n})+\pi N_x}{N_{x|n}},$$

$$=\frac{s.d.N_{x+n}+\pi N_x}{N_{x|n}},$$

where $d=1-v$, the discount of £1 for a year. The last expression is a very convenient form for practical use, requiring only the column N for its calculation.

113. *Example* 2.—Suppose the original policy in the last example to remain unaltered in its conditions as regards the payment of the sum assured at death, but that the premium π for the whole of life has to be commuted to an annual payment, p, which is to cease after n years.

Here, the present value of the premium p for n years is $p\cdot\frac{N_{x|n}}{D_x}$ and the present value of the premium π for the whole of life is $\pi\cdot\frac{N_x}{D_x}$, by equating these, we get

$$p=\pi\cdot\frac{N_x}{N_{x|n}},$$

which is therefore the new premium to be charged.

114. If a life aged x', on being allowed after n years from the date of his policy, a cash bonus B′, desires that instead of the bonus, payment of his annual premiums at some unknown future age $x'+n$, may cease altogether, the age required can be found thus (x=age at entry, x'=age at *first* valuation):—

$$B'=\frac{N_{x'+n}}{D_{x'}}.$$

whence,
$$N_{x'+n}=\frac{B'D_{x'}}{p_x},$$

which gives $N_{x'+n}$ by comparing this result with a table of N. Similarly, the next bonus will bring the age $x''+n$ earlier.

115 If the bonus be applied to make not only the payment of the premiums cease, but the amount assured itself become payable to the assured at age $x'+u$, or in case of previous death, B′ must suffice to meet the deferred annuity of $(d+p_x)$, or the annual interest on the amount assured as well as the premium.

$$\therefore B'=(d+p_x)\frac{N_{x'+u}}{D_{x'}}.$$

whence
$$N_{x'+u}=\frac{B'D_{x'}}{d+p_x},$$

and $x'+u$ can at once be determined by simple inspection of a Table N.

116. In case of a second bonus B″, the formula becomes

$$N_{x''+w}=\frac{B'D_{x''}+B''D_{x'}}{d+p_x};$$

and in case of a third bonus,

$$N_{x'''+y}=\frac{B'D_{x'}+B''D_{x''}+B'''D_{x'''}}{d+p_x}\text{; and so on.}$$

The following miscellaneous questions and solutions are added as useful exercises for the student:—

117. QUESTION 1. A person aged x has an annuity (P) on his life for n years, payable at the beginning of each year. If the annuity be forborne and accumulated till the end of the year of death, to find to what sum it will then amount.

The present value of the annuity payments is $\hat{a}_{x(1n}$ or $\frac{N_x - N_{x+n}}{D_x}$, and if π be the equivalent sum payable at death, its present value is $\pi . \frac{M_x}{D_x}$; hence, by equating these results, we get

$$\pi = \frac{N_x - N_{x+n}}{M_x},$$

which is therefore the answer to the question.

118. If the annuity were for the whole of life, we should have $\hat{a}_x$ or $\frac{N_x}{D_x} = \pi . \frac{M_x}{D_x}$ or $\pi = \frac{N_x}{M_x} =$ reciprocal of annual premium to assure £1 at the death of x.

119. QUESTION 2. To prove that $a_{1x)n} = \hat{a}_{x)1n} v c_{1x)1}$.[*]

Since

$$\begin{aligned} a_{x)n} &= c_{x)1} v + c_{x)2} v^2 + \ldots c_{x)n} v^n \\ \therefore a_{1x)n} &= c_{1x)1} v + c_{1x)2} v^2 + \ldots c_{1x)n} v^n \\ &= c_{1x)1} v + c_{1x)1} v (c_{x)1} v + c_{x)2} v^2 + \ldots c_{x)1n} v^{1n}). \\ &= c_{1x)1} v + c_{1x)1} v a_{x)1n}. \\ &= \hat{a}_{x)1n} v c_{1x)1}. \end{aligned}$$

This result may be obtained without the help of analysis, by considering that if a person aged $x-1$ were in possession of a temporary life annuity for n years, and if he could be quite sure that he would live one year and receive the first £1, the *present* value of his annuity would clearly be $\hat{a}_{x)1n} v$, but inasmuch as the chance of his surviving the first year is only $c_{1x)1}$, the present value of the annuity is reduced to $\hat{a}_{x)1n} v c_{1x)1}$, which is therefore equivalent to $a_{1x)n}$.

120. QUESTION 3. To show that the present value of £1 payable to a person aged x on his attaining the age $x+n$, or at the end of the year of death if that event should happen previously, is $\frac{1 - r a_{x)1n}}{1+r}$.

We have already found an expression in the columnar notation for the value of an endowment assurance; and the formula given in the present question might be deduced from it, but the following independent demonstration will be preferable.

The value of the temporary assurance

$$= \Sigma_n (c_{x)1n} - c_{x)n}) v^n = \frac{1}{c_{1x)1}} \Sigma_n c_{1x)n} v^n - \Sigma_n c_{x)n} v^n = \frac{a_{1x)n}}{c_{1x)1}} - a_{x)n},$$

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[*] The letter *c* stands for *Chance, i.e.*, probability.

and the value of the endowment is $\frac{l_{x+n}}{l_x}v^n$. The sum of these two results is the whole value of the risk. Now by the last question, $\frac{a_{1x)n}}{c_{1x)1}} = v\hat{a}_{x)1n}$ and, evidently, $a_{x)n} = a_{x)1n} + \frac{l_{x+n}}{l_x}.v^n$; hence, by substituting, we get for the present value of the temporary assurance $v\hat{a}_{x)1n} - a_{x)1n} - \frac{l_{x+n}}{l_x}.v^n$, to which adding the value of the endowment risk, the total is

$$v\hat{a}_{x)1n} - a_{x)1n} = \frac{1 - ra_{x)1n}}{1+r}.$$

The formula for the annual premium would be $\frac{1 - ra_{x)1n}}{(1+r)\hat{a}_{x)1n}}$. The results here obtained are of great practical importance, owing to the fact that we have tables* showing the values of the formulæ

$$\frac{1-ra}{1+r} \text{ and } \frac{1-ra}{(1+r)\hat{a}}$$

for all values of a from zero to 25, and for various rates of interest (r); so that it is only necessary to enter the tables referred to with the value of $a_{x)1n}$, taken from a table of temporary annuities, in order to pass at once to the single and annual premiums for an endowment assurance.

121. QUESTION 4. If $c_{x)1}$ denote the probability of living one year at age x, and e_x, the expectation of life at that age, show that

$$c_{x)1} = \frac{e_x - \frac{1}{2}}{e_{x1} + \frac{1}{2}}.$$

We have
$$\begin{aligned} e_x &= \tfrac{1}{2} + c_{x)1} + c_{x)2} + c_{x)3} + \&c. \\ &= \tfrac{1}{2} + c_{x)1} + c_{x)1}(c_{x1)1} + c_{x1)2} + \&c.) \\ &= \tfrac{1}{2} + c_{x)1} + c_{x)1}(e_{x1} - \tfrac{1}{2}); \end{aligned}$$

or,
$$c_{x)1} = \frac{e_x - \frac{1}{2}}{e_{x1} + \frac{1}{2}}.$$

122. QUESTION 5. To show that the value of an annuity when paid by m equal instalments every year, is greater than when paid annually, at the end of each year, by the $\frac{m-1}{2m}$ part of a year's purchase.

The ordinary tables of the values of life annuities are framed upon the supposition that the payments are made *annually*, and at the *end* of each year. From the results thus tabulated, we may easily deduce by the present theorem the equivalent half-yearly, quarterly, &c., instalments. Suppose an annuity of £1 to be payable by m instalments every year, each payment being $=\frac{1}{m}$, it is evident that we may look upon the whole annuity as composed of m separate annuities of $\frac{1}{m}$, the first payments of which are made at the end of the $\left(\frac{1}{m}\right)^{\text{th}}$, $\left(\frac{2}{m}\right)^{\text{th}}$, &c., parts of a year respectively. Now we know that the value of an annuity of £1, payable at the beginning of every year, differs from the value of one payable at the end of each year by £1, therefore, by simple proportion, when the first payment is to be made $\left(\frac{1}{m}\right)^{\text{th}}$ of a year before the end of it, the annuity value will be increased by the $\left(\frac{1}{m}\right)^{\text{th}}$ part of a £; and when at the $\left(\frac{2}{m}\right)^{\text{th}}$ part of a year before the end of it, by the $\left(\frac{2}{m}\right)^{\text{th}}$ part of a £, and so on; therefore, the pre-payment of the first $m-1$ annuities above alluded to, will augment the tabular value by

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* *Single and Annual Assurance Premiums*, by William Orchard.

$$\frac{1}{m}\left(\frac{1}{m}+\frac{2}{m}+\frac{3}{m}\ldots\ldots\frac{m-1}{m}\right)$$
$$=\frac{m-1}{2m}.$$

If $m=2$, or the annuity be paid half yearly, this becomes $\frac{1}{4}$. If $m=4$, or the payments be quarterly, it becomes $\frac{3}{8}$. These additions respectively must therefore be made to the values given in the tables, when an annuity is paid half-yearly or quarterly.

In the columnar system, the value of an annuity on a life aged x, payable at the end of each year is $\frac{N_{x1}}{D_x}$, and at the beginning of each year $1+\frac{N_{x1}}{D_x}$ or $\frac{N_x}{D_x}$, it is evident, therefore, that we need only interpolate by equal differences the interval between N_x and N_{x1}, to arrive at a result similar to that already found.

123. Question 6. If the lives of two persons, A and B, now aged x and y respectively, should *both* fail in the nth year from the present time, then it is equally likely that X will die before Y, as that Y will die before X, upon the supposition that the deaths occurring in any one year, as given in the Table of Mortality, are equally distributed over that year.

Let d_{x+n} and d_{y+n} denote the tabular number of deaths in the nth year from the present time, and let t be any fractional part of a year; then, supposing the annual deaths to be distributed as in the question, the probability of Y dying in the nth year at the expiration of the tth part thereof, *i.e.*, at the instant between t and $t+dt$ will be $\frac{d_{y+n}}{l_y}\,dt$, and the probability of X dying within the time t will be $\frac{d_{x+n}}{l_x}t$, hence the probability that Y will survive X at some time within the limits of the whole year will be

$$=\frac{d_{x+n}}{l_x}\cdot\frac{d_{y+n}}{l_y}\int_0^1 t.dt$$

$$=\tfrac{1}{2}\cdot\frac{d_{x+n}}{l_x}\cdot\frac{d_{y+n}}{l_y};$$

and as the same result would be found for the probability of X surviving Y, it is *equally likely* that either will die first supposing both to die within the year.

If there were v lives, the chance of any specified one failing *before* the rest, supposing them all to drop in the nth year, would be $=\frac{1}{v}\times$ probability of all dying in that year; that is to say, it would be an equal chance that any one of them in particular would die first, for the probability in question is expressed by

$$\frac{d_{x+n}}{l_x}\cdot\frac{d_{y+n}}{l_y}\cdots\cdots\cdots\cdots\frac{d_{k+n}}{l_k}\int_0^1(1-t)^{v-1}dt \qquad (a),$$

and $\int(1-t)^{1v}dt=C-\frac{1}{v}(1-t)^v$, which from $t=0$ to $t=1$ is $\frac{1}{v}$, hence (a) becomes $\frac{1}{v}\times$ probability that all will die in the year.*

124. QUESTION 7. To determine the single and annual premiums for assuring the sum of £1, to be paid on the death of X aged x, provided Y aged y be alive at that time.

The £1 will be payable if X die in the nth year and Y live to the end of it, the probability of which is $c_{y)n}(c_{x)1n}\cdot c_{x)n})$, and also if both should die in the nth year, X dying first, the probability of which is $\frac{1}{2}\cdot\frac{d_{x+n}}{l_x}\cdot\frac{d_{y+n}}{l_y}$, which may be written $\frac{1}{2}(c_{x)1n}-c_{x)n})(c_{y)1n}-c_{y)n})$, adding these results together we get the whole probability of Y surviving X in the nth year

$$=\tfrac{1}{2}(c_{x)1n}-c_{x)n})(c_{y)1n}+c_{y)n})$$

$$=\tfrac{1}{2}(c_{xy)1n}-c_{xy)n}+c_{y)n}\cdot c_{x)1n}-c_{x)n}\cdot c_{y)1n}).$$

Now $$c_{x)1n}=\frac{c_{1x)n}}{c_{1x}} \text{ and } c_{y)1n}=\frac{c_{1y)n}}{c_{1y}},$$

therefore substituting these values for $c_{x)1n}$ and $c_{y)1n}$ in the last two terms of the above expression, and multiplying it by v^n, the present value of £1 due at the end of n years, we get for the whole value of the risk, or the amount of the single premium,

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* This demonstration was communicated by Mr. Samuel Younger, F.I.A.

$$\Pi_{x)y}=\tfrac{1}{2}\Sigma v^n c_{xy(1n}-c_{xy(n}+\frac{c_{1x,y(n}}{c_{1x}}-\frac{c_{x,1y(n}}{c_{1y}}$$

$$=\tfrac{1}{2}(\Pi_{xy}+\frac{a_{1x,y}}{c_{1x}}-\frac{a_{x,1y}}{c_{1y}})$$

where Π_{xy} is the single premium on the *joint* lives.

125. Let $\pi_{x)y}$ be the annual premium for the survivorship risk, then since its payment will cease on the failure of the first life, we have

$$\pi_{x)y}=\frac{\Pi_{x)y}}{\hat{a}_{xy}}.$$

Example for an approximate solution of this question:—

In the simple case of survivorship which forms the subject of Question 7, in last page, if we omit the small contingency of *both* lives failing in the same year, x dying first, we should have

$$\begin{aligned}\Pi_{x)y}&=\Sigma v^n(c_{x(1n}-c_{x(n})c_{y(n}\\&=\Sigma v^n(c_{x(1n}c_{y(n}-c_{xy(n})\\&=\Sigma v^n\left(\frac{c_{1x,\ y(n}}{c_{1x(1}}-c_{xy(n}\right)\\&=\frac{a_{1x,\ y}}{c_{1x(n}}-a_{xy},\end{aligned}$$

a much more simple result than obtained before, and nearly as accurate.

SECT. III.

ANNUITIES ON THREE LIVES.

SELECTED FROM DODSON, SIMPSON, BAILY, MILNE, PROF. DE MORGAN, DR. FARR, ETC.

126. An annuity of £1 to cease on the failure of the joint existence of the last two survivors of three lives, x, y, z.

$$=a_{xy}+a_{xz}+a_{yz}-2a_{xyz}.$$

127. Value of an annuity of £1 payable during the life of the last survivor of three lives, x, y, z.

$$a_{\overline{xyz}}=a_x+a_y+a_z+a_{xyz}-(a_{xy}+a_{xz}+a_{yz}).$$

128. An annuity of £1 on x if it fail second out of three lives, x, y, and z.

$$\Pi=a_x-a_{xyz}.\quad \pi=\frac{\Pi}{\hat{a}_{xyz}}.$$

129. If x does not enter on possession until after the death of the survivor of y and z.
$$\Pi = a_x + a_{xyz} - (a_{xy} + a_{xz}).$$
$$\pi = \frac{\Pi}{\hat{a}_{xy} + a_{xz} - a_{xyz}}.$$

130. An annuity on the joint lives of x and y after the death of z.
$$\Pi = a_{xy} - a_{xyz}. \quad \pi = \frac{\Pi}{\hat{a}_{xyz}}.$$

131. An annuity on the joint lives and the survivor of two lives, x, y, after the death of z.
$$\Pi = a_x + a_y + a_{xyz} - (a_{xy} + a_{xz} + a_{yz}).$$
$$\pi = \frac{\Pi}{\hat{a}_{xz} + a_{yz} - a_{xyz}}.$$

132. An annuity on the longest of x, y, and z, to be divided equally between them during their joint lives, and on the decease of either of them it is to be equally divided between the survivors during their joint lives, and then to belong entirely to the last survivor during his life. Required the value of X's interest therein.
$$\text{X's share } = a_x + \frac{a_{xyz}}{3} - \frac{a_{xy} + a_{xz}}{2}.$$

133. An annuity on the longest of x, y, z, to be divided equally between x and y during their joint lives; if x die first, y and z are to enjoy it equally during their joint lives, and the survivor is to have the whole; but if y die first, x is to enjoy the whole during his life, and after his decease the whole annuity goes to z.
$$\text{X's share} = a_x - \frac{a_{xy}}{2}.$$
$$\text{Y's share} = a_y + \frac{a_{xyz}}{2} - \left(\frac{a_{xy} + a_{yz}}{2}\right).$$
$$\text{Z's share} = a_z + \frac{a_{xyz}}{2} - \left(a_{xz} + \frac{a_{yz}}{2}\right).$$

134. An annuity on the longest of x, y, z, to be enjoyed by x during his life, and after his decease is to be divided equally between y and z during their joint lives, and the survivor of them is to have the whole. Required to find Y's interest.
$$\text{Y's share } = a_y + \frac{a_{xyz}}{2} - \left(a_{xy} + \frac{a_{yz}}{2}\right).$$

135. An annuity for the term of $n+m$ years to be enjoyed by z and his heirs during the joint existence of two lives, x and y; and if that joint

existence fail before the expiration of n years, the annuity is to go to k and his heirs for the remainder of the term of m years. Required the value of k's interest.

$$k\text{'s share} = \frac{1}{r}\left[1 - v^n + \left(1 - \frac{l_x l_{y|n}}{l_x l_y}\right)(v^n - v^{n+m})\right] - a_{xy} + \frac{a_{xy|n} v^n l_x l_{y|n}}{l_x l_y}.$$

136. An annuity on x after the failure of the joint existence of y and z, provided that event take place by the death of y.

If x be the eldest.

$$= \tfrac{1}{2}(a_x + a_{xz} - a_{xy} - a_{xyz}).$$

If y be the eldest.

$$= \tfrac{1}{2}(a_x + a_{xz)q-y} - a_{xy} - a_{xyz}) - (\tfrac{1}{2} - f_{y)z}) a_{x(q-y}.$$

If z be the eldest.

$$= \tfrac{1}{2}(a_x + a_{xz} - a_{xy)q-z} - a_{xyz} - (\tfrac{1}{2} - f_{y)z}) a_{x(q-z}.$$

When the ages of y, z, are equal in all respects the formula becomes

$$= \tfrac{1}{2}(a_x - a_{xyy}).$$

(q = highest tabular age attainable.)

137. An annuity on the survivor of $x\,y$, to be shared equally between z and that survivor, provided z be then alive. Required z's interest.

$$\text{Z's share} = \frac{a_{xz} + a_{yz}}{2} - a_{xyz}.$$

138. An annuity dependent upon the joint existence of the two last survivors out of three lives, x, y, z, to be divided equally amongst them while they are all living, and after the decease of any of them, is to be divided equally between the two survivors during the remainder of their joint lives. Required, the interest of x.

$$\text{X's share} = \frac{a_{xy} + a_{xz}}{2} - \frac{2a_{xyz}}{3}.$$

139. An annuity after the decease of x, to be equally divided between y and z during their joint lives, and is then to go entirely to the last survivor for his life. To find the value of Y's interest.

$$\text{Y's share} = a_y + \frac{a_{xyz}}{2} - \left(a_{xy} + \frac{a_{yz}}{2}\right).$$

140. An annuity dependent on the last survivor of three lives, x, y, z, to be divided equally between x and y during their joint existence, and after the decease of either of them, to be divided equally between z, if then alive, and the survivor during their joint existence; and whenever there may be but one survivor left, it is to go entirely to him for the remainder of his life. Required X's interest.

$$\text{X's share} = a_x + \frac{a_{xyz}}{2} - \frac{a_{xy} + a_{xz}}{2}.$$

141. An annuity to be entered upon on the extinction of two out of three lives, and to continue during the remainder of the survivor's life.

$$=a_x+a_y+a_z+3a_{xyz}-2(a_{xy}+a_{xz}+a_{yz}).$$

142. An annuity dependent upon the life of the last survivor x out of x, y, z, to be divided equally during the joint existence of any of them.

$$\text{X's share }=a_x+\frac{a_{xyz}}{3}-\frac{a_{xy}+a_{xz}}{2}.$$

143. An annuity on x, after the extinction of the survivor of y and z, provided y be that survivor.

If x be the eldest.

$$=\tfrac{1}{2}(a_x+a_{xyz}-a_{xy}-a_{xz}).$$

If y be the eldest.

$$=\tfrac{1}{2}(a_x+a_{xyz}-a_{xy}-a_{xz)q-y}+(\tfrac{1}{2}-f_{y)z})a_{x(q-y}.$$

If z be the eldest.

$$=\tfrac{1}{2}(a_x+a_{xyz}-a_x-a_{xz})-\tfrac{1}{2}a_{xy(q-z}+(\tfrac{1}{2}-f_{y)z})a_{x(q-z}.$$

When the ages of y, z are equal the formula becomes

$$=\tfrac{1}{2}(a_x-2a_{xy}+a_{xyy}).$$

SURVIVORSHIP ASSURANCES ON THREE LIVES.

X failing.	An insurance of £1 payable on the failure of x out of three lives, x, y, z, in the following order:—	Equal ages.
1st.	144. $\Pi_{x'yz}=\frac{1}{3}\Pi_{xyz}+\frac{1}{6}c_{1x}(2a_{1x\cdot yz}+\frac{a_{1x\cdot 1y\cdot z}}{c_{1y}}+\frac{a_{1x\cdot y\cdot 1z}}{c_{1z}})$ $-\frac{1}{6}c_{1y}(a_{x\cdot 1y\cdot z}+\frac{2a_{x\cdot 1y\cdot 1z}}{c_{1z}})-\frac{a_{xy\cdot 1z}}{6c_{1z}}.$	$=\frac{\Pi_{xxx}}{3}.$
2d.	145. $\Pi_{x''yz}=\Pi_{x'y}+\Pi_{x'z}-2\Pi^{144}.$	$=\Pi_{xx}-\frac{2}{3}\Pi_{xxx}.$
3d.	146. $\Pi_{x'''yz}=\Pi_x-(\Pi^{144}+\Pi^{145}).$	$=\frac{\Pi_{\overline{xxx}}}{3}.$
1st or 2d.	147. $\Pi_{x'\,''yz}=\Pi^{144}+\Pi^{145}.$	$=\frac{\Pi_{xxx}+\Pi_{\overset{2}{\overline{xxx}}}}{3}.$
1st or 3d.	148. $\Pi_{x'\,'''yz}=\Pi_x-\Pi^{145}.$	$=\frac{\Pi_{xxx}+\Pi_{\overline{xxx}}}{3}.$
2d or 3d.	149. $\Pi_{x''\,'''yz}=\Pi_x-\Pi^{144}.$	$=\frac{\Pi_{\overset{2}{\overline{xxx}}}+\Pi_{\overline{xxx}}}{3}.$

X or Y failing.	An insurance of £1 payable on either *x* or *y* failing out of three lives, *x*, *y*, *z*.	Equal ages.
1st.	150. $\Pi_{x'y'z}=\frac{2}{3}\Pi_{xyz}+\frac{1}{6c_{1x}}(a_{1x\cdot yz}+\frac{2a_{1x\cdot 1y\cdot z}}{c_{1y}}-\frac{a_{1x\cdot y\cdot 1z}}{c_{1z}})+\frac{1}{6c_{1y}}(a_{x\cdot 1y\cdot z}-\frac{a_{x\cdot 1y\cdot 1z}}{c_{1z}})-\frac{a_{xy\cdot 1z}}{3c_{1z}}.$	$=\frac{2\Pi_{xxx}}{3}.$
2d.	151. $\Pi_{x''y''z}=\Pi_{xy}+\Pi_{x'z}+\Pi_{y'z}-\Pi^{150}.$	$=\frac{2}{3}\Pi_{\overset{2}{\overline{xxx}}}.$
3d.	152. $\Pi_{x'''y'''z}=\Pi_{\overline{xy}}+\Pi^{150}-(\Pi_{x'z}+\Pi_{y'z}).$	$=\frac{2}{3}\Pi_{\overline{xxx}}.$

X failing.	An insurance of £1 payable on *x* failing of three lives, *x*, *y*, *z*.	Equal Ages.
2d.	153. Provided *z* fail first: $\Pi_{x''yz'}=\Pi_{x'y}-\Pi^{144}.$	$=\frac{1}{6}\Pi_{\overset{2}{\overline{xxx}}}.$
2d or 3d.	154. Provided in the latter case *y* fail first. $\Pi_{x'\,''yz}=\Pi_{x'z}.$	$=\frac{1}{2}\Pi_{xx}.$
3d.	155. Provided *a* is the eldest, and *y* fail second. $\Pi_{x'''y'z}=\frac{1}{2}\Pi_{x}+\frac{1}{6}\Pi_{xyz}+\frac{1}{2}a_{xy}-\frac{1}{6}c_{1x}(3a_{1x\cdot y}-a_{1x\cdot yz}+\frac{a_{1x\cdot 1y\cdot z}}{c_{1y}}+3a_{1x\cdot 1z}-\frac{2a_{1x\cdot y\cdot 1z}}{c_{1z}})+\frac{1}{6}c_{1y}(a_{x\cdot 1y\cdot z}-\frac{a_{x\cdot 1y\cdot 1z}}{c_{1z}}+\frac{3a_{x\cdot 1z}-2a_{xy\cdot 1z}}{6c_{1z}}.$	$=\frac{1}{6}\Pi_{\overline{xxx}}.$
Do.	156. Provided *y* is the eldest, and that it fail second. $=\Pi^{155}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}+\frac{a_{1x\cdot 1z(q-y}}{2c_{1x\cdot 1z}}-\frac{a_{x\cdot 1z}}{2c_{1z}}.$	
Do.	157. Provided *z* be the eldest, and that it fail second. $=\Pi^{155}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}+-\frac{a_{1x\cdot 1y(q-z}}{2c_{1x\cdot 1y}}+\frac{a_{x\cdot 1y}}{2c_{1y}}.$	
1st or 3d.	158. Provided in the latter case *z* fail first, and *x* be the eldest. $\Pi_{x'''yz}=\Pi^{144}+\Pi^{155}.$	$=\frac{\Pi_{x}+\Pi_{xxx}-\Pi_{xx}}{2}.$
Do.	159. Provided *y* be the eldest, and *z* fail first. $=\Pi^{158}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}+\frac{a_{1x\cdot 1z(q-y}}{2c_{1x\cdot 1z}}-\frac{a_{x\cdot 1z(q-y}}{2c_{1z}}.$	

X failing	160. Provided *z* be the eldest, and that it fail first.	Equal ages.
1st or 3d.	$=\Pi^{158}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}-\frac{a_{1x\cdot1y(q-z}}{2c_{1x\cdot1y}}+\frac{a_{x\cdot1y(q-z}}{2c_{1y}}$.	
2d or 3d.	161. Provided *z* fail first, and that *x* be the eldest. $\Pi_{x'\,'''yz'}=\Pi^{158}+\Pi^{155}$.	$=\frac{1}{2}\Pi_x-\frac{1}{6}\Pi_{xxx}$.
Do.	162. Provided *y* be the eldest, and that *z* fail first. $=\Pi^{161}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}+\frac{a_{1x\cdot1z(q-y}}{2c_{1x\cdot1z}}-\frac{a_{x\cdot1z(q-y}}{2c_{1z}}$.	
Do.	163. Provided *z* be the eldest, and that it fail first. $=\Pi^{161}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}-\frac{a_{1x\cdot1y(q-z}}{2c_{1x\cdot1y}}+\frac{a_{x\cdot1y(q-z}}{2c_{1y}}$.	

Eldest life.	164. An insurance of £1 payable on the failure of the last survivor of *x*, *y*, provided both survive *z*.	Equal ages.
	$=\frac{1}{2}\Pi_x+\frac{1}{2}\Pi_y+\frac{1}{3}\Pi_{xyz}+a_{xy}-\frac{1}{6}c_{1x}(3a_{1x\cdot y}-2a_{1x\cdot yz}+\frac{2a_{1x\cdot1y\cdot z}}{c_{1y}}+\frac{3a_{1x\cdot1y}-a_{1x\cdot y\cdot1z}}{c_{1z}})-\frac{1}{6}c_{1y}(3a_{x\cdot1y}-2a_{x\cdot1y\cdot z}+\frac{3a_{1y\cdot1z}-a_{x\cdot1y\cdot1z}}{c_{1z}})+\frac{1}{6}c_{1z}(3a_{x\cdot1z}+3a_{y\cdot1z}-4a_{xy\cdot1z})$.	$=\frac{1}{3}\Pi_{\overline{xxx}}$.
X	165. If *x* be the eldest. $=\Pi^{164}+(\frac{1}{2}-f_{x)z})\Pi_{y(q-x}+\frac{a_{1y\cdot1z(q-x}}{2c_{1y\cdot1z}}-\frac{a_{y\cdot1z(q-x}}{2c_{1z}}$.	
Y	166. If *y* be the eldest. $=\Pi^{164}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}+\frac{a_{1x\cdot1z(q-y}}{2c_{1x\cdot1y}}-\frac{a_{x\cdot1z(q-y}}{2c_{1z}}$.	
Z	167. If *z* be the eldest. $=\Pi^{164}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}+(\frac{1}{2}-f_{x)z})\Pi_{y(q-z}-\frac{a_{1x\cdot1y(q-z}}{c_{1x\cdot1y}}+\frac{a_{1x\cdot y(q-z}}{2c_{1x}}+\frac{a_{x\cdot1y(q-z}}{2c_{1y}}$.	
	168. An insurance of £1 payable on the failure of the last survivor of *x*, *y*, provided both fail *before* *z*. $\Pi_{\overline{xy})z}=\Pi_{x'z}+\Pi_{y'z}-\Pi^{150}$.	$=\frac{1}{3}\Pi^{2}_{xxx}$.

Eldest life.	169. An insurance of £1 payable on the failure of the last survivor of x, y, provided that one fail before, and the other after z.	Equal ages.
	$=\frac{1}{2}(\Pi_x+\Pi_y+a_{xz}+a_{yz})+\frac{1}{3}\Pi_{xyz}+\frac{1}{6c_{1x}}(3a_{1x\cdot y}-3a_{1x\cdot z}$ $-a_{1x\cdot yz}-\frac{6a_{1x\cdot 1y}-4a_{1x\cdot 1y\cdot z}}{c_{1y}}-\frac{2a_{1x\cdot y\cdot 1z}}{c_{1z}})+\frac{1}{6c_{1y}}(3a_{x\cdot 1y}$ $-3a_{1y\cdot z}-a_{x\cdot 1y\cdot z}-\frac{2a_{x\cdot 1y\cdot 1z}}{c_{1z}})+\frac{a_{xy\cdot 1z}}{3c_{1z}}.$	$=\frac{1}{3}\Pi_{\overline{xxx}}.$
X	170. If x be the eldest. $=\Pi^{169}-(\frac{1}{2}-f_{x)z})\Pi_{y(q-x}-\frac{a_{1y\cdot 1z(q-x}}{2c_{1y\cdot 1z}}+\frac{a_{y\cdot 1z(q-x}}{2c_{1z}}.$	
Y	171. If y be the eldest. $=\Pi^{169}=(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}-\frac{a_{1x\cdot 1y(q-y}}{2c_{1x\cdot 1z}}+\frac{a_{x\cdot 1y(q-y}}{2c_{1z}}.$	
Z	172. If z be the eldest. $=\Pi^{169}-(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}-(\frac{1}{2}-f_{x)z})\Pi_{y(q-z}+\frac{a_{1x\cdot 1yq-z}}{c_{1x\cdot 1y}}$ $-\frac{a_{1x\cdot y(q-z}}{2c_{1x}}-\frac{a_{x\cdot 1y(q-z}}{2c_{1y}}.$	
Z failing.	173. An insurance of £1 payable on the failure of the survivor of $x\,y$, provided z fail either first or second of x, y, z.	$=\frac{2}{3}\Pi_{xxx}.$
1st or 2d.	$\Pi_{\overline{xy}\,z^{\prime\,\prime\prime}}=\Pi_{\overline{xy}}-\Pi^{168}.$	
1st or 3d.	174. Provided z fail either first or last. $\Pi_{\overline{xy}\,z^{\prime\,\prime\prime\prime}}=\Pi_{\overline{xy}}-\Pi^{169}.$	$=\Pi_x-\frac{1}{3}\Pi_{xxx}.$
2d or 3d.	175. Provided z fail either second or last. $\Pi_{\overline{xy}\,z^{\prime\prime\,\prime\prime\prime}}=\Pi_{\overline{xy}}-\Pi^{164}.$	$=\Pi_x-\frac{1}{3}\Pi_{xxx}.$
Eldest life. X	176. Provided y fail *after* z, and x be the eldest. $=\frac{1}{2}(\Pi_x+2\Pi_y-\Pi_{xy}-\Pi_{yz}+\Pi_{xyz})+\frac{1}{2}a_{xy}-\frac{a_{1x\cdot 1z}}{2c_{1x\cdot 1z}}$ $-\frac{a_{x\cdot 1y}+a_{1y\cdot z}-a_{x\cdot 1y\cdot z}}{2c_{1y}}+\frac{a_{x\cdot 1z}+a_{y\cdot 1z}-a_{xy\cdot 1z}}{2c_{1z}}.$	$=\frac{1}{2}\Pi_{\overline{xxx}}.$

Eldest life. Y	177. If *y* be the eldest. $=\Pi^{176}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}+\frac{a_{1x\cdot 1z(q-y}}{2c_{1x\cdot 1z}}-\frac{a_{x\cdot 1z(q-y}}{2c_{1z}}.$	Equal ages.
Z	178. If *z* be the eldest. $=\Pi^{176}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}-\frac{a_{1x\cdot 1y(q-z}}{2c_{1x\cdot 1y}}+\frac{a_{x\cdot 1y(q-z}}{2c_{1y}}.$	
X	179. Provided *y* fail *before* *z*, and that *x* be the eldest. $\Pi_{\overline{xy}\,1y)z}=\Pi_{\overline{xy}}-\Pi^{176}.$	$=\Pi_{\overline{xx}}=\frac{1}{2}\Pi_{\overline{xxx}}.$
Y	180. If *y* be the eldest. $=\Pi^{179}-(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}-\frac{a_{1x\cdot 1z(q-y}}{2c_{1x\cdot 1z}}+\frac{a_{x\cdot 1z(q-y}}{2c_{1z}}.$	
Z	181. If *z* be the eldest. $=\Pi^{179}-(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}+\frac{a_{1x\cdot 1y(q-z}}{2c_{1x\cdot 1y}}-\frac{a_{x\cdot 1y(q-z}}{2c_{1y}}.$	
X or Y failing	182. An insurance of £1 payable on either *x* or *y* failing first or second of *x*, *y*, *z*.	Equal ages.
1st or 2d.	$\Pi_{x'\,''y'\,''z}=\Pi_{xy}.$	
1st or 3d.	183. Provided either *x* or *y* fail either first or last. $=\frac{\Pi_x+\Pi_y}{2}+\Pi_{xyz}+a_{xy}-\frac{1}{2c_{1x}}(a_{1x\cdot y}-a_{1x\cdot yz}+\frac{a_{1x\cdot 1z}}{c_{1z}})$ $-\frac{1}{2c_{1y}}(a_{x\cdot 1y}-a_{x\cdot 1y\cdot z}+\frac{a_{1y\cdot 1z}}{c_{1z}}+\frac{1}{2c_{1z}}(a_{x\cdot 1z}+a_{y\cdot 1z}-$ $2a_{xy\cdot 1z}).$	$=\Pi_x+\Pi_{xxx}-\Pi_{xx}.$
Eldest life. X	184. If *x* be the eldest. $=\Pi^{183}+(\frac{1}{2}-f_{x)z})\Pi_{y(q-x}+\frac{a_{1y\cdot 1z(q-x}}{2c_{1y\cdot 1z}}-\frac{a_{y\cdot 1z(q-x}}{2c_{1z}}.$	
Y	185. If *y* be the eldest. $=\Pi^{183}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}+\frac{a_{1x\cdot 1z(q-y}}{2c_{1x\cdot 1z}}-\frac{a_{x\cdot 1z(q-y}}{2c_{1z}}.$	
Z	186. If *z* be the eldest. $=\Pi^{183}+(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}+(\frac{1}{2}-f_{x)z})\Pi_{y(q-z}-\frac{a_{1x\cdot 1y(q-z}}{c_{1x\cdot 1y}}$ $+\frac{a_{1x\cdot y(q-z}}{2c_{1x}}+\frac{a_{x\cdot 1y(q-z}}{2c_{1y}}.$	

X or Y failing. 2d or 3d.	187. Provided either *x* or *y* fail either second or last. $=\frac{\Pi_x+\Pi_y}{2}-\Pi_{xyz}-a_{xy}+\frac{1}{2c_{1x}}(a_{1x\cdot y}-a_{1x\cdot yz}+\frac{a_{1x1z}}{c_{1x}})+\frac{1}{2c_{1y}}(a_{x\cdot 1y}-a_{x\cdot 1y\cdot z}+\frac{a_{1y\cdot 1z}}{c_{1z}})-\frac{1}{2c_{1z}}(a_{x\cdot 1z}+a_{y\cdot 1z}-2a_{xy\cdot 1z})$.	$=\Pi_x+\Pi_{xx}-\Pi_{xxx}$.
Eldest life. X	188. If *x* be the eldest. $=\Pi^{187}-(\frac{1}{2}-f_{x)z})\Pi_{y(q-x}-\frac{a_{1y\cdot 1z(q-x}}{2c_{1y\cdot 1z}}+\frac{a_{y\cdot 1z(q-x}}{2c_{1z}}$.	
Y	189. If *y* be the eldest. $=\Pi^{187}-(\frac{1}{2}-f_{y)z})\Pi_{x(q-y}-\frac{a_{1x\cdot 1z(q-y}}{2c_{1x\cdot 1z}}+\frac{a_{x\cdot 1z(q-y}}{2c_{1z}}$.	
Z	190. If *z* be the eldest. $=\Pi^{187}-(\frac{1}{2}-f_{y)z})\Pi_{x(q-z}-(\frac{1}{2}-f_{x)z})\Pi_{y(q-z}+\frac{a_{1x\cdot 1y(q-z}}{c_{1x\cdot 1y}}-\frac{a_{1x\cdot y(q-z}}{2c_{1x}}-\frac{a_{x\cdot 1y(q-z}}{2c_{1y}}$.	

191. When an assurance is effected upon an assigned order of survivorship, the question, whether it will produce a claim or not, may be determined before the sum becomes payable. Thus, if an assurance be effected on the life of *x*, provided that life fail after another *y*, the claim must necessarily be determined at the failure of their joint continuance; and if *y* die first, the determination being in favour of the assured, the sum must be paid, but not until the decease of *x*.

Also, if an assurance be effected on the life of *x*, provided that life fail either second or third of the three, *x*, *y*, *z*, and that either *y* or *z* die first, the claim will then be certain, but will not take place until the decease of *x*.

Or, if the sum is to be paid upon *x* failing last of the three lives, *x*, *y*, *z*, and that *y* and *z* both fail before *x*, the claim will be determined at the decease of the survivor of these two, although the money will not be payable until the death of *x*.

192. When, even in the event of the claim being determined in favour of the assured, the sum does not necessarily become payable at that determination; the annual premium may either be payable only until the determination of the claim; or it may be stipulated that, in the event of the claim being determined in favour of the assured, the premium shall continue to be paid annually until the sum becomes due.

The annual premiums for such assurances do not admit of a simpler method of determination than that of finding the number of years' purchase in the value of an annuity on the lives during which the annual payments are to be made, and then dividing the total present value of the assurance by that number, increased by unity.

That is, upon the supposition of the payments being made at the commencement of each year; if they are to be made at the end of each year, then the addition of unity must be omitted.

It will be sufficient to show here, by a few examples, how the proper divisor may in every case be determined.

193. *Example* 1. The divisor for the annual premium equivalent to $\Pi_{x''y}$, if the premium be payable only until the claim is determined, will evidently be $\hat{a}_{xy}$; but if, in the event of the claim being determined in favour of the assured, the premium is to be continued until the sum becomes due, the divisor will be $\hat{a}_{x}$.

194. *Example* 2. The divisor for determining the annual premium equivalent to $\Pi_{x''\,'''yz}=\Pi^{149}$, if that premium be only payable until the claim is determined, will be $\hat{a}_{xyz}$, as that determination must take place at the failure of the first life.

But if, in the event of either y or z failing first, the annual premium is to be continued until the sum becomes payable, the divisor must be $\hat{a}_{x}$.

195. *Example* 3. In the case of the assurance expressed by $\Pi_{x''yz}=\Pi^{145}$, it is evident that the sum assured cannot be payable, neither can the claim be determined during the joint existence of x, and both of either of the other two lives; the premium therefore must be payable at the end of the nth year, if there be then

dead	living	whereof the probability is
none	xyz	$\ldots\ldots\ldots\ldots+c_{xyz)n}$
y	xz	$\ldots\ldots+c_{xz)n}-c_{xyz)n}$
z	xy	$c_{xy)n}\ldots\ldots\ldots\ldots c_{xyz)n}$;

and the sum of these probabilities being $c_{xy)n}+c_{xz)n}-c_{xyz)n}$, the divisor for the annual premium will be $1+a_{xy}+a_{xz}-a_{xyz}$.

And here, if the claim be determined in favour of the assured, the sum must necessarily be payable at the time of that determination.

196. *Example* 4. When the present value of the assurance is $\Pi_{x'y''z}$, the claim will remain undetermined while z and both or either of the other

two lives subsist; so that the divisor for the annual premium payable until the claim is determined may be found, in a manner, exactly similar to that of the last article. But as the algebraical expression for the divisor is already given in that case, we have only to interchange x and z therein to obtain that which is here sought, $1+a_{xz}+a_{yz}-a_{xyz}$.

And if, in the event of the claim being determined in favour of the assured, by z failing first of the three lives, the annual premium is to be continued until the sum becomes due; that is, until the failure of the joint existence of x and y; the present value of all the premiums that may be received between the time of the claim being ascertained and that of its taking place, will be $a_{xy(z}=a_{xy}-a_{xyz}$; which being added to that before obtained, the sum is $1+a_{xy}+a_{xz}+a_{yz}-2a_{xyz}=1+a_{\frac{2}{xyz}}$, the divisor for the annual premium payable until the sum assured becomes due, but subject to failure upon z failing second of the three lives.

That divisor, indeed, may easily be determined without algebra; for it is evident that the claim can only fail to take place by z dying second, and that the sum assured can only become due upon x or y dying second; therefore, the annual premium to be continued until one or the other of those events takes place, must be paid until the second death happens, and will depend upon the joint existence of the last two survivors out of the three lives.

197. *Example* 5. When the present value of the assurance is expressed by $\Pi_{x''y'z}$, the claim will be determined at the failure of the joint existence of x and y, and the divisor for the annual premium payable until that determination will be $\hat{a}_{xy}$.

If, in the event of the claim being determined in favour of the assured, the annual premium is to be continued from the time of such determination until the sum becomes due, the present value of an annuity of one pound, payable during that interval, will be $a_{x(y'z}=\hat{a}_{x}-a_{xy}-a_{x(y'z}$, to which adding a_{xy}, that of an annuity payable until the determination of the claim, we have $a_{x}-a_{x(y'z}$, the present value of £1 payable at the end of every year until the sum becomes due or the claim is determined against the assured.

Therefore the divisor for the annual premium payable at the beginning of each year, until the one or the other of those events shall take place, will be $\hat{a}_{x}-a_{x(y'z}$.

198. *Example* 6. When the present value of the assurance is $\Pi_{\overline{xy}\,z'''}$, the claim will be undetermined while z and both or either of the other two lives subsist; and therefore the divisor for the annual premium, to be paid only until that determination, will be $1+a_{xz}+a_{yz}-a_{xyz}$.

But if, in the event of the claim being determined in favour of the assured by the life z failing first of three, the annual payments are to be continued from the time of that determination until the sum becomes due; that is, until the extinction of the last survivor of the two remaining lives; the present value of £1, to be received at the end of every year during that interval, will be $a_{xy(z}+a_{x(y''z}+a_{y(x''z}=a_{\overline{xy}}-a_{xyz}-a_{x(y'z}-a_{y(x'z}$, to which adding the divisor before found, we have

$$1+a_{\overline{xy}}+a_{xz}+a_{yz}-2a_{xyz}-a_{x(y'z}-a_{y(x'z},$$

the divisor for determining the annual premium payable until the decease of the survivor of the two lives x, y, but subject to failure upon z failing the second of the three lives.

ON THE APPROXIMATION OF THE VALUE OF ANNUITIES IF INTEREST BE CONVERTED INTO PRINCIPAL TWICE A YEAR.

By PROFESSOR A. DE MORGAN, F.R.A.S.

199. The values of life annuities are usually calculated on the supposition that interest can be converted into principal only once a year, the truth being that such conversion always takes place twice a year at least. Mr. Milne has given a rule (p. 367) for making the requisite correction, but I believe the following will be found to be more easy of application: it is given for the case of half-yearly conversions only. Let the value of an annuity (for lives or a term of years) be A at c per cent, A_1 at $c+1$ per cent, A_2 at $c+2$ per cent, and so on. Take the differences of A, A_1, &c., as follows:—

$$\begin{array}{llll} A & & & \\ & B & & \\ A_1 & & C & \\ & B_1 & & D \\ A_2 & & C_1 & \\ & B_2 & & \\ A_3 & & & \end{array} \qquad \begin{array}{lll} B=A-A_1 & B_1=A_1-A_2 & B_2=A_2-A_3 \\ C=B-B_1 & C_1=B_1-B_2 & \\ D=C-C_1 & & \end{array}$$

Then the value of the same annuity at c per cent, interest being convertible twice a year, is

$$A-\frac{c^2}{400}\left(B+\frac{C}{2}+\frac{D}{3}-\frac{c^2(C+D)}{800}\right), \text{ very nearly.}$$

Example 1. Required the value of a perpetual annuity of £1 at 4 per cent, payable yearly, on the supposition that money is convertible half-yearly.

$$\begin{array}{llll} A\ =25{\cdot}000 & & & \\ & 5{\cdot}000 & & \\ A_1=20{\cdot}000 & & 1{\cdot}667 & \\ & 3{\cdot}333 & & {\cdot}715 \\ A_2=16{\cdot}667 & & {\cdot}952 & \\ & 2{\cdot}381 & & \\ A_3=14{\cdot}286 & & & \end{array}$$

$$\begin{array}{lr}
(1{\cdot}667+{\cdot}715)\frac{16}{800}={\cdot}048 & 5{\cdot}000 \\
 & {\cdot}834 \\
6{\cdot}024\times\frac{16}{400}={\cdot}241 & {\cdot}238 \\ \cline{2-2}
 & 6{\cdot}072 \\
 & {\cdot}048 \\ \cline{2-2}
 & 6{\cdot}024
\end{array}$$

$$25{\cdot}000-{\cdot}241=24{\cdot}759 \text{ answer.}$$

The real answer is 24·75225, and a perpetual annuity is the severest trial which can be made of the rule.

Example 2. (Milne, p. 367.) Required the value of a life annuity on (60), at 5 per cent, interest being convertible twice a year:

$$\begin{array}{llccc}
A & 8{\cdot}940 & & & \\
 & & {\cdot}636 & & \\
A_1 & 8{\cdot}304 & & {\cdot}075 & \\
 & & {\cdot}561 & & {\cdot}012 \\
A_2 & 7{\cdot}743 & & {\cdot}063 & \\
 & & {\cdot}498 & & \\
A_3 & 7{\cdot}245 & & &
\end{array}$$

$$\begin{array}{lr}
({\cdot}075+{\cdot}012)\frac{25}{800}={\cdot}0027 & {\cdot}6360 \\
 & {\cdot}0375 \\
{\cdot}6748\times\frac{25}{400}={\cdot}04218 & {\cdot}0040 \\ \cline{2-2}
 & {\cdot}6775 \\
 & {\cdot}0027 \\ \cline{2-2}
8{\cdot}940-{\cdot}04218=8{\cdot}89782 & {\cdot}6748
\end{array}$$

Mr. Milne's answer is 8·8977.

200. When an annuity is to be paid, not yearly, but half-yearly or quarterly, &c., interest being convertible into principal only once a year, the common correction of A, the present value of a yearly annuity, is to write $A+\frac{n-1}{2n}$ instead of A, n being the number of times which the annuity is payable yearly. This correction is strictly true when we speak of a perpetual annuity, on the supposition that by interest being convertible once a year, we mean that money may be invested at any one time of the year, so that the grantor of the annuity must be held to lose simple interest for the rest of the year on each of the prepaid portions. When this correction is to be investigated for a life annuity, it would generally be supposed that those who die in each year die at equal intervals. This is an insufficient supposition; for if the mortality of the table be increasing from year to year, it ought to be supposed that the mortality of the latter part of a year is greater than that of the former. We have therefore taken the supposition that the mortality of each year is to be constructed on the

hypothesis of constant third differences, which has the effect of distributing the deaths of each year in a manner depending on the mortality of the preceding and following years.

Proceeding on this plan we find the following result:—Let the annuity be payable n times a year, let the age be k, and let a_k be the number in the table alive at that age, A_k being the tabular yearly annuity. Also let

$$\beta=\frac{n-1}{2n},\quad \beta'=\frac{n^2-1}{72n^2},\quad \beta''=\frac{n^4-1}{180n^4}.$$

The value* of the annuity then is, accurately, so far as the suppositions are accurate (r being the interest of £1 for one year),

$$\left(1+6\beta'\frac{r^2}{1+r}-(\beta'+\beta'')\frac{r^4}{(1+r)^2}\right)A_k$$

$$+\beta-(8\beta'+2\beta'')\frac{r}{1+r}-(\beta'+\beta'')\frac{r^2}{(1+r)^2}$$

$$-3\beta'\frac{a_{k-1}-a_{k+1}}{a_k}+(\beta'+\beta'')r\left(\frac{a_{k+1}}{a_k}+\frac{1}{1+r}\,\frac{a_{k-1}}{a_k}\right).$$

This expression contains a coefficient for A_k depending (n apart) on the rate of interest only; certain terms which depend on the rate of interest only; another depending on the mortality only; and others depending jointly on the rate of interest and the mortality. To form an idea of the utmost value of these corrections, let us take the extreme case of an annuity payable momently (or n infinite) and a high rate of interest, 10 per cent. We have then

$$\beta=\frac{1}{2},\quad \beta'=\frac{1}{72},\quad \beta''=\frac{1}{180};$$

and the value of the annuity is

$$1{\cdot}00076\,A_k+{\cdot}48873-{\cdot}03989\frac{a_{k-1}}{a_k}+{\cdot}04361\frac{a_{k+1}}{a_k},$$

$A_k+{\cdot}5$ being the common approximation.

We may say then that the common approximation is generally correct to the first place of decimals, but by no means to the second or third. Now, it may fairly be insisted, looking at the manner in which such calculations are usually conducted, that the second place at least in the decimal part of an annuity shall be made as correct as can be conveniently done; not that this is, generally speaking, of very great consequence, but because there are other things by the dozen of no greater consequence, which are always attended to.

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* As we give no demonstration, we should say that, besides obtaining this expression from demonstration, we have tried several verifications: the simplest is that in which a_k is a constant, and $A_k = 1 : r$, which gives the perpetual annuity.

THE END.

APPENDIX

TO THE

DOCTRINE OF LIFE ANNUITIES AND ASSURANCES

CONTAINING

A PAPER READ BEFORE THE ROYAL SOCIETY,

ON

A NEW METHOD OF CALCULATING THE VALUE OF LIFE ANNUITIES.

BY FRANCIS BAILY.

1813.

PREFACE.

The following *paper* was read before the *Royal Society* during the course of the last year; but the Committee* (to whom it is customary to refer the papers which are presented to the Society within the year, in order that they may *select* for publication such as they conceive to be not only the most *curious*, but likewise the most *useful*) have not thought proper to publish it in their annual volume of the *Philosophical Transactions*. The *motives* which induced them to come to this decision I shall not stop to inquire into, since such a discussion would be foreign to my purpose; but perhaps they may be evident to the reader before he closes the present work.

The importance and utility of the method which has been investigated and explained in the paper here alluded to, will be apparent to every person who has studied with attention the subject of Life Annuities and Assurances, and will be a sufficient apology (if any apology indeed were necessary) for my submitting it, in its present shape, to the public opinion. The want of a more numerous collection of Tables than those which we at present possess, for determining the value of Annuities on Lives, according to the observations of human mortality at *various* places, has long been experienced and lamented. It was hoped and expected that when any person should come forward, skilful and bold enough to undertake the arduous task, he would meet with sufficient encouragement from the public to accomplish his design, and with sufficient patronage to remunerate him for the time, the labour, and the expense which must necessarily be incurred in so laudable and useful a pursuit. With still greater confidence might he hope for assistance and support from the various Life Assurance Companies that have been lately established; and who (it was natural to expect) would be desirous of obtaining all the information and help which they possibly could, on a subject in which they are so immediately interested. Nothing, perhaps, tended so much to destroy the numerous *Bubble* Societies which sprung up about forty years ago; nothing, probably, opened so effectually the eyes of the public with respect to their *delusive* schemes, as the publication of more correct and comprehensive Tables of the value of Life Annuities: whereby the true value, which *ought* to be given in such cases, was more accurately determined. And, as one improvement in science generally leads on to another, this naturally opened the way to a more complete and comprehensive investigation of the subject; so that, at the present day, a new, a distinct, and an interesting branch of analysis has arisen, which was unknown to mathematicians of a former period.*

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* This Committee consists of the Council for the current year; a list of whom, for the year 1812, will be found in page 186.

To this important circumstance the *present* Societies (who have established their plans on a more correct, and consequently a more firm and extensive basis) are indebted not only for their origin, but also for their very existence and success at this moment. For, by the variety of Tables of the value of Life Annuities which have been formed from the observations of human mortality at *different* places, they have had an opportunity of judging of the *proper* values on which the business of such societies ought to be conducted; and by a more enlarged analysis of the subject they have been able to facilitate their computations and extend their concerns to the more complicated cases that arise, and which formerly were not considered capable of an accurate solution. This knowledge is now more generally diffused and understood; and the public are not again likely to become the dupes of any artful impostor.

These advantages have kept pace with, and indeed are principally owing to, the gradual extension and publication of the Tables of Life Annuities; or, in other words, with our opportunities of detecting error, and of enlarging the boundaries of the science. For the labours of the analyst in this department would have been of very little *practical* utility if they had not been supported by such Tables. Those Tables, however, which we at present possess, are by no means sufficiently comprehensive for all the useful purposes of life. The observations made in *Sweden* seem to be the most correct for mankind *in general;* and those given by *M. De Parcieux* appear to be peculiarly adapted for a set of *life annuitants.* But the Tables of *Life Annuities,* which are formed from these observations, are so limited as to be inapplicable to many cases which now arise in the ordinary course of business. On the other hand, the observations made at *Northampton* are on too contracted a scale to give the true rate of human mortality amongst mankind in general; and, although their use has been continued, it is principally on account of the more numerous Tables of the value of Life Annuities which have been computed therefrom. In the present advanced state of the science, however, even these Tables are too confined; and it would be highly desirable to possess Tables of the value of annuities on two joint lives for every possible combination of age, not only according to the observations made at *Northampton*, but also according to those made in *Sweden*, and by *M. De Parcieux* in France. I now, however, despair of ever seeing this accomplished; and my reason for entertaining so discouraging an opinion arises from the want of patronage and support which an industrious and intelligent author has lately experienced from the public.

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* It is a singular fact, that the subject of Life Annuities has never yet formed a part of any elementary treatise on mixed mathematics. Its increasing utility and importance surely may lay claim to some notice from future compilers.

The name of this gentleman is Mr. GEORGE BARRETT of Petworth, in Sussex, who (at an immense expense of time and labour) has calculated and formed the most numerous and comprehensive set of Life Annuity Tables that ever were, or probably ever *will* be produced. The nature, the extent, and the design of those Tables will appear in the course of the following paper. It will be sufficient here to observe, that the author, flattered with the prospect of success, issued his proposals, about eighteen months ago, for publishing them by subscription. The work was to have been comprised in two large quarto volumes; containing, in the whole, about thirteen or fourteen hundred pages closely printed; the price not to be less than four guineas each volume. In order to promote his views as far as lay in *my* power, I addressed a letter to each of the Life Assurance Companies, urging his claims to *their* patronage and support. But his hopes and expectations have been defeated in every quarter; and after an ineffectual attempt to derive assistance from the public in support of his plan, and not wishing to involve himself in any serious expenses therein, he has been reluctantly obliged to abandon his design altogether; and thus the world has been deprived of one of the most extraordinary and successful efforts of patience, perseverance, and talent that ever was produced by one man. Out of all the Life Assurance Companies now established (fifteen in number), only two* consented to patronize the work; eleven† declined sending any answer to the application that was made; and two of them, the *Atlas* and the *Westminster* Societies (their names *shall* be made conspicuous) absolutely *refused* to subscribe for a single copy.

It is in vain for these several Life Assurance Companies to maintain that they are already possessed of every information which they require, and of every Table which may be useful to them in their business. They must know (or rather they *ought* to know) that they are very materially interested in every improvement in the science, and that even a re-calculation

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* The Albion and the Provident Institution.

† The Amicable, the Eagle, the Equitable, the Globe, the Hope, the London Assurance, the London Life Assurance, the Pelican, the Rock, the Royal Exchange, and the Sun.

(by another hand) of those Tables which they now enjoy, might serve to correct many errors which are *known* to exist in them. Besides, it cannot be necessary to tell them that they are *not* possessed of every Table requisite for carrying on their business to its fullest extent. For all these Companies are in the practice of granting assurances, on various contingencies, for *terms of years;* and I am informed that some of them, in order to abridge the necessary labour of computation, adopt a singular (but, at the same time, a very *inaccurate* and *unjust*) mode of determining the values in such cases. Moreover, in the granting and purchasing of *annuities* on lives, they want *other* Tables than those which are deduced from the *Northampton* observations. In determining also the value of Reversionary Sums and Annuities for *Widows*, it would be desirable to know how far the result is affected by the superior longevity of *females;* a circumstance which might be ascertained with the greatest accuracy by means of the new Tables here alluded to. Indeed, upon a variety of points connected with this subject (both of curiosity and real utility), these Tables were capable of throwing a very considerable degree of light and interest. But I am ashamed to insist further on so obvious a circumstance, and I now regret that I should have so much mistaken the nature and object of these *trading* companies, as ever to have addressed them as the patrons and promoters of literature and science.*

Besides the Life Assurance Companies, most of the nobility and gentlemen possessed of landed estates, together with several of the colleges and other corporate bodies, were addressed also by Mr. Barrett on this occasion. But the list of his subscribers was so small that he has not thought himself justified in sending the work to the press; and the public are thus for ever deprived of this singular monument of industry and abilities.

In order to preserve some record of Mr. Barrett's prodigious labour, as well as to explain the principles upon which his Tables are constructed—principles which open a *new* and a wide field to the analyst, and which greatly abridge the labour of computation in some of the most intricate problems that occur in the science—I drew up the following memoir, and presented it to the Royal Society, under the impression that they might think it worthy of publication in the *Philosophical Transactions*. They had, in the preceding year, given a favourable reception to a paper which I sent to them on a subject *less deserving* their attention;[*] but I had then to learn, that whilst the volumes which are annually published by the Royal Society are open to such curious yet fruitless speculations, they are shut to investigations of a more serious and important nature: that whilst the dark and doubtful researches of the theorist are received with indulgence, the labours of those who are desirous of rendering themselves *practically* useful to society, and of enlarging the boundaries of *true science*, are rejected with coldness and indifference. Nevertheless, as far as my humble efforts can avail, the astonishing labours of Mr. Barrett *shall not* sink into oblivion; and I am happy in having his permission to print the following paper, together with the accompanying Tables, by way of *Appendix* to my *Doctrine of Life Annuities and Assurances*, whereby some record of his wonderful exertions in this particular branch of science may *still* be preserved amongst those who are interested in the subject.

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* Dr. Price, in his account of the Equitable Society, observes that "it is indeed furnished with Tables, by which a great part of its business is conducted; but there are some important Tables which it wants, and with which it should be supplied. These, when composed, together with all its other Tables, should be subject to the revisal and examination of the best judges, and afterwards published, with a minute account of the principles assumed and the method taken in composing them. Such a publication would be a valuable addition to this part of science, and it would also be the means of increasing and establishing the credit of the Society."—*Obs. on Rev. Pay.*, vol. i. p. 180.

I cannot omit the present opportunity which is afforded me of noticing one of the most singular instances of *perseverance in error* that perhaps ever was witnessed in the republic of letters; and as the case is in some measure connected with the present subject, I hope I may be excused for introducing it in this place. In the Preface to my *Doctrine of Life Annuities and Assurances* (page 10), I have exposed the *inaccuracy* and *absurdity* of the formulæ which have been given by Mr. Morgan in the *Philosophical Transactions*, for the solution of those cases in Contingent Assurances where three lives are concerned. That work not being within the reach of every person, Mr. Morgan has inserted the formulæ at the end of the first volume of Dr. Price's *Observations on Reversionary Payments*, Note P. But I have asserted that "the errors of the original are multiplied in the copy; and Mr. Morgan, if he studies his own reputation as a mathematician, had better expunge them altogether in future than suffer them to remain in their present *worse than useless state.*" Yet, notwithstanding all that I have there said, and although I have given ample references to the body of the work for a *demonstration* of the absurdity and errors of his formulæ, he has (within these last three months) given a new edition of Dr. Price's work above mentioned, wherein *all the erroneous parts of the former editions are faithfully retained!!!*

Mr. Morgan may perhaps flatter himself that no one will follow him through his investigations, and that he shall thereby escape the censure which he so justly deserves for his extraordinary conduct; but I shall briefly point out a case or two where the absurdity of his formulæ may be readily detected, even by those who are unacquainted with the principles from which they are deduced.

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[*] On the solar eclipse which is said to have been predicted by Thales. See the *Phil. Trans.* for 1811, part ii. p. 220.

A singular instance of this kind occurs in the formula for the solution of his fourteenth problem, when the ages are equal.* For if we take any case where the lives are under 52 years of age (the rate of interest 4 per cent, and according to the *Northampton* Tables) we shall find that the value will always come out *negative* quantity!!! It is surely unnecessary to make any comment upon so singular a result as this; and yet, if the reader is desirous of amusing himself with any further proofs of the futility of this formula, he may take any case where the lives are from 52 to 57 years of age (both inclusive), and having calculated the values according to the two rates of 4 and 5 per cent interest, he will find that all the results, according to the *former* rate, will be *positive*, but that, according to the *latter* rate, they will be still all negative!!! The same circumstance will occur in other instances which might be adduced.

Now, this is one of his formulæ which I had particularly noticed as erroneous;† and the remarks which I have just made are sufficient to show its *absurdity*. Indeed this case, from its simplicity, readily points out the inaccuracy of the method of solution which has been adopted; but there are *many* others, wherein the quantities involved are so numerous, so intricate, and so complex, that the absurdity of the formula cannot be shown by a practical example without employing much time and labour.

One case, however, will suffice for my present purpose. Let the reader attempt to solve the eleventh problem given by Mr. Morgan,‡ by assuming the ages of A, B, C to be respectively 50, 40, and 30 years (the rate of interest 4 per cent, and according to the *Northampton* observations), and he will find that the value of an assurance of £100, payable on the contingency therein mentioned, will come out equal to the sum of *six hundred and twelve pounds!!!* and there is this further remarkable absurdity attending the formula that the *smaller* the sum to be received, the *greater* is the value of the assurance, and *vice versa*. Thus, the assurance of £1 payable on that contingency, is *six hundred and ninety-five pounds;* whereas the assurance of £800 is only *twenty-nine* pounds; and the assurance of £900 and all higher sums comes out a *negative quantity!!!*§

This is another of his formulæ which, amongst many others, I had also particularly noticed as incorrect: and I had even given the *true* quantities which (according to his own notation) *ought* to have been used instead of the erroneous ones which he has retained.* Mr. Morgan must be aware that there is a duty which he owes not only to the *public*, but also to his *employers;* and I leave it to his own sense of propriety to justify the re-publication of such *manifest errors.*

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* See Price's *Obs. on Rev. Pay.*, vol. i. p. 395. The formulæ there given $\frac{S(r-1)}{r} \times (V-CC-C-CCC)$.

† In my *Doctrine of Life Annuities*, &c., page 184, note.

‡ See Dr. Price's *Obs. on Rev. Pay.*, vol. i. page 392, case 2d.

§ I here allude to the formula in the *seventh* edition of Dr. Price's treatise; but if the question be solved by the formulæ given in the *sixth* edition, the value will in *all* cases be negative!!!

These are a *few* only of the *numerous cases* which I might adduce; but it is needless for me to insist further upon this point; since I have already said enough, both here and in another place, to enable the public to draw a just and proper inference on the subject. Mr. Morgan is known to the world, as a mathematician, by those papers only (in the PHILOSOPHICAL TRANSACTIONS) which he has written on CONTINGENT ASSURANCES. Those papers have been the subject of much comment in the preceding pages, and, therefore, I shall not add another remark upon them in this place. I wish it, however, to be understood that this branch of the science is not a subject of CURIOSITY only, but one of REAL UTILITY. Property to a very large amount (particularly in this country) is constantly depending on the contingencies mentioned in those problems to which I have referred; and it is, therefore, of material consequence that a proper, a correct, and a simple mode of determining the value of such property should be laid down. But every mathematician who has attended to the subject must know that *all* the formulæ which Mr. Morgan has given for this purpose only tend to perplex and *mislead* the public; and will not, except in very few cases, give a true and accurate (nor in *any* case, the most simple and concise) method of solving the different questions that arise. The natural consequence of which is, that in the distribution of such property, and according to such rules, one party must be *defrauded;* whilst another is *unjustly* enriched at the expense of such innocent sufferer.

As the formulæ above alluded to are inserted in Dr. Price's *Observations on Reversionary Payments* without the investigations, Mr. Morgan ought to have been more particular in seeing that they were correctly deduced, and properly stated; since the reader has no opportunity of immediately detecting any error. More especially ought he to have given greater attention to this point in any *subsequent* edition of that work, after he had been publicly attacked upon their want of accuracy. The world, however, will learn, perhaps with some surprise and indignation, that *he has not corrected a single* error!!! and that in the seventh edition of Dr. Price's treatise, which has just been published, *all* these *absurd and inaccurate* formulæ are still retained,—a disgrace to the editor, and an *insult* as well as an *injury* to the public at large.

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* See *Doctrine of Life Annuities*, &c., page 166, note.

Names of the Council of the Royal Society for the year 1812.

Sir Joseph Banks, Bart., *Pres.*
Sir Charles Blagden.
Admiral William Bligh.
Bishop of Carlisle.
Anthony Carlisle, Esq.
Dr. (now Sir) Humphrey Davy, *Sec.*
Lord Dundas.
Edward Howard, Esq.
Samuel Lysons, Esq., *Treas.*
Joseph de Mendoza Rios, Esq.
Matthew Montagu, Esq.
William Morgan, Esq.
Earl of Morton.
John Pond, Esq., *Astr. Roy.*
Lord Redesdale.
Edward Rudge, Esq.
Joseph Sabine, Esq.
Lord Webb Seymour.
Viscount Valentia.
Dr. Wm. Hyde Wollaston, *Sec.*
Dr. Thomas Young, *For. Sec.*

ON A NEW METHOD OF CALCULATING THE VALUE OF LIFE ANNUITIES.

(*Read June* 4, 1812.)

The trouble attending the calculation of Tables for determining the value of Life Annuities, particularly where two or more lives are concerned, is so great, that (notwithstanding the importance of the subject, and its extensive application and utility at the present day) few persons have been disposed to undertake the laborious task of forming them. If we except those Tables which have been published by Dr. Price and Mr. Baron Maseres, there are no others that, in the present state of the science, can properly be applied to any practical use. The *former* has given, in his *Observations on Reversionary Payments*, several valuable Tables deduced from the bills of mortality as observed at *Northampton* and in *Sweden;* and the *latter* has given, in his *Principles of the Doctrine of Life Annuities*, similar Tables deduced from the observations of *M. De Parcieux* in France.

At the same time that Dr. Price published the Tables above alluded to, he gave "A Specimen of an easy and expeditious method of calculating the values of Annuities on Single or Joint Lives," which greatly abridged the labour attending such calculations; and in my *Doctrine of Life Annuities and Assurances*, p. 31, I have given an improvement of that method. The *practical* application, however, of this method is capable of being rendered much more simple and easy, as I shall attempt to show in the course of this paper. But the more immediate object that I have in view, is to lay before the Society a *new* method of arranging the Tables for determining the value of Life Annuities, whereby a considerable portion of the time and labour employed in such calculations may be avoided, and their application to the solution of various problems connected with the subject rendered more extensive and easy.

Let A be the life upon which the annuity is granted, and let the number of persons living at the age of A, according to any given Table of observation, and at 1, 2, 3, &c., years older, be denoted respectively by $a, b, c, d, \ldots\ldots\ldots w, x, y, z:$ z denoting the number of persons living at the age of the oldest life in the given Table, and in general equal to unity. Moreover, let the *amount* of £1 in a year, according to the given rate of interest, be denoted by r. This being premised, it is well known (by those who are conversant with the subject) that the value of an annuity on the single life A is expressed by the following series:—

$$\frac{1}{a}\left(\frac{b}{r}+\frac{c}{r^2}+\frac{d}{r^3}+\cdots\cdots\frac{x}{r^{n-2}}+\frac{y}{r^{n-1}}+\frac{z}{r^n}\right);$$

n denoting the number of years from the age of A to the age of the oldest life in the Table of observations; and the *sum* of this series, numerically expressed according to the age of A, forms the common Tables of the value of Life Annuities. But the above series may be expressed, more conveniently for our present purpose, by the following one:—

$$\frac{1}{ar^n}(br^{n-1}+cr^{n-2}+dr^{n-3}+\cdots\cdots xr^2+yr+z);$$

which is evidently the same as the former.

In the plan proposed, therefore, A is supposed to be a child just born, or one whose age is equal to 0; and *each* term of the series (beginning at the end) is to be numerically expressed and arranged in regular order in the same column, at the bottom of which must be placed the numerical value of the common divisor ar^n. In a collateral column are to be placed the *sums* of the first *one*, first *two*, first *three*, first *four*, &c., values in the former column, which I shall denote by Z, Y, X, &c., and by the help of these two columns we shall be enabled to solve every question relative to Life Annuities and Assurances, not only with less liability to error, but frequently in a more expeditious and easy manner than by the usual method of proceeding.

In order to render these remarks the more intelligible, I shall insert the following specimen, which shows the manner in which the values, above alluded to, are to be arranged, and where I have supposed the age of the oldest life in the Table of observations to be 96 years:—

Age.	Column 𝔄.	Column 𝔅.
96	z	Z
95	yr	Y
94	xr^2	X
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
3	dr^{n-3}	D
2	cr^{n-2}	C
1	br^{n-1}	B
0	ar^n	A

Consequently we shall have

$$Z=z$$
$$Y=yr+z$$
$$X=xr^2+yr+z.$$

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$$C=cr^{n-2}+dr^{n-3}+er^{n-4}\ldots\ldots\ldots\ldots xr^2+yr+z$$
$$B=br^{n-1}+cr^{n-2}+dr^{n-3}+\ldots\ldots\ldots xr^2+yr+z$$
$$A=ar^{n}\ \ +br^{n-1}+cr^{n-2}+\ldots\ldots\ldots xr^2+yr+z.$$

Whence it follows, that the value of an annuity on the life of a child *just* born is denoted by $\frac{B}{ar^n}$; that the value of an annuity on a life *one* year old is denoted by $\frac{C}{br^{n-1}}$; on a life *two* years old by $\frac{D}{cr^{n-2}}$; on a life *three* years old by $\frac{E}{dr^{n-3}}$; and so on to the extremity of human life.

It may be objected to this mode of arrangement that a *numerical operation* is necessary for determining the value of the annuity; whereas, in the usual method of arranging the Tables, such value is found by *inspection.** This objection, however, is of weight only as far as regards *single* lives, and a few cases of *two joint* lives; because, in the cases of those joint lives whose ages are *not* given in the existing Tables, the values are not so readily found by the method now in use as by the process here laid down. Moreover, the present arrangement will enable us to solve, with great facility and expedition, a variety of intricate problems that frequently occur in practice (particularly in cases of *deferred* or *temporary* annuities and assurances), which, according to the present method of solution, require much time and labour, and are consequently liable to considerable error in the numerical enunciation.

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* The mode of arranging Annuity Tables, as hitherto adopted by most *English* writers on this subject (that is, by showing the *number of years' purchase* that the annuities are worth), is certainly accompanied with many advantages, since it facilitates the solution of several problems that frequently arise. The *French* writers, however, give the *reciprocals* of those values; that is, they form their Tables so as to show the *annuity which £1 will purchase.* But this is not so convenient for general use as the former method; nevertheless it is frequently adopted, even in this country, and particularly by most of the public offices. The new method here suggested is, in many cases, superior to both; and may, in all cases, be readily converted into either by changing the numerator for the denominator; and *vice versâ.* Thus, $\frac{B}{ar^n}$ is the value of an annuity on the life of A just born: and $\frac{ar^n}{B}$ is the annuity which £1 will purchase on the same life.

For instance, the value of an annuity on the life of a child just born, *deferred* for 1, 2, 3, etc., years, is respectively denoted by $\frac{C}{ar^n}$, $\frac{D}{ar^n}$, $\frac{E}{ar^n}$, &c. The value of a similar annuity on a life *one* year old is respectively denoted by $\frac{D}{br^{n-1}}$, $\frac{E}{br^{n-1}}$, $\frac{F}{br^{n-1}}$, &c. On a life *two* years old, by $\frac{E}{cr^{n-2}}$, $\frac{F}{cr^{n-2}}$, $\frac{G}{cr^{n-2}}$, &c., and so on.

In like manner, the value of a *temporary* annuity for 1, 2, 3, &c., years on the life of a child just born is respectively denoted by $\frac{B-C}{ar^n}$, $\frac{B-D}{ar^n}$, $\frac{B-E}{ar^n}$, &c. The value of a similar annuity on a life *one* year old is respectively denoted by $\frac{C-D}{br^{n-1}}$, $\frac{C-E}{br^{n-1}}$, $\frac{C-F}{br^{n-1}}$, &c. On a life *two* years old, by $\frac{D-E}{cr^{n-2}}$, $\frac{D-F}{cr^{n-2}}$, $\frac{D-G}{cr^{n-2}}$, &c., and so on.

Moreover, if an annuity be *deferred* for any number of years, and to be paid for in *equal annual payments*,* the value of those annual payments may be determined in the same easy and expeditious manner. Thus, let the annuity be deferred for five years, on the life of a child just born, to be paid for in five equal annual payments, the value of those equal annual payments will be denoted by $\frac{G}{A-F}$; if the life had been *one* year old, the value would be denoted by $\frac{H}{B-G}$; and so on. Had the period been for six years, to be paid for in six equal annual payments, the values would be respectively $\frac{H}{A-G}$, and $\frac{I}{B-H}$; and so on, for any other interval.

The value of *Assurances* on any life may also be determined with nearly equal facility. Thus, the value of an assurance of £1 on the life of a child just born will be denoted by $\frac{A-B.r}{r.ar^n}$; on a life *one* year old, by $\frac{B-C.r}{r.br^{n-1}}$; on a life *two* years old, by $\frac{C-D.r}{r.cr^{n-2}}$; and so on.

If the assurance be paid for *in equal annual payments*,† the value of those payments may be determined by substituting A, B, C, &c., instead of ar^n, br^{n-1}, cr^{n-2}, &c., in the several denominators above stated. Thus, the value of an assurance of £1 on the life of a child just born, to be paid for *in one present payment*, is equal to $\frac{\text{A}-\text{B}.r}{r.ar^n}$; and the annual payment which ought to be made as an equivalent thereto during that life, is $\frac{\text{A}-\text{B}.r}{\text{A}.r}=\frac{1}{r}-\frac{\text{B}}{\text{A}}$; which is a much more simple formula for such cases than any that has ever yet been given.

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* The first payment to be made immediately, and the remaining ones at the beginning of each succeeding year.

† See the preceding note.

In a similar manner we may find the value of *deferred assurances.* Thus, the value of an assurance of £1 on the life of a child just born, *deferred* for 1, 2, 3, &c., years is respectively denoted by $\frac{\text{B}-\text{C}.r}{r.ar^n}$, $\frac{\text{C}-\text{D}.r}{r.ar^n}$, $\frac{\text{D}-\text{E}.r}{r.ar^n}$, &c. And on a life *one* year old, deferred for the same periods, it is respectively $\frac{\text{C}-\text{D}.r}{r.br^{n-1}}$, $\frac{\text{D}-\text{E}.r}{r.b^{n-1}}$, $\frac{\text{E}-\text{F}.r}{r.br^{n-1}}$, &c., and so on.

A similar method of proceeding will lead us to the value of *temporary assurances.* Thus, the value of an assurance of £1 for four years on the life of a child just born, is equal to $\frac{\text{A}-\text{E}-(\text{B}-\text{F})r}{r.ar^n}$.

If this temporary assurance be paid for in *annual payments*. the value of those annual payments will be $\frac{1}{r}-\frac{\text{B}-\text{F}}{\text{A}-\text{E}}$: which is a much more simple and expeditious mode of obtaining the value than any hitherto known.

If what has been stated respecting *single* lives be well understood, no difficulty is likely to occur in the case of any number of *joint* lives. For, if we take *two* joint lives, A, B, and make the number of persons living at the age of A, and at the age of 1, 2, 3, &c. years older, equal to a, a', a'', a''', &c.; and likewise make the number of persons living at the age of B, and at the age of 1, 2, 3, &c. years older, equal to b, b', b'', b''', &c., then, by substituting ab for a, $a'b'$ for b, $a''b''$ for c, $a'''b'''$ for d, &c., in the above specimen, we shall find that the same rule will enable us to determine, with equal facility, the value of an annuity on *two* joint lives of any ages. And the like method may be extended also to the case of *three* joint lives.

In order to find the *Expectation of Life*, we have only to make $r=1$ in the specimen above given and the same rule will still hold good, with this exception, however, that we must add $\frac{1}{2}$ to the values thus found, in order to obtain the true expectation of life for every *moment* of human existence, agreeably to the method laid down by all the writers on this subject. Thus if we make

$$\begin{aligned}\text{A}&=a+b+c+\ldots\ldots\ldots x+y+z\\ \text{B}&=b+c+d+\ldots\ldots\ldots x+y+z\\ \text{C}&=c+d+e+\ldots\ldots\ldots x+y+z\end{aligned}$$

then will the expectation of a *single* life A be equal to $\frac{E}{a}+\frac{1}{2}=\frac{A}{a}-\frac{1}{2}$; the expectation of a single life B will be equal to $\frac{C}{b}+\frac{1}{2}=\frac{B}{b}-\frac{1}{2}$; and so on for any other life. And in like manner we might proceed to find the expectation of any *two joint* lives, after making the substitutions above alluded to. But it is presumed that enough has been said to show the application of the method.

Having thus given a general view of the principles upon which this new method is founded, I shall proceed to give some practical examples of its utility and superior advantage. For this purpose I have annexed to this paper four Tables, which have been calculated agreeably to the principles above alluded to, and which will more fully illustrate the rules above given. All these Tables have been deduced from the *Sweden* observations of life *in general.*

The *first* Table is for determining the *expectation* of a *single* life at every age. The ages are inserted in the first column in an inverted order, beginning with the oldest life in the Table of observations. In the column 𝔄 and opposite to the respective ages, are inserted the number of persons living at those ages; and in the column 𝔅 are set down the *sums* of the values in the preceding columns, according to the method above explained.

The *second* Table is for determining the *expectation* of *two joint* lives, whose difference of age is 9, 10, and 11 years; these three differences being requisite for the solution of some of the following questions. The age of the *youngest* of the two lives is inserted in the first column; and the values in the column 𝔄 are found by multiplying together the number of persons living at the respective ages of the two given lives. Thus, the number of persons living at the ages of 71 and 80 are 1622 and 558, the product of which is 905076, and which forms the seventeenth value in this column. The column 𝔅 is formed as in the preceding Table, by taking the *sums* of the values in column 𝔄. The column marked x in this and the subsequent Tables will be explained hereafter.

The *third* Table is for determining the value of an *annuity* on a *single* life at every age; its construction will be sufficiently evident if compared with the specimen given in page 188.

The *fourth* Table is for determining the value of an *annuity* on *two joint* lives, whose difference of age is 9, 10, and 11 years; it is formed upon similar principles to the preceding one. The age of the *youngest* of the two lives is inserted in the first column, as already explained in the second Table.

The rate of interest made use of in the computation of the last two Tables is 4 per cent.

These are all the Tables which are necessary to be inserted for the purpose of solving the following cases; but I ought previously to explain the meaning of the column x in the last three Tables. The figures in that column are of use in enabling us to *point off* the proper number of *integers* that arise from the process; they are the logarithmic indices of the numbers before which they stand, and, consequently indicate the *places* of *figures* of which such numbers would consist if fully expressed. Thus, in the second Table, the number against the age of 0, in the column 𝔄 ought to be 61770000; but as the first *six* figures are all that are necessary for our purpose, the last two figures are rejected, in order to abridge the size of the Table. The logarithmic index 7, which is prefixed, shows therefore that the number ought to consist of *eight* places of figures, and that we must affix ciphers in order to make up that number. Where the index is *less* than the number of figures, we have only to point off the proper number of integers, and to consider the rest as decimals.

This being premised, it is presumed that the principles upon which the following cases are solved, will be sufficiently clear and intelligible without any further explanation:—

The *expectation* of a single life aged 40 is $\frac{110935}{4591}+\frac{1}{2}=24\cdot664$; the *expectation* of a life of 50 is $\frac{69086}{3846}+\frac{1}{2}=18\cdot463$; the *expectation* of two joint lives aged 40—50 is $\frac{246594800}{17657000}+\frac{1}{2}=14\cdot466$.

The value of an *annuity* on a *single* life aged 40 is $\frac{579207\cdot1}{41283\cdot3}=14\cdot030$; and the value of an *annuity* on a *single* life aged 50 is $\frac{272201\cdot4}{23363\cdot8}=11\cdot651$. On the two *joint* lives aged 40 — 50, the value is $\frac{1034031000}{107263000}=9\cdot640$.

The annuity *which* £1 *would purchase* on each of these lives would be respectively $\frac{41283\cdot3}{579207\cdot1}=\cdot0713$, $\frac{23363\cdot8}{272201\cdot4}=\cdot0858$, and $\frac{107263000}{1034031000}=\cdot1037$.

The value of an annuity on a life of 40 *deferred* for 20 years is $\frac{105513\cdot5}{41283\cdot3}=2\cdot556$; and on the life of 50 *deferred* for 30 years it is $\frac{3411\cdot076}{23363\cdot8}=\cdot146$. On the two joint lives 40—50 *deferred* for 15 years the value is $\frac{158765800}{107263000}=1\cdot480$.

The value of an annuity on a life of 40, deferred for 20 years, and to be paid for by *equal annual payments* during that interval, is $\frac{105513\cdot5}{620490\cdot4-117538\cdot0}=\cdot2098$; and the value of a similar annuity on a life of 50 deferred for 30 years, is $\frac{3411{\cdot}076}{295565{\cdot}2-4456{\cdot}200}={\cdot}0117$. On the two joint lives 40—50, deferred for 15 years and payable in a similar manner, the value is $\frac{158765800}{1141294000-185786900}={\cdot}1661$.

The value of a *temporary* annuity for 20 years on a life aged 40 is $\frac{579207{\cdot}1-105513{\cdot}5}{41283{\cdot}3}=11{\cdot}474$; and of a *temporary* annuity for 30 years on a life aged 50 is $\frac{272201{\cdot}4-3411{\cdot}076}{23363{\cdot}8}=11{\cdot}505$. On the two joint lives 40—50, for 15 years, the value is $\frac{1034031000-158765800}{107263000}=8{\cdot}160$.

The value of an *assurance* of £1 on a life of 40 is $\frac{620490{\cdot}4-579207{\cdot}1\times1{\cdot}04}{41283{\cdot}3\times1{\cdot}04}$ $={\cdot}4219$; and the value of a similar *assurance* on a life of 50 is $\frac{295565{\cdot}2-272201{\cdot}4\times1{\cdot}04}{23363{\cdot}8\times1{\cdot}04}={\cdot}5134$. On the two joint lives 40—50, it is $\frac{1141294000-1034031000\times1{\cdot}04}{107263000\times1{\cdot}04}={\cdot}5908$.

The value of an *assurance* of £1 in *annual payments* on a single life aged 40, is equal to ${\cdot}96154-\frac{579207{\cdot}1}{620490{\cdot}4}={\cdot}02807$; the value of a similar assurance on a life aged 50, is equal to ${\cdot}96154-\frac{272201{\cdot}4}{295565{\cdot}2}={\cdot}01059$. On two joint lives 40—50, it is equal to ${\cdot}96154-\frac{1034031000}{1141294000}={\cdot}05552$.

The value of an *assurance* of £1 on a life of 40, *deferred* for 20 years, is $\frac{117538{\cdot}0-105513{\cdot}5\times1{\cdot}04}{41283{\cdot}3\times1{\cdot}04}={\cdot}1818$; and on a life of 50, *deferred* for 30 years, it is $\frac{4456{\cdot}200-3411{\cdot}076\times1{\cdot}04}{23363{\cdot}8\times1{\cdot}04}={\cdot}0374$. On the two joint lives 40-50, *deferred* for 15 years, the value is $\frac{185786900-158765800\times1{\cdot}04}{107263000\times1{\cdot}04}$ $={\cdot}1853$.

The value of a *temporary assurance* of £1 on a single life aged 40, for 20 years, is equal to $\frac{620490{\cdot}4-117538{\cdot}0-1{\cdot}04(579207{\cdot}1-105513{\cdot}5)}{41283{\cdot}3\times1{\cdot}04}$ $={\cdot}2401$; and on a life aged 50, for 30 years, it is equal to $\frac{295565{\cdot}2-4456{\cdot}200-1{\cdot}04(272201{\cdot}4-3411{\cdot}076)}{23363{\cdot}8\times1{\cdot}04}={\cdot}4760$. On two joint lives 40—50 for 15 years, it is equal to

$$\frac{1141294000-185786900-1{\cdot}04(1034031000-158765800)}{107263000\times1{\cdot}04}={\cdot}4055.$$

If we apply these Tables to the solution of any cases in *Contingent* annuities and assurances, the advantage of their form and arrangement will be much more evident, since we can, by a very simple numerical operation, solve some of the most intricate cases on this subject. In questions relative to contingent annuities, it is always required to know the *probability of survivorship* between the given lives, but according to the methods hitherto adopted, this information cannot be obtained except at an expense of time and labour which few persons are disposed to give to the subject. Neither are there any Tables in existence that will suit all cases that may occur. By the present plan, however, this may be very readily obtained.

Let us take the case of two lives, A, B, of which A is supposed to be the youngest. And let us suppose A′ to be a life one year older than A, and B′ to be a life one year older than B. Then if we make AB, A′B, AB′, A′B′, to denote the values in Table II. standing against the ages of the joint lives AB, A′B, AB′, A′B′, respectively in the column 𝕭; and if we make *ab* to denote the value standing against the ages of the joint lives AB, in the column 𝕬, we shall have the following formula:—

$$\frac{\mathrm{AB}-\mathrm{A'B}-(\mathrm{A'B'}-\mathrm{AB'})}{2ab}$$

for the value of the probability that the life A shall die before the life B. So that, if the life of A were 40, and of B 50, the value would be equal to

$$\frac{264251800-257791100-(246594800-252670600)}{2\times17657000}=\frac{12536500}{35314000}=\cdot3550.$$

If we retain the same characters to denote the similar and corresponding values in Table IV., we shall have the following formula:—

$$\frac{\mathrm{AB}-\mathrm{A'B}-r(\mathrm{A'B'}-\mathrm{AB'})}{2r.abr^{n})}$$

for the value of an assurance of £1 on the life of A provided he dies *before* B; the numerical solution of which formula is readily performed by the help of the present Tables. For instance, let us suppose the age of A to be 40, and of B 50; and that it were required to determine the value of an assurance of £1 on the life of 40, provided he dies *before* the life of 50. This value would be equal to

$$\frac{1141294000-1116533000-1{\cdot}04(1034031000-1056721000)}{2{\cdot}08\times107263000}$$

$$=\frac{48358600}{223107040}=\cdot2168.$$

If the value of the assurance be required in *annual payments*, we have only to substitute AB instead of abr^{n} in the denominator of the given formula. Thus $\frac{48358600}{223107040}$ being the value of the assurance in one present payment, the value of the same in equal annual payments will be $\frac{48358600}{2373891520}=\cdot 02037$.

In like manner we might proceed to find the value of an assurance depending on any order of survivorship amongst *three* lives;* for if a Table were formed for *three joint* lives upon similar principles to those which have been already explained, and if we retained the same characters as before to denote the similar and corresponding values in that Table, the formulæ for such cases might be rendered equally simple and easy. Thus, the present value of £1, payable on the decease of A, provided he be the *first* that fails of three given lives, A, B, C, would be denoted by

$$\frac{2\mathrm{ABC}-2\mathrm{A'BC}+\mathrm{AB'C}-\mathrm{A'B'C}-r(2\mathrm{A'B'C'}-2\mathrm{AB'C'}+\mathrm{A'BC'}-\mathrm{ABC'})}{6r.abcr^n},$$

which produces the same value as the formula given in page 131 of my *Doctrine of Life Annuities*, &c. And the present value of £1, payable on the decease of A or B, provided *either* of them be the *first* that fails of three given lives, A, B, C, would be denoted by

$$\frac{4\mathrm{ABC}-\mathrm{A'BC}-\mathrm{AB'C}-2\mathrm{A'B'C}-r(4\mathrm{A'B'C'}-\mathrm{AB'C'}-\mathrm{A'BC'}-2\mathrm{ABC'})}{6r.abcr^n}$$

which produces the same value as the formula given in page 147 of my work above mentioned. These two formulæ, together with those already given, will serve for the solution of *most* cases in Contingent Assurances where three joint lives are concerned;† and in those cases where we are obliged to resort to other methods, the process will be found to be equally simple and easy.

If the assurance be required for a *term of years* only, the value may *in all cases* be determined with *equal* ease and expedition.

Let any person attempt to solve the cases above given, by the common rules laid down for that purpose, and by any of the existing Tables upon this subject, and he will immediately be convinced of the superior advantage of the plan here proposed. Almost all of them, particularly where two or more joint lives are concerned, would require the aid of logarithms and a troublesome calculation, and the solution would consequently be exposed to considerable error, whereas, agreeably to the present plan, the greater part of the process is performed simply by subtraction, and *only one* operation by *division* is required.

If we add another column, $\mathfrak{C}$, to the only one already given in the specimen in page 186, and take $\mathfrak{z}$, $\mathfrak{y}$, $\mathfrak{x}$, &c., to denote the *sums* of the values in column

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* This part relative to *three* lives was not inserted in the paper when it was sent to the Royal Society.

† They will serve for the solution of Problems 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 43, and 46 in my *Doctrine of Life Annuities*, &c.

$\mathfrak{B}$, then by arranging them in a similar manner, as in the following specimen :—

Age.	$\mathfrak{A}$.	$\mathfrak{B}$.	$\mathfrak{C}$.
96	z	Z	$\mathfrak{z}$
95	yr	Y	$\mathfrak{y}$
94	xr^2	X	$\mathfrak{x}$
.	.		
.			
.			
.			
.			
3	dr^{n-3}	D	$\mathfrak{d}$
2	cr^{n-2}	C	$\mathfrak{c}$
1	br^{n-1}	B	$\mathfrak{b}$
0	ar^n	A	$\mathfrak{a}$

we shall consequently have

$\mathfrak{z}=Z$

$\mathfrak{y}=Y+Z$

$\mathfrak{x}=X+Y+Z$

&c.

In this case, $\frac{\mathfrak{b}}{ar^n}$ will be the value of an *increasing* annuity on the life of a child *just born;* that is, of an annuity of *one* pound for the first year, *two* pounds for the second year, *three* pounds for the third year, and so on. Moreover, $\frac{\mathfrak{c}}{br^{n-1}}$ will be the value of a similar annuity on the life of a child *one* year old; $\frac{\mathfrak{d}}{cr^{n-2}}$ will be the value of a similar annuity on the life of a child *two* years old; and so on for any other age.

These are some of the principal advantages attending the new arrangement of the Tables. There are others of inferior moment which it would be needless to insist upon in this place, since they will immediately occur to such persons as have turned their attention to this subject. Enough has probably been said to warrant the adoption of this method in the formation of any *new* Tables which may be hereafter published: and the following directions may be of use to those persons who are employed in such calculations.

Provide a set of smooth deal lathes or rods, one edge of which should be about one-fifth of an inch in thickness; and divide the whole length of each rod into a convenient number of equal parts. On the edge of the rod thus divided (or on a slip of paper to be pasted thereon), write down within the several divisions, the logarithms of the number of persons living at every age of human existence, but in an *inverse* order, that is, beginning at the age of the *oldest* life in the Table of observations, and proceeding upwards to the *youngest*. If it be required to calculate the value of an annuity on *two joint* lives, *two* similar sets of such rods must be formed; and if *three joint* lives, then *three* similar sets; and so on. Let the computist also prepare another set of rods, divided in a similar manner, whereon he must insert the logarithms of the amount of £1 at the end of 0, 1, 2, 3, &c., years; according to the given rate of interest. Thus, let it be required to calculate the value of an annuity on two joint lives, according to the *Northampton* Table of Observations, and at 4 per cent interest; in which case the logarithms on the respective rods will be arranged in the following manner :—

·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c.
·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c.
·0000000	·0170333	·0340667	·0511000	·0681334	·0851667	&c.

And when we operate with these rods, they must be brought together (either by the hand, or by means of a groove in the table), for the more convenient addition of the numbers in the respective columns. If the two lives are of equal ages, the ends of all the rods must coincide with each other, in which case the logarithms in the several divisions will stand directly under each other, as in the following manner :—

·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c.
·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c.
·0000000	·0170333	·0340667	·0511000	·0681334	·0851667	&c.

and the natural numbers, corresponding to the sums of the logarithms in each perpendicular division, will be the values that are to be inserted in the column 𝔄 in the Table.

If there be *one* year's difference between the ages of the two lives, the upper rod must be moved *one* division towards the left, as in the following manner :—

·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c.
	·0000000	·6020600	·9542425	1·2041200	1·3802112	&c.
	·0000000	·0170333	·0340667	·0511000	·0681334	&c.

and the natural numbers, corresponding to the sums of the logarithms in each perpendicular division, will be the values that are, in this case, to be inserted in the column 𝔄 in the Table.

If there be *two* years' difference between the ages of the two lives, the upper rod must be moved *two* divisions towards the left; if *three* years, *three* divisions, and so on; and the natural numbers, corresponding to the sums in these cases, will be the values that ought to be inserted in the column 𝔄. So that these three sets of rods will serve for *every possible combination* of ages between two lives; and thereby not only save an immense deal of trouble, but, moreover, prevent those errors which are so likely to occur when we have to transcribe the same numbers over and over again with the pen. It may be further remarked, that the rod which contains the logarithms of the amount of £1, may be used with *any other* Table of Observations where the same rate is required, in the same way that the rods containing the logarithms of the number of persons living, may be used with any other *rate* that is required.

In order to detect, by a different process, any error in taking out and operating by the logarithms, we may adopt the following method :*—Let Z, Y, X, &c., respectively denote the *logarithm* of the number of people living, in the Table of Observations, at the age of the oldest life, and at 1, 2, 3, &c. years younger than the oldest; and let R denote the *logarithm* of the amount of £1 in a year, according to the given rate of interest. Then the logarithms for finding the value of an annuity on any two *equal* joint lives will be

$$Z+Z,\ Y+Y+\mathrm{R},\ X+X+2\mathrm{R},\ W+W+3\mathrm{R},\ V+V+4\mathrm{R},\ \&c.$$

Having thus found the logarithms for such cases where the ages are equal, or where the difference of age is $=0$, we may find the logarithms for those cases where the difference is 1, 2, 3, 4, &c. years, by the following method. Take out the *differences* between the logarithms Z, Y, X, &c., thus,

$$Y-Z,\ X-Y,\ W-X,\ V-W,\ T-V,\ \&c.$$

and *add* them respectively to the values above found, in the manner stated below; and the results will give the proper logarithms for finding the value of an annuity on any two joint lives whose difference of age is respectively 1, 2, 3, &c. years. Thus, the logarithms, when the difference of age is

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* This method of proving the accuracy of the calculation has been communicated to the author *since* the paper was read before the Royal Society. See the note in page 201.

$$\begin{array}{lllll}
\text{0 year are} = & Z+Z, & Y+Y+\mathrm{R}, & X+X+2\mathrm{R}, & W+W+3\mathrm{R}, \text{ \&c.} \\
\textit{add} & Y-Z & X-Y & W-X & V-W \\ \cline{2-5}
\text{1 year are} = & Z+Y, & Y+X+\mathrm{R}, & X+W+2\mathrm{R}, & W+V+3\mathrm{R}, \text{ \&c.} \\
\textit{add} & X-Y & W-X & V-W & T-V \\ \cline{2-5}
\text{2 years are} = & Z+X, & Y+W+\mathrm{R}, & X+V+2\mathrm{R}, & W+T+3\mathrm{R}, \text{ \&c.} \\
\textit{add} & W-X & V-W & T-V & S-T \\ \cline{2-5}
\text{3 years are} = & Z+W, & Y+V+\mathrm{R}, & X+T+2\mathrm{R}, & W+S+3\mathrm{R}, \text{ \&c.}
\end{array}$$

It will be readily be seen, that the results obtained by this process will be the same as by the rods above alluded to, provided they are both accurate. But if the results should differ there must be some error, which ought to be corrected.

The public are indebted to Mr. GEORGE BARRETT of Petworth, in Sussex, for this easy, expeditious, and ingenious mode of determining and arranging the value of Life Annuities; and it is with his concurrence that I have drawn up this paper to explain the principles upon which it is founded, and to point out the advantages which attend it. I have been the more induced to enter upon this subject since it gives me an opportunity of perpetuating the name of one who, by his labours on this subject, has deserved so much from every person that is interested in the science; and, at the same time, of making known to the world the astonishing and beneficial effects that may be produced by perseverance and attention in so laudable a pursuit. Amidst a variety of avocations, Mr. Barrett has never lost sight of his favourite object. At an early period of his life he became attached to this branch of analysis; and without the aid of a master he made considerable progress in the mathematics, with a view to his further and more complete study of the subject. He very soon saw, and lamented how much the application of the science was abridged, through the want of a more numerous and extensive set of Life Annuity Tables than those we at present possess. In order to remedy this defect, he at first devoted his leisure hours to the formation of a few Tables, which were intended for a new society that was about to be established in his neighbourhood. Pleased with the subject, and probably intending to render his labours more serviceable to the public at large, he extended his plan; and after twenty-five years' close application he has calculated the most complete set of Tables relative to this subject that probably will ever be again produced by any one person. These Tables are eighteen in number, and may be arranged in the following manner:—

For the Expectation of Lives.

Tables I., II., III., IV., V.—Showing the number of persons living at the several ages therein mentioned; according to the observations made in *Sweden* amongst *males*, amongst *females*, and amongst *lives in general;* according to the observations made at Northampton; and according to the observations made by *M. De Parcieux* in France. From which may be easily deduced the expectation of life at the several ages in each place respectively.

Tbble VI.—For ascertaining the expectation of any *two joint* lives, and also the probability of one life dying *before* or *after* another: deduced from the *Sweden* Table of *lives in general.*

Table VII.—For ascertaining the expectation of a *single* life, or of any *two*, *three*, or *four* joint lives: deduced from M. De Moivre's hypothesis of an equal decrement of life.

For Annuities on Single Lives.

Table VIII.—For ascertaining the value of an annuity on the life of a *male* at every age, interest at 3, 4, 5, and 6 per cent: deduced from the *Sweden* Table of Observations.

Table IX.—For ascertaining the value of an annuity on the life of a *female* at every age, interest at 3, 4, 5, and 6 per cent: deduced from the *Sweden* Table of Observations.

Table X.—For ascertaining the value of an annuity of a single life at every age, interest at 3, 3½, 4, 4½, 5, 5½, 6, 6½, 7, 7½, and 8 per cent: deduced from the *Sweden* Table of Lives *in general.*

Table XI.—For ascertaining the value of an annuity on a single life at every age, interest at 3, 3½, 4, 4½, 5, 5½, and 6 per cent: deduced from the *Northampton* Table of Observations.

Table XII.—For ascertaining the value of an annuity on a single life at every age, interest 3, 3½, 4, 4½, 5, 5½, and 6 per cent: deduced from the observations of M. De Parcieux in France.

For Annuities on Two Joint Lives.

Table XIII.—For ascertaining the value of an annuity on two joint lives, consisting of a *male* and a *female* for every possible combination of age, interest at 4 per cent: deduced from the *Sweden* Table of Observations.

Table XIV.—For ascertaining the value of an annuity on two joint lives for every possible combination of age, interest at 3, 4, 5, and 6 per cent: deduced from the *Sweden* Table of *lives in general.*

Table XV.—For ascertaining the value of an annuity on two joint lives for every possible combination of age, interest at 3, 4, and 5 per cent: deduced from the Northampton Table of Observations.

Table XVI.—For ascertaining the value of an annuity on two joint lives for every possible combination of age, interest at 3, 4, and 5 per cent: deduced from the observations of *M. De Parcieux.*

For Annuities on Three Joint Lives.

Table XVII.—For ascertaining the value of an annuity on three joint lives for every possible combination of age, interest at 4 per cent: deduced from the *Sweden* Table of Lives *in general.*

For Annuities on M. De Moivre's Hypothesis.

Table XVIII.—For ascertaining the value of an annuity on a *single* life, and on *two*, *three*, and *four* joint lives, for every possible combination of age, interest 3, 3½, 4, 4½, 5, 5½, 6, 6½, 7, 7½, and 8 per cent: deduced from M. De Moivre's hypothesis of an equal decrement of life.

All the above Tables are arranged agreeably to the principles laid down in this paper, and will therefore show with *equal* facility the value of annuities, whether they be *temporary*, *deferred*, or for the *whole duration* of life. Tables VIII., IX., and XIII. have the additional column 𝕮 alluded to in page 195, whereby they show the value of *increasing* annuities, and are consequently of essential use to those societies which are formed for the benefit of *Old Age* and of *Widows.* Tables XI. and XV. will be extremely useful to the various public companies lately established for the purpose of granting *assurances* on lives; and will save much time and labour in estimating, at the stated intervals, the true assets of such companies agreeably to the method which I have laid down upon this subject in my *Doctrine of Life Annuities and Assurances*, page 264. Tables XII. and XVI. will be equally desirable to those societies which grant *annuities* on lives, since they are formed from those observations of human mortality which show more correctly than any other the probability of living amongst a set of *Life Annuitants.* Tables X., XIV., and XVII. will be of very considerable use and advantage to the various *colleges* and *corporate bodies* in this Kingdom, as well as to noblemen and other persons of landed property whose estates are *leased on lives*, since there are no Tables at present existing whereby the true value for the renewal of a life in such leases can be fairly estimated; and we are consequently oftentimes obliged to rest contented with an approximation which (there is much reason to fear) is frequently far from the truth. By the help of these Tables, however, this doubt will be removed, and the values may in all cases be determined with the greatest accuracy. The remaining Tables, which far exceed in extent anything of the kind as yet published, will tend to the solution of many problems which have hitherto remained inexplicable for want of such assistance. The whole of them have been carefully examined and corrected separately by two other persons, so that there is the greatest reason to believe that they are accurately and truly computed.*

It is needless for me to enlarge, in this place, upon the astonishing perseverance and industry displayed by Mr. Barrett in the formation of these Tables. Every one conversant with the subject must be aware of the immense time and trouble which must have been employed in the computation. It is indeed the most successful effort of this kind that has ever yet been made by any one individual; and every person interested in the science must regret not only that they have not yet been made public, but that there is a chance of their being lost *for ever* to the world. Whether Mr. Barrett, however, may meet with sufficient patronage to induce him to publish the *whole* of these Tables, or whether he may publish them *in part*, or whether indeed he may be prevailed upon to arrange them according to the usual method (for they are at all times capable of being thus converted), the object and design of the present paper will be the same; which is to point out some curious properties and advantages which such a mode would obtain over the present arrangement, and thus, by encouraging their adoption, to open a new field to the analyst. On the other hand, should Mr. Barrett fail in his object, it may not be considered altogether useless to hold out a *beacon* to posterity, lest they should pursue a similar hopeless course, and waste the best part of their lives in the same fruitless and delusive career.

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* One of these persons operated by means of the rods alluded to in page 196, and the other by the method explained in page 198; the similarity of the two results confirmed the correctness of each other.

TABLE I.—FOR THE EXPECTATION OF A SINGLE LIFE.

Age.	Column 𝔄.	Column 𝔅.	Age.	Column 𝔄.	Column 𝔅.
96	1	1			
95	2	3	45	4219	93284
94	5	8	44	4297	97581
93	11	19	43	4375	101956
92	21	40	42	4453	106409
91	33	73	41	4526	110935
90	47	120	40	4591	115526
89	62	182	39	4651	120177
88	82	264	38	4709	124886
87	109	373	37	4767	129653
86	144	517	36	4825	134478
85	189	706	35	4884	139362
84	244	950	34	4947	144309
83	309	1259	33	5010	149319
82	384	1643	32	5072	154391
81	468	2111	31	5132	159523
80	558	2669	30	5191	164714
79	648	3317	29	5246	169960
78	743	4060	28	5301	175261
77	848	4908	27	5355	180616
76	963	5871	26	5407	186023
75	1084	6955	25	5457	191480
74	1214	8169	24	5507	196987
73	1354	9523	23	5555	202542
72	1489	11012	22	5603	208145
71	1622	12634	21	5650	213795
70	1749	14383	20	5697	219492
69	1873	16256	19	5740	225232
68	1997	18253	18	5782	231014
67	2118	20371	17	5822	236836
66	2236	22607	16	5862	242698
65	2354	24961	15	5899	248597
64	2472	27433	14	5936	254533
63	2590	30023	13	5974	260507
62	2708	32731	12	6016	266523
61	2822	35553	11	6061	272584
60	2930	38483	10	6115	278699
59	3030	41513	9	6177	284876
58	3125	44638	8	6250	291126
57	3220	47858	7	6337	297463
56	3312	51170	6	6442	303905
55	3403	54573	5	6564	310469
54	3494	58067	4	6707	317176
53	3584	61651	3	6952	324128
52	3674	65325	2	7296	331424
51	3761	69086	1	7805	339229
50	3846	72932	0	10000	349229
49	3924	76856			
48	3997	80853			
47	4069	84922			
46	4143	89065			

TABLE II.—For the Expectation of Two Joint Lives.

Part 1.—*Difference of Age* 9 *years.*

Youngest Age.	*x*.	Column 𝔄.	*x*.	Column 𝔅.	Youngest Age.	*x*.	Column 𝔄.	*x*.	Column 𝔅.
87	2	109000	2	1090000					
86		288000		3970000					
85		945000	3	1342000	40	7	180151	8	2758062
84	3	268400		4026000	39		185900		2943963
83		648900	4	1051500	38		191609		3135572
82	4	126720		2318700	37		197497		3333069
81		219960		4518300	36		203567		3536635
80		345960		7977900	35		209865		3746501
79		531360	5	1329150	34		216431		3962932
78		809870		2139020	33		223095		4186027
77	5	122112		3360140	32		229559		4415586
76		182007		5180210	31		235610		4651196
75		264496		7825170	30		241433		4892630
74		375126	6	1157643	29		247034		5139664
73		519936		1677579	28		252699		5392362
72		696852		2374431	27		258379		5650741
71		905076		3279507	26		264078		5914819
70	6	113335		4412860	25		269958		6184777
69		139164		5804499	24		275901		6460677
68		169346		7497955	23		281750		6742427
67		203963		9537589	22		287546		7029973
66		242382	7	1196141	21		293291		7323264
65		285776		1481917	20		298865		7622129
64		334709		1816626	19		304277		7926406
63		385651		2202277	18		309626		8236033
62		439238		2641514	17		314796		8550828
61		493568		3135082	16		319889		8870717
60		548789		3683871	15		324858		9195575
59		605091		4288962	14		329745		9525320
58		661875		4950837	13		334123		9860043
57		719992		5670829	12		339904	9	1019995
56		779645		6450474	11		345295		1054524
55		841222		7291695	10		351001		1089624
54		904946		8196641	9		357154		1125340
53		970547		9167189	8		363875		1161727
52	7	103680	8	1020399	7		371475		1198875
51		110197		1130596	6		380014		1236876
50		116534		1247130	5		389639		1275840
49		122625		1369755	4		400676		1315908
48		128703		1498459	3		418232		1357731
47		134765		1633224	2		442210		1401952
46		140896		1774210	1		477276		1449679
45		147412		1921622	0		617700		1511449
44		154004		2075627					
43		160737		2236364					
42		167477		2403841					
41		174070		2577911					

TABLE II.—For the Expectation of Two Joint Lives—*continued.*

Part 2.—*Difference of Age* 10 *years.*

Youngest Age.	*x.*	Column 𝔄.	*x.*	Column 𝔅.	Youngest Age.	*x.*	Column 𝔄.	*x.*	Column 𝔅.
86	2	144000	2	1440000					
85		378000		5220000	40	7	176570	8	2642518
84	3	122000	3	1742000	39		182505		2825024
83		339900		5141000	38		188219		3013242
82		806400	4	1320500	37		193969		3207211
81	4	154440		2864900	36		199900		3407111
80		262260		5487500	35		206056		3613167
79		401760		9505100	34		212573		3825740
78		609260	5	1559770	33		219187		4044927
77		924320		2484090	32		225856		4270783
76	5	138672		3870810	31		232274		4503058
75		204876		5919570	30		238319		4741377
74		296216		8881730	29		243991		4985368
73		418386	6	1306559	28		249624		5234992
72		571776		1878335	27		255273		5490265
71		759096		2637431	26		260888		5751153
70		975942		3613373	25		266520		6017673
69	6	121370		4827077	24		272431		6290104
68		148377		6310848	23		278305		6568409
67		179606		8106912	22		284184		6852594
66		215327	7	1026018	21		289958		7142552
65		255174		1281192	20		295731		7438283
64		300101		1581292	19		301120		7739403
63		350686		1931978	18		306504		8045907
62		403221		2335200	17		311768		8357675
61		457728		2792928	16		216958		8674634
60		512457		3305385	15		321908		8996542
59		567519		3872904	14		326895		9323437
58		624062		4496966	13		331856		9655293
57		681996		5178962	12		337076		9992370
56		740563		5919526	11		342446	9	1033482
55		801066		6720592	10		348372		1068319
54		863717		7584309	9		354560		1103775
53		928256		8512565	8		361375		1139912
52		994919		9507484	7		368940		1176806
51	7	106135	8	1056884	6		377630		1214569
50		112688		1169572	5		387210		1253290
49		118897		1288469	4		398127		1293103
48		124906		1413375	3		415312		1334634
47		131022		1544397	2		438927		1378527
46		137216		1681613	1		473061		1425833
45		143573		1825186	0		611500		1486983
44		150137		1975323					
43		156800		2132122					
42		163603		2295726					
41		170223		2465948					

TABLE II.—For the Expectation of Two Joint Lives—*continued.*

Part 3.—*Difference of Age* 11 *years.*

Youngest Age.	*x.*	Column 𝔄.	*x.*	Column 𝔅.	Youngest Age.	*x.*	Column 𝔄.	*x.*	Column 𝔅.
85	2	189000	2	1890000	40	7	172667	8	2526706
84		488000		6770000	39		178877		2705584
83	3	154500	3	2222000	38		184781		2890365
82		422400		6446000	37		190537		3080902
81		982800	4	1627400	36		196329		3277231
80	4	184140		3468800	35		202344		3479575
79		304560		6514400	34		208714		3688289
78		460660	5	1112100	33		215280		3903569
77		695360		1807460	32		221900		4125469
76	5	104967		2857130	31		228528		4353997
75		156096		4418090	30		234945		4588941
74		229446		6712550	29		240844		4829785
73		330376	6	1001631	28		246549		5076335
72		460101		1461732	27		252167		5328502
71		622848		2084580	26		257752		5586253
70		818532		2903112	25		263300		5849554
69	6	104513		3948246	24		268902		6118515
68		129406		5242302	23		274706		6393321
67		157367		6815976	22		280710		6674032
66		189613		8712104	21		286568		6960600
65		226690	7	1097901	20		292370		7252970
64		267965		1365865	19		297963		7550933
63		314426		1680291	18		303324		7854257
62		366663		2046955	17		308624		8162881
61		420196		2467150	16		313910		8476791
60		475246		2942396	15		318959		8795750
59		529947		3472343	14		323927		9119677
58		585312		4057656	13		328988		9448666
57		643034		4700690	12		334189		9782854
56		701482		5402171	11		339598	9	1012245
55		760911		6163082	10		345497		1046795
54		822488		6985570	9		351904		1081985
53		885965		7871535	8		358750		1117860
52		951566		8823101	7		366405		1154501
51	7	101848		9841580	6		375053		1192006
50		108534	8	1092692	5		384782		1230484
49		114973		1207665	4		395646		1270049
48		121109		1328774	3		412671		1311316
47		127156		1455931	2		435863		1354902
46		133405		1589335	1		469549		1401857
45		139733		1729068	0		406100		1462467
44		146227		1875295					
43		152862		2028158					
42		159595		2187753					
41		166285		2354039					

TABLE III.—For Annuities on a Single Life.

Youngest Age.	x.	Column 𝔄.	x.	Column 𝔅.	Youngest Age.	x.	Column 𝔄.	x.	Column 𝔅.
96	0	100000	0	1000000					
95		208000		3080000	45	4	311824	5	4350490
94		540800		8488000	44		330293		4680783
93	1	123735	1	2086150	43		349740		5030522
92		245670		4542853	42		370214		5400737
91		401496		8557808	41		391335		5792071
90		594700	2	1450481	40		412833		6204904
89		815878		2266858	39		434957		6639862
88	2	112223		3388585	38		457997		7097858
87		155141		4939995	37		482183		7580042
86		213155		7071547	36		507572		8087614
85		290957		9981115	35		534330		8621943
84		390652	3	1388763	34		562871		9184814
83		514508		1903271	33		592841		9777655
82		664965		2568235	32		624185	6	1040184
81		842842		3411076	31		656831		1105867
80	3	104512		4456200	30		690958		1174963
79		126224		5718440	29		726210		1247584
78		150518		7223622	28		763176		1323901
77		178661		9010230	27		801789		1404080
76		211005	4	1112028	26		841957		1488272
75		247019		1359047	25		883733		1576649
74		287708		1646755	24		927503		1669400
73		333723		1980477	23		973011		1766701
72		381676		2362153	22	5	102068		1868768
71		432399		2794552	21		107041		1975809
70		484905		3279457	20		112249		2088058
69		540055		3819512	19		117619		2205677
68		598841		4418353	18		123219		2328896
67		660530		5078883	17		129034		2457931
66		725224		5804107	16		135118		2593049
65		794036		6598143	15		141410		2734458
64		867192		7465335	14		147988		2882447
63		944931		8710265	13		154803		3037340
62	4	102750		9437766	12		162221		3196561
61		111359	5	1055135	11		169972		3369534
60		120245		1175380	10		178346		3547880
59		129323		1304703	9		187361		3735240
58		138713		1443416	8		197158		3932398
57		148647		1592063	7		207898		4140296
56		159010		1751073	6		219797		4360093
55		169914		1920987	5		232918		4593011
54		181436		2102423	4		247512		4840522
53		193554		2295977	3		266815		5107337
52		206351		2502327	2		291218		5398556
51		219687		2722014	1		323996		5722552
50		233638		2955652	0		431718		6154270
49		247911		3203563					
48		262624		3466187					
47		278049		3744236					
46		294430		4038666					

TABLE IV.—For Annuities on Two Joint Lives.

Part 1.—*Difference of Age* 9 *years.*

Youngest Age.	*x*.	Column 𝔄.	*x*.	Column 𝔅.	Youngest Age.	*x*.	Column 𝔄.	*x*.	Column 𝔅.
87	2	109000	2	1090000					
86		299520		4085200					
85	3	102211	3	1430632	40	8	113816	9	1230349
84		301913		4449767	39		122146		1352495
83		759121	4	1204098	38		130933		1483429
82	4	154174		2745841	37		140355		1623783
81		278320		5529037	36		150455		1774239
80		455260	5	1008163	35		161315		1935554
79		727203		1735366	34		173016		2108570
78	5	115270		2888064	33		185477		2294047
77		180756		4695620	32		198485		2492532
76		280191		7497534	31		211866		2704398
75		423467	6	1173220	30		225786		2930184
74		624612		1797832	29		240265		3170449
73		900361		2698193	28		255605		3426055
72	6	125499		3953184	27		271805		3697860
71		169519		5648375	26		288912		6986772
70		220766		7856032	25		307159		4293930
69		281920	7	1067524	24		326477		4620408
68		356786		1424309	23		346734		4967142
67		446909		1871218	22		368022		5335165
66		552333		2423551	21		390391		5725556
65		677265		3100816	20		413722		6139278
64		824962		3925778	19		438063		6577341
63		988541		4914319	18		463594		7040935
62	7	117094		6085255	17		490188		7531123
61		136840		7453657	16		518044		8049167
60		158236		9036018	15		547134		8596301
59		181449	8	1085051	14		577579		9173881
58		206416		1291466	13		609752		9783632
57		233522		1524988	12		543957	10	1042759
56		262985		1787973	11		680337		1110793
55		295105		2083078	10		719243		1182717
54		330159		2413237	9		761125		1258829
53		368256		2781493	8		806466		1339479
52		409132		3190625	7		856242		1425100
51		452242		3642867	6		910961		1516196
50		497377		4140244	5		971396		1613336
49		544309		4684554	4	9	103887		1717223
48		594142		5278696	3		112776		1829999
47		647011		5925706	2		124012		1954011
46		703953		6629660	1		139199		2093210
45		765478		7395138	0		187360		2280571
44		831700		8226838					
43		902785		9129623					
42		978264	9	1010789					
41	8	105744		1116533					

TABLE IV.—For Annuities on Two Joint Lives—*continued.*

Part 2.—*Difference of Age* 10 *years.*

Youngest Age.	*x*.	Column 𝔄.	*x*.	Column 𝔅.
86	2	144000	2	1440000
85		393120		5371200
84	3	131955	3	1856672
83		382341		5680085
82		943374	4	1511382
81	4	187900		3390381
80		331843		6708807
79		528689	5	1199569
78		833814		2033384
77	5	131559		3348979
76		205268		5401663
75		315397		8555635
74		474251	6	1329815
73		696643		2026458
72		990131		3016589
71	6	136709		4383678
70		182792		6211599
69		236417		8575774
68		300585	7	1158162
67		378404		1536566
66		471808		2008373
65		581481		2589855
64		711214		3301069
63		864341		4165410
62	7	103358		5198989
61		122023		6419218
60		142077		7839990
59		163637		9476356
58		187138	8	1134773
57		212691		1347464
56		240194		1587658
55		270210		1857869
54		302997		2160865
53		338663		2499529
52		377504		2877032
51		418820		3295852
50		462463		3758315
49		507464		4265779
48		554435		4820215
47		604845		5425059
46		658778		6083837
45		716867		6800704
44		779630		7580334
43		846798		8427131
42		918880		9346011
41		994302	9	1034031
40	8	107263	9	1141294
39		115303		1256598
38		123670		1380267
37		132546		1512813
36		142062		1654876
35		152295		1807171
34		163396		1970566
33		175220		2145786
32		187773		2333559
31		200833		2534392
30		214301		2748693
29		228179		2976872
28		242784		3219656
27		258209		3477865
26		274444		3752309
25		291584		4043893
24		309973		4353866
23		329323		4683189
22		349730		5032920
21		371109		5404029
20		393639		5797668
19		416844		6214512
18		441268		6655780
17		466801		7122582
16		493555		7616137
15		521314		8137451
14		550566		8688017
13		581277		9269294
12		614038		9883333
11		648773	10	1053211
10		686399		1121850
9		726535		1194504
8		770120		1271516
7		817692		1353285
6		870430		1440328
5		928213		1533149
4		992558		1632405
3		107682		1740087
2		118357		1858444
1		132663		1991107
0		178346		2169453

TABLE IV.—For Annuities on Two Joint Lives—*continued.*

Part 3.—*Difference of Age* 11 *years.*

Youngest Age.	*x.*	Column 𝔄.	*x.*	Column 𝔅.
85	2	189000	2	1890000
84		507520		6965200
83	3	167107	3	2367592
82		475142		7119017
81	4	114974	4	1861639
80		224034		4101984
79		385366		7955640
78		606197	5	1401761
77		951648		2353409
76	5	149401		3847417
75		231060		6158019
74		353222		9690235
73		528943	6	1497966
72		766102		2264068
71	6	107857		3342639
70		147413		4816769
69		195752		6774285
68		252069		9294977
67		318797	7	1248295
66		399486		1647781
65		496706		2144487
64		610630		2755117
63		745164		3500281
62		903720		4404001
61	7	107709		5481091
60		126693		6748019
59		146926		8217281
58		168767		9904953
57		192827	8	1183322
56		218768		1402090
55		246794		1648883
54		277436		1926319
53		310802		2237121
52		347167		2584289
51		386443		2970732
50		428285		3399017
49		471842		3870859
48		516905		4387764
47		564423		4952187
46		615844		5568031
45		670862		6238893
44		730120		6969013
43		793782		7762795
42		861825		8624690
41		933943		9558634
40	8	100858	9	1056721
39		108665		1165386
38		116741		1282128
37		125193		1407321
36		134159		1541479
35		143799		1685278
34		154259		1839538
33		165477		2005014
32		177388		2182402
31		189994		2372396
30		203142		2575538
29		216572		2792110
28		230571		3022681
27		245257		3267938
26		260716		3528655
25		276982		3805637
24		294255		4099892
23		312675		4412567
22		332169		4744736
21		352664		5097400
20		374197		5471597
19		396610		5868207
18		419894		6288101
17		444321		6732422
16		470008		7202431
15		496671		7699101
14		524684		8223685
13		554090		8777776
12		585363		9363139
11		618631		9981771
10		654554	10	1063632
9		693358		1132968
8		735121		1206480
7		780840		1284564
6		831241		1367688
5		886914		1456380
4		948434		1551223
3	9	102881		1654105
2		113010		1767115
1		126614		1893729
0		169972		2063701

END.

CORRECTIONS TO VOL. I. (B signifies "from Bottom.")

PAGE	LINE	*for*	*Read*
84,	11,	$a_x - a_{xy} - a_{xz} + a_{xyz}$,	$g(a_x - a_{xy} - a_{xz} + a_{xyz})$.
95,	6B,	$_{m1}$; $_{m2}$; $_{m3}$;	$_{m}$; $_{m1}$; $_{m2}$.
97,	18,	l_{m},	$_{m}$.
100,	13,	$l_{z\vert 1}$,	$l_{z\vert 2}$.
101,	1,	$_{+1m}$,	$_{1+m}$.
	8B,		$_+$ to be raised.
109,	11B,	) (,	+.
116,	11,	$vl_{xz}\, l_y$,	$vl_{xz}\, l_{y1}$.
	4B,	$vl_x\, l_{y1}$,	$vl_{x1}\, l_{yz}$.
118,	9,	$a_{x\cdot y1}\, l_y$,	$a_{x\cdot 1y}\, l_{1y}$.
119,	6B,	$v^m l_{x1}\, l_y$, $a_{1x\cdot y}$, $v^m l_{1x}\, l_y$,	$v^m l_{x1}\, l_{y\vert m}$, $a_{1x\cdot y\vert m}$, $v^m l_{1x}\, l_{y\vert m}$.
120,	10,	$a_{1x\cdot y}$,	$a_{1x\cdot y}\,\frac{l_{1x}}{l_x}$.
124,	8,	1st l_{y1},	l_{1y}.
127,	3B,	$a_{y})_{m-1}$,	$a_{y})_{m-1}$.
129,	6B,	$\frac{l_{1z}}{6l_z}$,	$\frac{l_{x1}}{l_x} - a_{xy\cdot 1z}\frac{l_{1z}}{6l_z}$.
130,	2,	$a_{x\cdot y1\cdot z}$,	$a_{x1\cdot yz}$.
	4,	l_z, $a_{x\cdot 1y\cdot z}$,	l_{z1}, $a_{xy\cdot 1z}$.
135,	13,	$va_{y1\cdot z}$,	$va_{y1\cdot z1}$.
136,	6,	$a_{x1\cdot yz}\, l_x$,	$a_{x1\cdot yz}\, l_{x1}$.
139,	16,	$l_x\, l_y$,	$l_x\, l_z$.
	8B,	$a_{x1\cdot y}$,	$a_{x1\cdot z}$.
147,	1, 3, and 5,	sv, sv^2, sv^3,	$\frac{sv}{6}$, $\frac{sv^2}{6}$, $\frac{sv^3}{6}$.
	17,	$\frac{s}{6l_x}$,	$-\frac{s}{6l_x}$.
158,	9,	$a_{y1\cdot z}$, a_{xz},	$a_{x1\cdot z}$, $a_{xz\vert m}$.
	11,	$_{xz+m}$, $a_{y1\cdot z}$,	$_{xz\vert m}$, $a_{x1\cdot z}$.
159,	8,	ra_x,	ra_{x+m}.
162,	18,	$_{y+z}$,	$_{y+m}$.
168,	5,	a_{xy},	$a_{xy\vert m}$.
	17,	$l_x\, l_{y\vert m}$,	$l_x\, l_{y\vert m}\, v^m$.
	18,	$a_{1x\cdot y}$,	$a_{1x\cdot y\vert m}$.
169,	13B,	l_{x1},	l_{z1}.
177,	5B,	$_{1x\cdot yz}\, l_{1z}$,	$a_{1x\cdot yz}\, l_{1z}$.
180,	8,	$_{1z}$,	$_{1z}\, l_{1z}$.
182,	5B,	l_{z1},	l_{z2}.
183,	10B,	last term,	$\frac{l_{x1}\, l_z\, v^m}{l_x\, l_z}$.
185,	14,	$\frac{v(1-ra_y)}{2}$,	$v(1-ra_y)$.
186,	2,	$\mathbf{M}-v$,	$\mathbf{M}-\frac{v}{2}$.
	5B,	sTv,	$s \times v$.
203,	3,	$\frac{srv}{\mathrm{NL}}$,	$\frac{sr}{\mathrm{NL}(1+r)}$.
204,	8,	$(r-a_x)]$,	$(1-ra_x)]$.

TABLES.

	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
1	·00005479	·00006849	·00008219	·00009589	·00010959	·00012329
2	00010959	00013699	00016438	00019178	00021918	00024658
3	00016438	00020548	00024658	00028767	00032877	00036986
4	00021918	00027397	00032877	00038356	00043836	00049315
5	00027397	00034247	00041096	00047945	00054795	00061644
6	00032877	00041096	00049315	00057534	00065753	00073973
7	00038356	00047945	00057534	00067123	00076712	00086301
8	00043836	00054795	00065753	00076712	00087671	00098630
9	00049315	00061644	00073973	00086301	00098630	00110959
10	00054795	00068493	00082192	00095890	00109589	00123288
20	00109589	00136986	00164384	00191781	00219178	00246575
30	00164384	00205479	00246575	00287671	00328767	00369863
40	00219178	00273973	00328767	00383562	00438356	00493151
50	00273973	00342466	00410959	00479452	00547945	00616438
60	00328767	00410959	00493151	00575342	00657534	00739726
70	00383562	00479452	00575342	00671233	00767123	00863014
80	00438356	00547945	00657534	00767123	00876712	00986301
90	00493151	00616438	00739726	00863014	00986301	01109589
100	00547945	00684932	00821918	00958904	01095890	01232877
110	00602740	00753425	00904110	01054795	01205479	01356164
120	00657534	00821918	00986301	01150685	01315068	01479452
130	00712329	00890411	01068493	01246575	01424658	01602740
140	00767123	00958904	01150685	01342466	01534247	01726027
150	00821918	01027397	01232877	01438356	01643836	01849315
160	00876712	01095890	01315068	01534247	01753425	01972603
170	00931507	01164384	01397260	01630137	01863014	02095890
180	00986301	01232877	01479452	01726027	01972603	02219178
190	01041096	01301370	01561644	01821918	02082192	02342466
200	01095890	01369863	01643836	01917808	02191781	02465753
210	01150685	01438356	01726027	02013699	02301370	02589041
220	01205479	01506849	01808219	02109589	02410959	02712329
230	01260274	01575342	01890411	02205479	02520548	02835616
240	01315068	01643836	01972603	02301370	02630137	02958904
250	01369863	01712329	02054795	02397260	02739726	03082192
260	01424658	01780822	02136986	02493151	02849315	03205479
270	01479452	01849315	02219178	02589041	02958904	03328767
280	01534247	01917808	02301370	02684932	03068493	03452055
290	01589041	01986301	02383562	02780822	03178082	03575342
300	01643836	02054795	02465753	02876712	03287671	03698630
310	01698630	02123288	02547945	02972603	03397260	03821918
320	01753425	02191781	02630137	03068493	03506849	03945205
330	01808219	02260274	02712329	03164384	03616438	04068493
340	01863014	02328767	02794521	03260274	03726027	04191781
350	01917808	02397260	02876712	03356164	03835616	04315068
360	01972603	02465753	02958904	03452055	03945205	04438356

	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
1	·00013699	·00016438	·00019178	·00021918	·00024658	·00027397
2	00027397	00032877	00038356	00043836	00049315	00054795
3	00041096	00049315	00057534	00065753	00073973	00082192
4	00054795	00065753	00076712	00087671	00098630	00109589
5	00068493	00082192	00095890	00109589	00123288	00136986
6	00082192	00098630	00115068	00131507	00147945	00164384
7	00095890	00115068	00134247	00153425	00172603	00191781
8	00109589	00131507	00153425	00175342	00197260	00219178
9	00123288	00147945	00172603	00197260	00221918	00246575
10	00136986	00164384	00191781	00219178	00246575	00273973
20	00273973	00328767	00383562	00438356	00493151	00547945
30	00410959	00493151	00575342	00657534	00739726	00821918
40	00547945	00657534	00767123	00876712	00986301	01095890
50	00684932	00821918	00958904	01095890	01232877	01369863
60	00821918	00986301	01150685	01315068	01479452	01643836
70	00958904	01150685	01342466	01534247	01726027	01917808
80	01095890	01315068	01534247	01753425	01972603	02191781
90	01232877	01479452	01726027	01972603	02219178	02465753
100	01369863	01643836	01917808	02191781	02465753	02739726
110	01506849	01808219	02109589	02410959	02712329	03013699
120	01643836	01972603	02301370	02630137	02958904	03287671
130	01780822	02136986	02493151	02849315	03205479	03561644
140	01917808	02301370	02684932	03068493	03452055	03835616
150	02054795	02465753	02876712	03287671	03698630	04109589
160	02191781	02630137	03068493	03506849	03945205	04383562
170	02328767	02794521	03260274	03726027	04191781	04657534
180	02465753	02958904	03452055	03945205	04438356	04931507
190	02602740	03123288	03643836	04164384	04684932	05205479
200	02739726	03287671	03835616	04383562	04931507	05479452
210	02876712	03452055	04027397	04602740	05178082	05753425
220	03013699	03616438	04219178	04821918	05424658	06027397
230	03150685	03780822	04410959	05041096	05671233	06301370
240	03287671	03945205	04602740	05260274	05917808	06575342
250	03424658	04109589	04794521	05479452	06164384	06849315
260	03561644	04273973	04986301	05698630	06410959	07397261
270	03698630	04438356	05178082	05917808	06657534	07397260
280	03835616	04642740	05369863	06136986	06904110	07671233
290	03972603	04767123	05561644	06356164	07150685	07945205
300	04109589	04931507	05753425	06575342	07397260	08219178
310	04246575	05095890	05945205	06794521	07643836	08493151
320	04383562	05260274	06136986	07013699	07890411	08767123
330	04520548	05424658	06328767	07232877	08136986	09041096
340	04657534	05589041	06520548	07452055	08383562	09315068
350	04794521	05753425	06712329	07671233	08630137	09589041
360	04931507	05917808	06904110	07890411	08876712	09863014

TABLE II.—THE DISCOUNT OF £1 FOR ANY NUMBER OF DAYS, ETC.

(See page 7.)

DAYS.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
1	·00005479	·00006849	·00008218	·00009588	·00010958	·00012327
2	00010958	00013697	00016436	00019174	00021913	00024651
3	00016436	00020544	00024651	00028759	00032866	00036973
4	00021913	00027390	00032866	00038341	00043816	00049291
5	00027390	00034235	00041079	00047922	00054765	00061606
6	00032866	00041079	00049291	00057501	00065710	00073918
7	00038341	00047922	00057501	00067078	00076653	00086227
8	00043816	00054765	00065710	00076653	00087594	00098533
9	00049291	00061606	00073918	00086227	00098533	00110836
10	00054765	00068446	00082124	00095798	00109469	00123136
11	00060238	00075286	00090329	00105368	00120403	00135433
12	00065710	00082124	00098533	00114936	00131334	00147727
13	00071182	00088962	00106735	00124502	00142263	00160018
14	00076653	00095798	00114936	00134066	00153190	00172305
15	00082124	00102634	00123136	00143629	00164114	00184590
16	00087594	00109469	00131334	00153190	00175035	00196872
17	00093064	00116303	00139531	00162748	00185955	00209151
18	00098533	00123136	00147727	00172305	00196872	00221426
19	00104001	00129968	00155921	00181860	00207786	00233699
20	00109469	00136799	00164114	00191414	00218699	00245969
21	00114936	00143629	00172305	00200965	00229608	00258235
22	00120403	00150458	00180495	00210515	00240516	00270499
23	00125869	00157286	00188684	00220062	00251421	00282760
24	00131334	00164114	00196872	00229608	00262324	00295017
25	00136799	00170940	00205058	00239153	00273224	00307272
26	00142263	00177766	00213243	00248695	00284122	00319524
27	00147727	00184590	00221426	00258235	00295017	00331772
28	00153190	00191414	00229608	00267774	00305910	00344018
29	00158652	00198236	00237789	00277311	00316801	00356260
30	00164114	00205058	00245969	00286846	00327690	00368500
31	00169575	00211879	00254147	00296379	00338576	00380737
32	00175035	00218699	00262324	00305910	00349459	00392970
33	00180495	00225518	00270499	00315440	00360341	00405201
34	00185955	00232336	00278673	00324968	00371219	00417428
35	00191414	00239153	00286846	00334494	00382096	00429653
36	00196872	00245969	00295017	00344018	00392970	00441874
37	00202329	00252784	00303187	00353540	00403842	00454093
38	00207786	00259598	00311356	00363060	00414711	00466308
39	00213243	00266412	00319524	00372579	00425578	00478521
40	00218699	00273224	00327690	00382096	00436443	00490731
41	00224154	00280035	00335855	00391611	00447305	00502937
42	00229608	00286846	00344018	00401124	00458165	00515141
43	00235062	00293656	00352180	00410635	00469023	00527341
44	00240516	00300464	00360341	00420145	00479878	00539539
45	00245969	00307272	00368500	00429653	00490731	00551733
46	00251421	00314079	00376658	00439159	00501581	00563925
47	00256873	00320885	00384815	00448663	00512429	00576114
48	00262324	00327690	00392970	00458165	00523275	00588299
49	00267774	00334494	00401124	00467666	00534118	00600482
50	00273224	00341297	00409277	00477164	00544959	00612662

TABLE II.—THE DISCOUNT OF £1 FOR ANY NUMBER OF DAYS, ETC.

(See page 7.)

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
1	·00013697	·00016436	·00019174	·00021913	·00024651	·00027390
2	00027390	00032866	00038341	00043816	00049291	00054765
3	00041079	00049291	00057501	00065710	00073918	00082124
4	00054765	00065710	00076653	00087594	00098533	00109469
5	00068446	00082124	00095798	00109469	00123136	00136799
6	00082124	00098533	00114936	00131334	00147727	00164114
7	00095798	00114936	00134066	00153190	00172305	00191414
8	00109469	00131334	00153190	00175035	00196872	00218699
9	00123136	00147727	00172305	00196871	00221426	00245969
10	00136799	00164114	00191414	00218699	00245969	00273224
11	00150458	00180495	00210515	00240516	00270499	00300464
12	00164114	00196872	00229608	00262324	00295017	00327690
13	00177766	00213243	00248695	00284122	00319524	00354900
14	00191414	00229608	00267774	00305910	00344018	00382096
15	00205058	00245969	00286846	00327690	00368500	00409277
16	00218699	00262324	00305910	00349459	00392970	00436443
17	00232336	00278673	00324968	00371219	00417428	00463594
18	00245969	00295017	00344018	00392970	00441874	00490731
19	00259598	00311356	00363060	00414711	00466308	00517852
20	00273224	00327690	00382096	00436443	00490731	00544959
21	00286846	00344018	00401124	00458165	00515141	00572051
22	00300464	00360341	00420145	00479878	00539539	00599128
23	00314079	00376658	00439159	00501581	00563925	00626191
24	00327690	00392970	00458165	00523275	00588299	00653239
25	00341297	00409277	00477164	00544959	00612662	00680272
26	00354900	00425578	00496156	00566634	00637012	00707290
27	00368500	00441874	00515141	00588299	00661350	00734294
28	00382096	00458165	00534118	00609955	00685677	00761283
29	00395688	00474450	00553088	00631602	00709991	00788257
30	00409277	00490731	00572051	00653239	00734294	00815217
31	00422862	00507005	00591007	00674867	00758585	00842162
32	00436443	00523275	00609955	00696485	00782864	00869093
33	00450020	00539539	00628896	00718094	00807131	00896009
34	00463594	00555798	00647830	00739693	00831386	00922910
35	00477164	00572051	00666757	00761283	00855629	00949796
36	00490731	00588299	00685677	00782864	00879861	00976668
37	00504293	00604542	00704589	00804435	00904081	01003526
38	00517852	00620780	00723494	00825997	00928288	01030369
39	00531407	00637012	00742392	00847550	00952484	01057197
40	00544959	00653239	00761283	00869093	00976668	01084011
41	00558507	00669460	00780167	00890627	01000841	01110810
42	00572051	00685677	00799043	00912151	01025001	01137595
43	00585592	00701888	00817913	00933666	01049150	01164365
44	00599128	00718094	00836774	00955172	01073287	01191121
45	00612662	00734294	00855629	00976668	01097412	01217862
46	00626191	00750489	00874477	00998156	01121526	01244589
47	00639717	00766679	00893318	01019633	01145627	01271301
48	00653239	00782864	00912151	01041102	01169717	01297999
49	00666757	00799043	00930978	01062561	01193796	01324682
50	00680272	00815217	00949796	01084011	01217862	01351351

Table II.—The Discount of £1 for any number of Days, etc.

(See page 7.)

DAYS.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
51	·00278673	·00348099	·00417428	·00486661	·00555798	·00624838
52	00284122	00354900	00425578	00496156	00566634	00637012
53	00289570	00361701	00433727	00505649	00577468	00649183
54	00295017	00368500	00441874	00515141	00588299	00661350
55	00300464	00375298	00450020	00524630	00599128	00673515
56	00305910	00382096	00458165	00534118	00609955	00685677
57	00311356	00388893	00466308	00543604	00620780	00697836
58	00316801	00395688	00474450	00553088	00631602	00709991
59	00322246	00402483	00482591	00562570	00642421	00722144
60	00327690	00409277	00490731	00572051	00653239	00734294
61	00333133	00416070	00498869	00581530	00664054	00746441
62	00338576	00422862	00507005	00591007	00674867	00758585
63	00344018	00429653	00515141	00600482	00685677	00770726
64	00349459	00436443	00523275	00609955	00696485	00782864
65	00354900	00443232	00531407	00619427	00707290	00794999
66	00360341	00450020	00539539	00628896	00718094	00807131
67	00365780	00456808	00547669	00638364	00728895	00819260
68	00371219	00463594	00555798	00647830	00739693	00831386
69	00376658	00470380	00563925	00657295	00750489	00843509
70	00382096	00477164	00572051	00666757	00761283	00855629
71	00387533	00483948	00580176	00676218	00772075	00867747
72	00392970	00490731	00588299	00685677	00782864	00879861
73	00398406	00497512	00596421	00695134	00793650	00891972
74	00403842	00504293	00604542	00704589	00804435	00904081
75	00409277	00511073	00612662	00714043	00815217	00916186
76	00414711	00517852	00620780	00723494	00825997	00928288
77	00420145	00524630	00628896	00732944	00836774	00940388
78	00425578	00531407	00637012	00742392	00847550	00952484
79	00431011	00538184	00645126	00751839	00858322	00964578
80	00436443	00544959	00653239	00761283	00869093	00976668
81	00441874	00551733	00661350	00770726	00879861	00988756
82	00447305	00558507	00669460	00780167	00890627	01000841
83	00452735	00565279	00677569	00789606	00901390	01012922
84	00458165	00572051	00685677	00799043	00912151	01025001
85	00463594	00578822	00693783	00808479	00922910	01037077
86	00469023	00585592	00701888	00817913	00933666	01049150
87	00474450	00592360	00709991	00827344	00944420	01061220
88	00479876	00599128	00718094	00836774	00955172	01073287
89	00485304	00605896	00726195	00846203	00965921	01085351
90	00490731	00612662	00734294	00855629	00976668	01097412
91	00496156	00619427	00742392	00865054	00987413	01109470
92	00501581	00626191	00750489	00874477	00998156	01121526
93	00507005	00632954	00758585	00883898	01008896	01135578
94	00512429	00639717	00766679	00893318	01019633	01145627
95	00517852	00646478	00774772	00902735	01030369	01157674
96	00523275	00653239	00782864	00912151	01041102	01169717
97	00528697	00659999	00790954	00921565	01051832	01181758
98	00534118	00666757	00799043	00930978	01062561	01193796
99	00539539	00673515	00807131	00940388	01073287	01205830
100	00544959	00680272	00815217	00949796	01084011	01217862

TABLE II.—THE DISCOUNT OF £1 FOR ANY NUMBER OF DAYS, ETC.

(See page 7.)

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
51	·00693783	·00831386	·00968608	·01105451	·01241917	·01378006
52	00707290	00847550	00987413	01126883	01265960	01404646
53	00720794	00863708	01006211	01148305	01289991	01431272
54	00734294	00879861	01025001	01169717	01314011	01457883
55	00747791	00896009	01043785	01191121	01338019	01484480
56	00761283	00912151	01062561	01212515	01362015	01511063
57	00774772	00928288	01081330	01233900	01385999	01537631
58	00788257	00944420	01100092	01255275	01409973	01564185
59	00801739	00960547	01118847	01276642	01433934	01590725
60	00815217	00976668	01137595	01297999	01457883	01617251
61	00828692	00992785	01156335	01319347	01481821	01643762
62	00842162	01008896	01175069	01340685	01505748	01670259
63	00855629	01025001	01193796	01362015	01529662	01696741
64	00869093	01041102	01212515	01383335	01553566	01723209
65	00882553	01057197	01231227	01404646	01577457	01749663
66	00896009	01073287	01249932	01425948	01601337	01776103
67	00909461	01089372	01268630	01447240	01625205	01802529
68	00922910	01105451	01287321	01468524	01649062	01828940
69	00936355	01121526	01306005	01489798	01672907	01855337
70	00949796	01137595	01324682	01511063	01696741	01881720
71	00963234	01153659	01343352	01532319	01720563	01908089
72	00976668	01169717	01362015	01553566	01744374	01934444
73	00990099	01185771	01380671	01574803	01768173	01960784
74	01003526	01201819	01399319	01596031	01791960	01987111
75	01016949	01217862	01417961	01617251	01815736	02013423
76	01030369	01233900	01436595	01638461	01839501	02039721
77	01043785	01249932	01455223	01659661	01863254	02066005
78	01057197	01265960	01473843	01680853	01886995	02092275
79	01070606	01281982	01492457	01702036	01910725	02118530
80	01084011	01297999	01511063	01723209	01934444	02144772
81	01097412	01314011	01529662	01744374	01958151	02171000
82	01110810	01330017	01548255	01765529	01988146	02197213
83	01124204	01346019	01566840	01786675	02005530	02223413
84	01137595	01362015	01585418	01807812	02029203	02249598
85	01150982	01378006	01603990	01828940	02052864	02275770
86	01164365	01393992	01622554	01850059	02076514	02301927
87	01177745	01409973	01641111	01871169	02100153	32328071
88	01191121	01425948	01659661	01892270	02123780	02354200
89	01204493	01441918	01678205	01913361	02147395	02380316
90	01217862	01457883	01696741	01934444	02171000	02406417
91	01231227	01473843	01715270	01955517	02194592	02432505
92	01244589	01489798	01733793	01976582	02218174	02458578
93	01257947	01505748	01752308	01997637	02241744	02484638
94	01271301	01521692	01770816	02018684	02265303	02510684
95	01284652	01537631	01789318	02039721	02288850	02536716
96	01297999	01553566	01807812	02060749	02312386	02562734
97	01311342	01569494	01826300	02081768	02335911	02588738
98	01324682	01585418	01844780	02102779	02359424	02614728
99	01338019	01601337	01863254	02123780	02382926	02640704
100	01351351	01617251	01881720	02144772	02406417	02666667

(See page 7.)

DAYS.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
101	·00550379	·00687028	·00823302	·00959203	·01094732	·01229891
102	00555798	00693783	00831386	00968608	01105451	01241917
103	00561216	00700537	00839469	00978011	01116168	01253940
104	00566634	00707290	00847550	00987413	01126883	01265960
105	00572051	00714043	00855629	00996813	01137595	01277977
106	00577468	00720794	00863708	01006211	01148305	01289991
107	00582884	00727545	00871785	01015607	01159012	01302002
108	00588299	00734294	00879861	01025001	01169717	01314011
109	00593714	00741043	00887935	01034394	01180420	01326016
110	00599128	00747791	00896009	01043785	01191121	01338019
111	00604542	00754537	00904081	01053174	01201819	01350018
112	00609955	00761283	00912151	01062561	01212515	01362015
113	00615368	00768028	00920220	01071946	01223208	01374009
114	00620780	00774772	00928288	01081330	01233900	01385999
115	00626191	00781515	00936355	01090712	01244589	01397987
116	00631602	00788257	00944420	01100092	01255275	01409973
117	00637012	00794999	00952484	01109470	01265960	01421955
118	00642421	00801739	00960547	01118847	01276642	01433934
119	00647830	00808479	00968608	01128222	01287321	01445910
120	00653239	00815217	00976668	01137595	01297999	01457883
121	00658647	00821955	00984727	01146966	01308674	01469854
122	00664054	00828692	00992785	01156335	01319347	01481821
123	00669460	00835428	01000841	01165703	01330017	01493786
124	00674867	00842162	01008896	01175069	01340685	01505748
125	00680272	00848896	01016949	01184433	01351351	01517707
126	00685677	00855629	01025001	01193796	01362015	01529662
127	00691081	00862362	01033052	01203156	01372676	01541615
128	00696485	00869093	01041102	01212515	01383335	01553566
129	00701888	00875823	01049150	01221872	01393992	01565513
130	00707290	00882553	01057197	01231227	01404646	01577457
131	00712692	00889281	01065243	01240580	01415298	01589399
132	00718094	00896009	01073287	01249932	01425948	01601337
133	00723494	00902735	01081330	01259282	01436595	01613273
134	00728895	00909461	01089372	01268630	01447240	01625205
135	00734294	00916186	01097412	01277977	01457883	01637135
136	00739693	00922910	01105451	01287321	01468524	01649062
137	00745092	00929633	01113489	01296664	01479162	01660986
138	00750489	00936355	01121526	01306005	01489798	01672907
139	00755887	00943076	01129561	01315345	01500432	01684826
140	00761283	00949796	01137595	01324682	01511063	01696741
141	00766679	00956516	01145627	01334018	01521692	01708654
142	00772075	00963234	01153659	01343352	01532319	01720563
143	00777470	00969952	01161689	01352684	01542943	01732470
144	00782864	00976668	01169717	01362015	01553566	01744374
145	00788257	00983384	01177745	01371344	01564185	01756275
146	00793650	00990099	01185771	01380671	01574803	01768173
147	00799043	00996813	01193796	01389996	01585418	01780068
148	00804435	01003526	01201819	01399319	01596031	01791960
149	00809827	01010238	01209841	01408641	01606642	01803850
150	00815217	01016949	01217862	01417961	01617251	01815736

(See page 7.)

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
101	·01364680	·01633159	·01900180	·02165755	·02429897	·02692615
102	01378006	01649062	01918633	02186729	02453365	02718550
103	01391328	01664960	01937078	02207695	02476822	02744471
104	01404646	01680853	01955517	02228651	02500267	02770378
105	01417961	01696741	01973949	02249598	02523701	02796272
106	01431272	01712624	01992374	02270537	02547124	02822151
107	01444579	01728501	02010792	02291466	02570536	02848017
108	01457883	01744374	02029203	02312386	02593937	02873869
109	01471184	01760241	02047608	02333298	02617326	02899707
110	01484480	01776103	02066005	02354200	02640704	02925532
111	01497774	01791960	02084395	02375093	02664071	02951343
112	01511063	01807812	02102779	02395978	02687427	02977140
113	01524349	01823659	02121155	02416854	02710771	03002923
114	01537631	01839501	02139525	02437720	02734104	03028693
115	01550910	01855337	02157888	02458578	02757426	03054449
116	01564185	01871169	02176243	02479427	02780737	03080191
117	01577457	01886995	02194592	02500267	02804037	03105920
118	01590725	01902817	02212935	02521098	02827325	03131635
119	01603990	01918633	02231270	02541920	02850603	03157336
120	01617251	01934444	02249598	02562734	02873869	03183024
121	01630508	01950250	02267920	02583538	02897124	03208698
122	01643762	01966051	02286234	02604333	02920368	03234358
123	01657012	01981846	02304542	02625120	02943601	03260005
124	01670259	01997637	02322843	02645898	02966823	03285639
125	01683502	02013423	02341137	02666667	02990033	03311258
126	01696741	02029203	02359424	02687427	03013233	03336864
127	01709977	02044979	02377705	02708178	03036421	03362457
128	01723209	02060749	02395978	02728920	03059598	03388036
129	01736438	02076514	02414245	02749654	03082765	03413601
130	01749663	02092275	02432505	02770378	03105920	03439153
131	01762885	02108030	02450758	02791094	03129064	03464692
132	01776103	02123780	02469004	02811801	03152197	03490217
133	01789318	02139525	02487243	02832499	03175319	03515728
134	01802529	02155265	02505476	02853188	03198430	03541226
135	01815736	02171000	02523701	02873869	03221530	03566711
136	01828940	02186729	02541920	02894541	03244619	03592182
137	01842141	02202454	02560132	02915204	03267697	03617639
138	01855337	02218174	02578338	02935858	03290763	03643083
139	01868531	02233889	02596536	02956503	03313819	03668514
140	01881720	02249598	02614728	02977140	03336864	03693931
141	01894906	02265303	02632913	02997768	03359898	03719335
142	01908089	02281002	02651091	03018387	03382921	03744726
143	01921268	02296697	02669262	03038997	03405933	03770103
144	01934444	02312386	02687427	03059598	03428934	03795466
145	01947616	02328071	02705584	03080191	03451924	03820817
146	01960784	02343750	02723735	03100775	03474904	03846154
147	01973949	02359424	02741880	03121350	03497872	03871477
148	01987111	02375093	02760016	03141917	03520829	03896788
149	02000268	02390758	02778148	03162475	03543775	03922085
150	02013423	02406417	02796272	03183024	03566711	03947368

(See page 7.)

DAYS.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
151	·00820608	·01023659	·01225881	·01427279	·01627857	·01827620
152	00825997	01030369	01233900	01436595	01638461	01839501
153	00831386	01037077	01241917	01445910	01649062	01851379
154	00836774	01043785	01249932	01455223	01659661	01863254
155	00842162	01050491	01257947	01464534	01670259	01875126
156	00847550	01057197	01265960	01473843	01680853	01886995
157	00852936	01063902	01273971	01483151	01691446	01898862
158	00858322	01070606	01281982	01492457	01702036	01910725
159	00863708	01077309	01289991	01501761	01712624	01922586
160	00869093	01084011	01297999	01511063	01723209	01934444
161	00874477	01090712	01306005	01520364	01733793	01946299
162	00879861	01097412	01314011	01529662	01744374	01958151
163	00885244	01104112	01322015	01538959	01754953	01970000
164	00890627	01110810	01330017	01548255	01765529	01981846
165	00896009	01117508	01338019	01557548	01776103	01993690
166	00901390	01124204	01346019	01566840	01786675	02005530
167	00906771	01130900	01354017	01576130	01797245	02017368
168	00912151	01137595	01362015	01585418	01807812	02029203
169	00917531	01144288	01370011	01594705	01818377	02041035
170	00922910	01150982	01378006	01603990	01828940	02052864
171	00928288	01157674	01385999	01613273	01839501	02064691
172	00933666	01164365	01393992	01622554	01850059	02076514
173	00939044	01171055	01401983	01631834	01860615	02088335
174	00944420	01177745	01409973	01641111	01871169	02100153
175	00949796	01184433	01417961	01650387	01881720	02111968
176	00955172	01191121	01425948	01659661	01892270	02123780
177	00960547	01197807	01433934	01668934	01902817	02135589
178	00965921	01204493	01441918	01678205	01913361	02147395
179	00971295	01211178	01449901	01687474	01923904	02159199
180	00976668	01217862	01457883	01696741	01934444	02171000
181	00982041	01224545	01465864	01706007	01944982	02182797
182	00987413	01231227	01473843	01715270	01955517	02194592
183	00992785	01237908	01481821	01724532	01966051	02206385
184	00998156	01244589	01489798	01733793	01976582	02218174
185	01003526	01251268	01497774	01743051	01987111	02229960
186	01008896	01257947	01505748	01752308	01997637	02241744
187	01014265	01264624	01513721	01761563	02008162	02253525
188	01019633	01271301	01521692	01770816	02018684	02265303
189	01025001	01277977	01529662	01780068	02029203	02277078
190	01030369	01284652	01537631	01789318	02039721	02288850
191	01035735	01291326	01545599	01798266	02050236	02300620
192	01041102	01297999	01553566	01807812	02060749	02312386
193	01046467	01304671	01561531	01817057	02071260	02324150
194	01051832	01311342	01569494	01826300	02081768	02335911
195	01057197	01318013	01577457	01835541	02092275	02347669
196	01062561	01324682	01585418	01844780	02102779	02359424
197	01067924	01331351	01593378	01854018	02113280	02371177
198	01073287	01338019	01601337	01863254	02123780	02382926
199	01078649	01344685	01609295	01872488	02134277	02394673
200	01084011	01351351	01617251	01881720	02144772	02406417

TABLE II.—THE DISCOUNT OF £1 FOR ANY NUMBER OF DAYS, ETC.

(See page 7.)

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
151	·02026574	·02422071	·02814389	·03203564	·03589635	·03972639
152	02039721	02437720	02832499	03224096	03612549	03997896
153	02052864	02453365	02850603	03244619	03635452	04023140
154	02066005	02469004	02868700	03265133	03658343	04048370
155	02079142	02484638	02886790	03285639	03681224	04073587
156	02092275	02500267	02904873	03306135	03704095	04098791
157	02105404	02515891	02922950	03326624	02726954	04123982
158	02118530	02531510	02941020	03347103	03749802	04149160
159	02131653	02547124	02959083	03367574	03772640	04174324
160	02144772	02562734	02977140	03388036	03795466	04199475
161	02157888	02578338	02995189	03408489	03818282	04224613
162	02171000	02593937	03013233	03428934	03841087	04249738
163	02184108	02609531	03031269	03449370	03863882	04274849
164	02197213	02625120	03049299	03469798	03886665	04299947
165	02210315	02640704	03067322	03490217	03909438	04325033
166	02223413	02656283	03085338	03510627	03932200	04350105
167	02236507	02671857	03103347	03531029	03954951	04375164
168	02249598	02687427	03121350	03551422	03977691	04400209
169	02262686	02702991	03139347	03571806	04000421	04425242
170	02275770	02718550	03157336	03592182	04023140	04450262
171	02288850	02734104	03175319	03612549	04045848	04475268
172	02301927	02749654	03193295	03632907	04068545	04500262
173	02315001	02765198	03211265	03653257	04091232	04525242
174	02328071	02780737	03229227	03673598	04113907	04550209
175	02341137	02796272	03247183	03693931	04136572	04575163
176	02354200	02811801	03265133	03714255	04159227	04600104
177	02367259	02827325	03283076	03734571	04181871	04625033
178	02380316	02842845	03301012	03754878	04204504	04649948
179	02393368	02858359	03318941	03775177	04227126	04674850
180	02406417	02873869	03336864	03795466	04249738	04699739
181	02419463	02889374	03354781	03815748	04272339	04724615
182	02432505	02904873	03372690	03836021	04294929	04749478
183	02445543	02920368	03390593	03856285	04317509	04774328
184	02458578	02935858	03408489	03876541	04340077	04799165
185	02471610	02951343	03426379	03896788	04362636	04823989
186	02484638	02966823	03444262	03917026	04385184	04848801
187	02497663	02982297	03462139	03937256	04407721	04873599
188	02510684	02997768	03480008	03957478	04430247	04898385
189	02523701	03013233	03497872	03977691	04452763	04923157
190	02536716	03028693	03515728	03997896	04475268	04947916
191	02549726	03044148	03533578	04018092	04497763	04972663
192	02562734	03059598	03551422	04038279	04520247	04997397
193	02575737	03075044	03569258	04058459	04542720	05022118
194	02588738	03090484	03587089	04078629	04565183	05046826
195	02601734	03105920	03604912	04098791	04587636	05071521
196	02614728	03121350	03622729	04118945	04610077	05096204
197	02627718	03136776	03640540	04139090	04632508	05120873
198	02640704	03152197	03658343	04159227	04654929	05145530
199	02653687	03167613	03676141	04179355	04677339	05170174
200	02666667	03183024	03693931	04199475	04699739	05194805

(See page 7.)

DAYS.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
201	·01089372	·01358016	·01625205	·01890951	·02155265	·02418158
202	01094732	01364680	01633159	01900180	02165755	02429897
203	01100092	01371344	01641111	01909407	02176243	02441632
204	01105451	01378006	01649062	01918633	02186729	02453365
205	01110810	01384667	01657012	01927856	02197213	02465095
206	01116168	01391328	01664960	01937078	02207695	02476822
207	01121526	01397987	01672907	01946299	02218174	02488546
208	01126883	01404646	01680853	01955517	02228651	02500267
209	01132239	01411304	01688798	01964734	02239126	02511986
210	01137595	01417961	01696741	01973949	02249598	02523701
211	01142950	01424617	01704683	01983163	02260069	02535414
212	01148305	01431272	01712624	01992374	02270537	02547124
213	01153659	01437926	01720563	02001584	02281002	02558832
214	01159012	01444579	01728501	02010792	02291466	02570536
215	01164365	01451232	01736438	02019999	02301927	02582238
216	01169717	01457883	01744374	02029203	02312386	02593937
217	01175069	01464534	01752308	02038406	02322843	02605633
218	01180420	01471184	01760241	02047608	02333298	02617326
219	01185771	01477832	01768173	02056807	02343750	02629017
220	01191121	01484480	01776103	02066005	02354200	02640704
221	01196470	01491127	01784032	02075201	02364648	02652389
222	01201819	01497774	01791960	02084395	02375093	02664071
223	01207167	01504419	01799887	02093588	02385537	02675750
224	01212515	01511063	01807812	02102779	02395978	02687427
225	01217862	01517707	01815736	02111968	02406417	02699100
226	01223208	01524349	01823659	02121155	02416854	02710771
227	01228554	01530991	01831581	02130341	02427288	02722439
228	01233900	01537631	01839501	02139525	02437720	02734104
229	01239245	01544271	01847420	02148707	02448150	02745767
230	01244589	01550910	01855337	02157888	02458578	02757426
231	01249932	01557548	01863254	02167066	02469004	02769083
232	01255275	01564185	01871169	02176243	02479427	02780737
233	01260618	01570822	01879083	02185419	02489848	02792388
234	01265960	01577457	01886995	02194592	02500267	02804037
235	01271301	01584092	01894906	02203764	02510684	02815683
236	01276642	01590725	01902817	02212935	02521098	02827325
237	01281982	01597358	01910725	02222103	02531510	02838965
238	01287321	01603990	01918633	02231270	02541920	02850603
239	01292660	01610621	01926539	02240435	02552328	02862237
240	01297999	01617251	01934444	02249598	02562734	02873869
241	01303337	01623880	01942347	02258760	02573137	02885498
242	01308674	01630508	01950250	02267920	02583538	02897124
243	01314011	01637135	01958151	02277078	02593937	02908747
244	01319347	01643762	01966051	02286234	02604333	02920368
245	01324682	01650387	01973949	02295389	02614728	02931986
246	01330017	01657012	01981846	02304542	02625120	02943601
247	01335352	01663636	01989742	02313693	02635510	02955213
248	01340685	01670259	01997637	02322843	02645898	02966823
249	01346019	01676880	02005530	02331991	02656283	02978429
250	01351351	01683502	02013423	02341137	02666667	02990033

(See page 7.)

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
201	·02679643	·03198430	·03711715	·04219586	·04722128	·05219423
202	02692615	03213831	03729493	04239689	04744506	05244029
203	02705584	03229227	03747264	04259784	04766874	05268622
204	02718550	03244619	03765028	04279870	04789232	05293202
205	02731512	03260005	03782786	04299947	04811579	05317769
206	02744471	03275387	03800538	04320017	04833916	05342324
207	02757426	03290763	03818282	04340077	04856242	05366865
208	02770378	03306135	03836021	04360130	04878557	05391394
209	02783327	03321502	03853752	04380174	04900862	05415911
210	02796272	03336864	03871477	04400209	04923157	05440414
211	02809213	03352221	03889196	04420237	04945441	05464905
212	02822151	03367574	03906908	04440255	04967715	05489384
213	02835086	03382921	03924614	04460266	04989978	05513849
214	02848017	03398264	03942313	04480268	05012231	05538302
215	02860945	03413601	03960005	04500262	05034474	05562743
216	02873869	03428934	03977691	04520247	05056706	05587170
217	02886790	03444262	03995371	04540224	05078927	05611585
218	02899707	03459585	04013044	04560192	05101139	05635987
219	02912621	03474904	04030710	04580153	05123340	05660377
220	02925532	03490217	04048370	04600104	05145530	05684754
221	02938439	03505525	04066024	04620048	05167710	05709119
222	02951343	03520829	04083670	04639983	05189880	05733471
223	02964243	03536128	04101311	04659910	05212039	05757810
224	02977140	03551422	04118945	04679829	05234188	05782137
225	02990033	03566711	04136572	04699739	05256327	05806452
226	03002923	03581995	04154194	04719641	05278455	05830753
227	03015810	03597274	04171808	04739534	05300573	05855043
228	03028693	03612549	04189416	04759420	05322681	05879319
229	03041572	03627818	04207018	04779297	05344778	05903583
230	03054449	03643083	04224613	04799165	05366865	05927835
231	03067322	03658343	04242202	04819026	05388942	05952074
232	03080191	03673598	04259784	04838878	05411009	05976301
233	03093057	03688849	04277360	04858722	05433065	06000515
234	03105920	03704095	04294929	04878557	05455111	06024717
235	03118779	03719335	04312492	04898385	05477146	06048906
236	03131635	03734571	04330048	04918203	05499171	06073083
237	03144487	03749802	04347598	04938014	05521186	06097247
238	03157336	03765028	04365142	04957817	05543191	06121399
239	03170182	03780250	04382679	04977611	05565186	06145539
240	03183024	03795466	04400209	04997397	05587170	06169666
241	03195863	03810678	04417734	05017175	05609144	06193780
242	03208698	03825885	04435252	05036944	05631108	06217883
243	03221530	03841087	04452763	05056706	05653062	06241973
244	03234358	03856285	04470268	05076459	05675005	06266050
245	03247183	03871477	04487767	05096204	05696938	06290115
246	03260005	03886665	04505259	05115940	05718861	06314168
247	03272823	03901848	04522744	05135669	05740774	06338209
248	03285639	03917026	04540224	05155389	05762677	06362237
249	03298450	03932200	04557697	05175101	05784569	06386253
250	03311258	03947368	04575163	05194805	05806452	06410256

(See page 7.)

DAYS.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT	4 PER CENT.	4½ PER CENT.
251	·01356683	·01690122	.02021314	·02350281	·02677048	·03001634
252	01362015	01696741	02029203	02359424	02687427	03013233
253	01367346	01703359	02037092	02368565	02697803	03024828
254	01372676	01709977	02044979	02377705	02708178	03036421
255	01378006	01716594	02052864	02386842	02718550	03048011
256	01383335	01723209	02060749	02395978	02728920	03059598
257	01388664	01729824	02068632	02405112	02739288	03071183
258	01393992	01736438	02076514	02414245	02749654	03082765
259	01399319	01743051	02084395	02423376	02760016	03094344
260	01404646	01749663	02062275	02432505	02770378	03105920
261	01409973	01756275	02100153	02441632	02780737	03117493
262	01415298	01762885	02108030	02450758	02791094	03129064
263	01420623	01769495	02115905	02459882	02801449	03140632
264	01425948	01776103	02123780	02469004	02811801	03152197
265	01431272	01782711	02131653	02478124	02822151	03163759
266	01436595	01789318	02139525	02487243	02832499	03175319
267	01441918	01795924	02147395	02496361	02842845	03186876
268	01447240	01802529	02155265	02505476	02853188	03198430
269	01452562	01809133	02163133	02514589	02863530	03209981
270	01457883	01815736	02171000	02523701	02873869	03221530
271	01463204	01822339	02178865	02532812	02884206	03233076
272	01468524	01828940	02186729	02541920	02894541	03244619
273	01473843	01835541	02194592	02551027	02904873	03256159
274	01479162	01842141	02202454	02560132	02915204	03267697
275	01484480	01848739	02210315	02569236	02925532	03279231
276	01489798	01855337	02218174	02578338	02935858	03290763
277	01495115	01861934	02226032	02587438	02946182	03302293
278	01500432	01868531	02233889	02596536	02956503	03313819
279	01505748	01875126	02241744	02605633	02966823	03325343
280	01511063	01881720	02249598	02614728	02977140	03336864
281	01516378	01888314	02257451	02623821	02987455	03348383
282	01521692	01894906	02265303	02632913	02997768	03359898
283	01527006	01901498	02273153	02642003	03008078	03371411
284	01532319	01908089	02281002	02651091	03018387	03382921
285	01537631	01914679	02288850	02660177	03028693	03394428
286	01542943	01921268	02296697	02669262	03038997	03405933
287	01548255	01927856	02304542	02678345	03049299	03417435
288	01553566	01934444	02312386	02687427	03059598	03428934
289	01558876	01941030	02320229	02696506	03069896	03440431
290	01564185	01947616	02328071	02705584	03080191	03451924
291	01569494	01954200	02335911	02714661	03090484	03463415
292	01574803	01960784	02343750	02723735	03100775	03474904
293	01580111	01967367	02351588	02732808	03111064	03486389
294	01585418	01973949	02359424	02741880	03121350	03497872
295	01590725	01980530	02367259	02750949	03131635	03509352
296	01596031	01987111	02375093	02760016	03141917	03520829
297	01601337	01993690	02382926	02769083	03152197	03532303
298	01606642	02000268	02390758	02778148	03162475	03543775
299	01611947	02006846	02398588	02787210	03172750	03555244
300	01617251	02013423	02406417	02796272	03183024	03566711

(See page 7.)

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
251	·03324063	·03962532	·04592623	·05214501	·05828324	·06434247
252	03336864	03977691	04610077	05234188	05850186	06458226
253	03349662	03992845	04627525	05253868	05872038	06482193
254	03362457	04007995	04644966	05273539	05893879	06506148
255	03375248	04023140	04662400	05293202	05915711	06530090
256	03388036	04038279	04679829	05312857	05937533	06554019
257	03400820	04053415	04697250	05332503	05959343	06577937
258	03413601	04068545	04714666	05352142	05981145	06601842
259	03426379	04083670	04732075	05371772	06002936	06625735
260	03439153	04098791	04749478	05391394	06024717	06649616
261	03451924	04113907	04766874	05411009	06046488	06673485
262	03464692	04129019	04784264	05430614	06068248	06697341
263	03477456	04144125	04801648	05450212	06089999	06721186
264	03490217	04159227	04819026	05469802	06111740	06745018
265	03502974	04174324	04836397	05489384	06133470	06768838
266	03515728	04189416	04853762	05508957	06155191	06792645
267	03528479	04204504	04871120	05528523	06176901	06816441
268	03541226	04219586	04888472	05548080	06198602	06840225
269	03553970	04234664	04905818	05567629	06220292	06863996
270	03566711	04249738	04923157	05587170	06241973	06887755
271	03579448	04264806	04940490	05606703	06263643	06911502
272	03592182	04279870	04957817	05626228	06285303	06935237
273	03604912	04294929	04975137	05645745	06306954	06958960
274	03617639	04309983	04992451	05665254	06328594	06982671
275	03630363	04325033	05009759	05684754	06350224	07006369
276	03643083	04340077	05027061	05704247	06371845	07030056
277	03655800	04355118	05044356	05723732	06393455	07053731
278	03668514	04370153	05061645	05743208	06415056	07077393
279	03681224	04385184	05078927	05762677	06436646	07101043
280	03693931	04400209	05096204	05782137	06458226	07124682
281	03706635	04415231	05113474	05801590	06479797	07148308
282	03719335	04430247	05130738	05821034	06501358	07171923
283	03732032	04445259	05147995	05840470	06522908	07195525
284	03744726	04460266	05165246	05859899	06544449	07219115
285	03757416	04475268	05182491	05879319	06565980	07242694
286	03770103	04490266	05199730	05898731	06587501	07266260
287	03782786	04505259	05216962	05918136	06609011	07289815
288	03795466	04520247	05234188	05937533	06630513	07313357
289	03808143	04535231	05251408	05956921	06652004	07336887
290	03820817	04550209	05268622	05976301	06673485	07360406
291	03833487	04565183	05285829	05995673	06694956	07383913
292	03846154	04580153	05303030	06015038	06716418	07407407
293	03858817	04595117	05320225	06034394	06737869	07430890
294	03871477	04610077	05337414	06053742	06759311	07454361
295	03884134	04625033	05354596	06073083	06780743	07477820
296	03896788	04639983	05371772	06092415	06802165	07501267
297	03909438	04654929	05388942	06111740	06823577	07524702
298	03922085	04669870	05406106	06131056	06844980	07548126
299	03934728	04684807	05423263	06150365	06866372	07571537
300	03947368	04699739	05440414	06169666	06887755	07594937

(See page 7.)

DAYS.	2 PER CENT.	½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
301	·01622554	·02019999	·02414245	·02805331	·03193295	·03578174
302	01627857	02026574	02422071	02814389	03203564	03589635
303	01633159	02033148	02429897	02823445	03213831	03601093
304	01638461	02039721	02437720	02832499	03224096	03612549
305	01643762	02046293	02445543	02841552	03234358	03624002
306	01649062	02052864	02453365	02850603	03244619	03635452
307	01654362	02059435	02461185	02859652	03254877	03646899
308	01659661	02066005	02469004	02868700	03265133	03658343
309	01664960	02072574	02476822	02877746	03275387	03669785
310	01670259	02079142	02484638	02886790	03285639	03681224
311	01675556	02085709	02492453	02895832	03295888	03692661
312	01680853	02092275	02500267	02904873	03306135	03704095
313	01686150	02098840	02508080	02913912	03316381	03715526
314	01691446	02105404	02515891	02922950	03326624	03726954
315	01696741	02111968	02523701	02931986	03336864	03738379
316	01702036	02118530	02531510	02941020	03347103	03749802
317	01707330	02125092	02539318	02950052	03357339	03761222
318	01712624	02131653	02547124	02959083	03367574	03772640
319	01717917	02138213	02554930	02968112	03377806	03784054
320	01723209	02144772	02562734	02977140	03388036	03795466
321	01728501	02151330	02570536	02986165	03398264	03806876
322	01733793	02157888	02578338	02995189	03408489	03818282
323	01739084	02164444	02586138	03004212	03418713	03829686
324	01744374	02171000	02593937	03013233	03428934	03841087
325	01749663	02177554	02601734	03022252	03439153	03852486
326	01754953	02184108	02609531	03031269	03449370	03863882
327	01760241	02190661	02617326	03040285	03459585	03875275
328	01765529	02197213	02625120	03049299	03469798	03886665
329	01770816	02203764	02632913	03058311	03480008	03898053
330	01776103	02210315	02640704	03067322	03490217	03909438
331	01781389	02216864	02648494	03076331	03500423	03920820
332	01786675	02223413	02656283	03085338	03510627	03932200
333	01791960	02229960	02664071	03094344	03520829	03943577
334	01797245	02236507	02671857	03103347	03531029	03954951
335	01802529	02243053	02679643	03112350	03541226	03966322
336	01807812	02249598	02687427	03121350	03551422	03977691
337	01813095	02256142	02695209	03130349	03561615	03989057
338	01818377	02262686	02702991	03139347	03571806	04000421
339	01823659	02269228	02710771	03148342	03581995	04011782
340	01828940	02275770	02718550	03157336	03592182	04023140
341	01834221	02282310	02726328	03166328	03602366	04034495
342	01839501	02288850	02734104	03175319	03612549	04045848
343	01844780	02295389	02741880	03184308	03622729	04057198
344	01850059	02301927	02749654	03193295	03632907	04068545
345	01855337	02308464	02757426	03202281	03643083	04079890
346	01860615	02315001	02765198	03211265	03653257	04091232
347	01865892	02321536	02772968	03220247	03663429	04102571
348	01871169	02328071	02780737	03229227	03673598	04113907
349	01876445	02334604	02788505	03238206	03683766	04125241
350	01881720	02341137	02796272	03247183	03693931	04136572

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
301	·03960005	·04714666	·05457559	·06188959	·06909128	·07618324
302	03972639	04729589	05474698	06208243	06930491	07641700
303	03985269	04744506	05491831	06227520	06951844	07665064
304	03997896	04759420	05508957	06246789	06973188	07688417
305	04010519	04774328	05526077	06266050	06994521	07711757
306	04023140	04789232	05543191	06285303	07015845	07735086
307	04035756	04804131	05560299	06304549	07037160	07758403
308	04048370	04819026	05577401	06323786	07058464	07781708
309	04060980	04833916	05594496	06343015	07079759	07805001
310	04073587	04848801	05611585	06362237	07101043	07828283
311	04086191	04863681	05628668	06381451	07122319	07851553
312	04098791	04878557	05645745	06400656	07143584	07874811
313	04111388	04893428	05662816	06419854	07164840	07898057
314	04123982	04908295	05679880	06439044	07186085	07921292
315	04136572	04923157	05696938	06458226	07207322	07944514
316	04149160	04938014	05713991	06477401	07228548	07967726
317	04161743	04952867	05731036	06496567	07249765	07990925
318	04174324	04967715	05748076	06515726	07270972	08014113
319	04186901	04982558	05765110	06534876	07292169	08037289
320	04199475	04997397	05782137	06554019	07313357	08060453
321	04212046	05012231	05799159	06573154	07334535	08083606
322	04224613	05027061	05816174	06592282	07355703	08106747
323	04237177	05041886	05833183	06611401	07376862	08129877
324	04249738	05056706	05850186	06630513	07398011	08152994
325	04262295	05071521	05867182	06649616	07419150	08176101
326	04274849	05086332	05884173	06668712	07440280	08199195
327	04287400	05101139	05901158	06687800	07461400	08222278
328	04299947	05115940	05918136	06706881	07482510	08245349
329	04312492	05130738	05935108	06725953	07503611	08268409
330	04325033	05145530	05952074	06745018	07524702	08291457
331	04337570	05160318	05969034	06764075	07545784	08314494
332	04350105	05175101	05985988	06783124	07566856	08337519
333	04362636	05189880	06002936	06802165	07587918	08360532
334	04375164	05204654	06019877	06821199	07608971	08383534
335	04387688	05219423	06036813	06840225	07630014	08406524
336	04400209	05234188	06053742	06859243	07651047	08429503
337	04412727	05248949	06070666	06878253	07672071	08452470
338	04425242	05263704	06087583	06897255	07693086	08475426
339	04437754	05278455	06104494	06916250	07714091	08498370
340	04450262	05293202	06121399	06935237	07735086	08521303
341	04462767	05307944	06138298	06954216	07756072	08544224
342	04475268	05322681	06155191	06973188	07777048	08567134
343	04487767	05337414	06172078	06992152	07798014	08590032
344	04500262	05352142	06188959	07011108	07818972	08612919
345	04512753	05366865	06205833	07030056	07839919	08635795
346	04525242	05381584	06222702	07048996	07860857	08658659
347	04537727	05396299	06239564	07067929	07881786	08681511
348	04550209	05411009	06256421	07086855	07902705	08704352
349	04562688	05425714	06273271	07105772	07923614	08727182
350	04575163	05440414	06290115	07124682	07944514	08750000

TABLE II.—THE DISCOUNT OF £1 FOR ANY NUMBER OF DAYS, ETC.

(See page 7.)

DAYS.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
351	·01886995	·02347669	·02804037	·03256159	·03704095	·04147901
352	01892270	02354200	02811801	03265133	03714255	04159227
353	01897543	02360730	02819564	03274105	03724414	04170550
354	01902817	02367259	02827325	03283076	03734571	04181871
355	01908089	02373788	02835086	03292045	03744726	04193189
356	01913361	02380316	02842845	03301012	03754878	04204504
357	01918633	02386842	02850603	03309977	03765028	04215816
358	01923904	02393368	02858359	03318941	03775177	04227126
359	01929174	02399893	02866115	03327904	03785323	04238433
360	01934444	02406417	02873869	03336864	03795466	04249738
361	01939713	02412940	02881622	03345823	03805608	04261040
362	01944982	02419463	02889374	03354781	03815748	04272339
363	01950250	02425984	02897124	03363736	03825885	04283635
364	01955517	02432505	02904873	03372690	03836021	04294929
365	01960784	02439024	02912621	03381643	03846154	04306220
YEARS.						
1	01960784	02439024	02912621	03381643	03846154	04306220
2	03846154	04761905	05660377	06542056	07407407	08256881
3	05660377	06976744	08256881	09502262	10714286	11894273
4	07407407	09090909	10714286	12280702	13793103	15254237
5	09090909	11111111	13043478	14893617	16666667	18364347
6	10714286	13043478	15254237	17355372	19354839	21259843
7	12280702	14893617	17355372	19678715	21875000	23954373
8	13793103	16666667	19354839	21875000	24242424	26470588
9	15254237	18367347	21259843	23954373	26470588	28825623
10	16666667	20000000	23076923	25925926	28571429	31034483
11	18032787	21568627	24812030	27797834	30555556	33110368
12	19354839	23076923	26470588	29577465	32432432	35064935
13	20634921	24528302	28057554	31271478	34210526	36908517
14	21875000	25925926	29577465	32885906	35897436	38650307
15	23076923	27272727	31034483	34426229	37500000	40298507
16	24242424	28571429	32432432	35897436	39024390	41860465
17	25373134	29824561	33774834	37304075	40476190	43342776
18	26470588	31034483	35064935	36650307	41860465	44751381
19	27536232	32203390	36305732	39939940	43181818	46091644
20	28571429	33333333	37500000	41176471	44444444	47368421
21	29577465	34426229	38650307	42363112	45652174	48586118
22	30555556	35483871	39759036	43502825	46808511	49748744
23	31506849	36507986	40828402	44598338	47916667	50859951
24	32432432	37500000	41860465	45652174	48979592	51923077
25	33333333	38461538	42857143	46666667	50000000	52941176
26	34210526	39393939	43820225	47643979	50980392	53917051
27	35064935	40298507	44751381	48586118	51923077	54853273
28	35897436	41176471	45652174	49494949	52830189	55752212
29	36708861	42028985	46524064	50372208	53703704	56616052
30	37500000	42857143	47368421	51219512	54545454	57446808

(See page 7.)

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
351	·04587636	·05455111	·06306954	·07143584	·07965405	·08772807
352	04600104	05469802	06323786	07162478	07986286	08795602
353	04612570	05484489	06340612	07181365	08007158	08818386
354	04625033	05499171	06357432	07200244	08028020	08841159
355	04637492	05513849	06374246	07219115	08048873	08863920
356	04649948	05528523	06391054	07237979	08069716	08886670
357	04662400	05543191	06407856	07256835	08090550	08909408
358	04674850	05557855	06424652	07275683	08111374	08932136
359	04687296	05572515	06441442	07294524	08132189	08954851
360	04699739	05587170	06458226	07313357	08152994	08977556
361	04712178	05601821	06475004	07332182	08173791	09000249
362	04724615	05616467	06491776	07351000	08194577	09022931
363	04737048	05631108	06508542	07369810	08215354	09045602
364	04749478	05645745	06525302	07388613	08236122	09068261
365	04761905	05660377	06542056	07407407	08256881	09090909
YEARS.						
1	04761905	05660377	06542056	07407407	08256881	09090909
2	09090909	10714286	12280702	13793103	15254237	16666667
3	13043478	15254237	17355372	19354839	21259843	23076923
4	16666667	19354839	21875000	24242424	26470588	28571429
5	20000000	23076923	25925926	28571429	31034483	33333333
6	23076923	26470588	29577465	32432432	35064935	37500000
7	25925926	29577465	32885906	35897436	38650307	41176471
8	28571429	32432432	35897436	39024390	41860465	44444444
9	31034483	35064935	36650307	41860465	44751381	47368421
10	33333333	37500000	41176471	44444444	47368421	50000000
11	35483871	39759036	43502825	46808511	49748744	52380952
12	37500000	41860465	45652174	48979592	51923077	54545455
13	39393939	43820225	47643979	50980392	53917051	56521739
14	41176471	45652174	49494949	52830189	55752212	58333333
15	42857143	47368421	51219512	54545455	57446808	60000000
16	44444444	48979592	52830189	56140351	59016393	61538462
17	45945946	50495049	54337900	57627119	60474308	62962963
18	47368421	51923077	55752212	59016393	61832061	64285714
19	48717949	53271028	57081545	60317460	63099630	65517241
20	50000000	54545455	58333333	61538462	64285714	66666667
21	51219512	55752212	59514170	62686567	65397924	67741935
22	52380952	56896552	60629921	63768116	66442953	68750000
23	53488372	57983193	61685824	64788732	67426710	69696970
24	54545455	59016393	62686567	65753425	68354430	70588235
25	55555556	60000000	63636364	66666667	69230769	71428571
26	56521739	60937500	64539007	67532468	70059880	72222222
27	57446808	61832061	65397924	68354430	70845481	72972973
28	58333333	62686567	66216216	69135802	71590909	73684210
29	59183673	63503650	66996700	69879518	72299169	74358974
30	60000000	64285714	67741935	70588235	72972973	75000000

SUPPLEMENTARY TABLE I.—AMOUNT OF £1 IN ANY NUMBER OF DAYS

(At Compound Interest.)

DAYS.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.
1	1·00005426	1·00006765	1·00008099	1·00009426	1·00010746	1·00012060
2	1·00010851	1·00013531	1·00016198	1·00018852	1·00021493	1·00024122
3	1·00016278	1·00020297	1·00024298	1·00028279	1·00032241	1·00036185
4	1·00021704	1·00027064	1·00032399	1·00037707	1·00042991	1·00048249
5	1·00027131	1·00033831	1·00040500	1·00047136	1·00053742	1·00060315
6	1·00032558	1·00040599	1·00048601	1·00056566	1·00064493	1·00072382
7	1·00037985	1·00047367	1·00056704	1·00065997	1·00075246	1·00084452
8	1·00043413	1·00054136	1·00064808	1·00075429	1·00086000	1·00096522
9	1·00048840	1·00060905	1·00072911	1·00084862	1·00096755	1·00108594
10	1·00054269	1·00067675	1·00081016	1·00094295	1·00107512	1·00120667
20	1·00108566	1·00135394	1·00162097	1·00188679	1·00215139	1·00241480
30	1·00162894	1·00203159	1·00243245	1·00283152	1·00322882	1·00362438
40	1·00217251	1·00270970	1·00324457	1·00377714	1·00430741	1·00483542
50	1·00271637	1·00338828	1·00405736	1·00472365	1·00538716	1·00604793
60	1·00326053	1·00406731	1·00487081	1·00567105	1·00646807	1·00726189
70	1·00380499	1·00474680	1·00568491	1·00661935	1·00755014	1·00847733
80	1·00434974	1·00542675	1·00649967	1·00756854	1·00863338	1·00969422
90	1·00489478	1·00610716	1·00731510	1·00851862	1·00971778	1·01091259
100	1·00544012	1·00678803	1·00813118	1·00946961	1·01080334	1·01213243
110	1·00598576	1·00746937	1·00894793	1·01042148	1·01189007	1·01335374
120	1·00653169	1·00815116	1·00976534	1·01137426	1·01297797	1·01457652
130	1·00707792	1·00883342	1·01058341	1·01232793	1·01406704	1·01580078
140	1·00762445	1·00951613	1·01140214	1·01328251	1·01515729	1·01702652
150	1·00817127	1·01019931	1·01222153	1·01423798	1·01624870	1·01825373
160	1·00871839	1·01088295	1·01304159	1·01519436	1·01734129	1·01948243
170	1·00926581	1·01156706	1·01386232	1·01615163	1·01843505	1·02071260
180	1·00981352	1·01225162	1·01468371	1·01710981	1·01952999	1·02194427
190	1·01036153	1·01293665	1·01550576	1·01806890	1·02062610	1·02317742
200	1·01090984	1·01362215	1·01632848	1·01902888	1·02172339	1·02441205
210	1·01145844	1·01430810	1·01715187	1·01998977	1·02282187	1·02564818
220	1·01200735	1·01499453	1·01797592	1·02095157	1·02392152	1·02688580
230	1·01255655	1·01568141	1·01880064	1·02191428	1·02502236	1·02812491
240	1·01310605	1·01636876	1·01962603	1·02287789	1·02612437	1·02936552
250	1·01365585	1·01705658	1·02045209	1·02384241	1·02722758	1·03060762
260	1·01420594	1·01774486	1·02127882	1·02480784	1·02833197	1·03185123
270	1·01475634	1·01843361	1·02210622	1·02577419	1·02943755	1·03309633
280	1·01530703	1·01912282	1·02293428	1·02674144	1·03054431	1·03434293
290	1·01585802	1·01981250	1·02376302	1·02770960	1·03165227	1·03559104
300	1·01640931	1·02050265	1·02459243	1·02867868	1·03276142	1·03684066
310	1·01696090	1·02119326	1·02542251	1·02964867	1·03387176	1·03809178
320	1·01751279	1·02188434	1·02625327	1·03061958	1·03498329	1·03934442
330	1·01806498	1·02257589	1·02708470	1·03159140	1·03609602	1·04059856
340	1·01861747	1·02326791	1·02791680	1·03256414	1·03720995	1·04185422
350	1·01917026	1·02396039	1·02874957	1·03353780	1·03832507	1·04311140
360	1·01972334	1·02465335	1·02958302	1·03451237	1·03944139	1·04437009
YEARS.						
¼	1·00496293	1·00619225	1·00741707	1·00863745	1·00985340	1·01106499
½	1·00995049	1·01242284	1·01488916	1·01734950	1·01980390	1·02225242
¾	1·01496281	1·01869209	1·02241666	1·02613680	1·02985245	1·03356362
1	1·02000000	1·02500000	1·03000000	1·03500000	1·04000000	1·04500000

TABLE III.—AMOUNT OF £1 IN ANY NUMBER OF DAYS.

(At Compound Interest.)

DAYS.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.	9 PER CENT.	10 PER CENT.
1	1·00013368	1·00015965	1·00018538	1·00021087	1·00023613	1·00026116
2	1·00026738	1·00031933	1·00037080	1·00042179	1·00047232	1·00052238
3	1·00040110	1·00047904	1·00055625	1·00063276	1·00070856	1·00078368
4	1·00053483	1·00063877	1·00074174	1·00084377	1·00094486	1·00104504
5	1·00066858	1·00079852	1·00092726	1·00105482	1·00118121	1·00130647
6	1·00080235	1·00095831	1·00111282	1·00126591	1·00141762	1·00156797
7	1·00093614	1·00111811	1·00129841	1·00147706	1·00165409	1·00182954
8	1·00106995	1·00127794	1·00148403	1·00168824	1·00189061	1·00209117
9	1·00120377	1·00143780	1·00166969	1·00189947	1·00212719	1·00235288
10	1·00133761	1.00159768	1·00185538	1·00211075	1·00236382	1·00261465
20	1·00267701	1·00319792	1·00371420	1·00422595	1·00473323	1·00523614
30	1·00401820	1·00480071	1·00557648	1·00634561	1·00710824	1·00786448
40	1·00536119	1·00640607	1·00744220	1·00846975	1·00948887	1·01049969
50	1·00670597	1·00801398	1·00931139	1·01059838	1·01187512	1·01314179
60	1·00805255	1·00962447	1·01118405	1·01273150	1·01426701	1·01579080
70	1·00940093	1·01123753	1·01306018	1·01486911	1·01666456	1·01844674
80	1·01075112	1·01285317	1·01493979	1·01701124	1·01906777	1·02110962
90	1·01210311	1·01447139	1·01682289	1·01915790	1·02147667	1·02377947
100	1·01345691	1·01609219	1·01870949	1·02130908	1·02389126	1·02645629
110	1·01481252	1·01771559	1·02059958	1·02346481	1·02631156	1·02914012
120	1·01616994	1·01934157	1·02249318	1·02562508	1·02873757	1·03183096
130	1·01752918	1·02097016	1·02439029	1·02778991	1·03116933	1·03452884
140	1·01889024	1·02260134	1·02629093	1·02995932	1·03360683	1·03723377
150	1·02025312	1·02423514	1·02819509	1·03213330	1·03605009	1·03994577
160	1·02161782	1·02587154	1·03010278	1·03431187	1·03849913	1·04266486
170	1·02298435	1·02751056	1·03201402	1·03649504	1·04095396	1·04539107
180	1·02435270	1·02915220	1·03392879	1·03868282	1·04341459	1·04812440
190	1·02572289	1·03079645	1·03584713	1·04087522	1·04588103	1·05086488
200	1·02709491	1·03244334	1·03776902	1·04307224	1·04835331	1·05361252
210	1·02846876	1·03409286	1·03969447	1·04527390	1·05083143	1·05636735
220	1·02984445	1·03574501	1·04162350	1·04748021	1·05331541	1·05912938
230	1·03122198	1·03739980	1·04355611	1·04969117	1·05580526	1·06189863
240	1·03260135	1·03905724	1·04549230	1·05190680	1·05830099	1·06467513
250	1·03398257	1·04071733	1·04743209	1·05412711	1·06080263	1·06745888
260	1·03536564	1·04238006	1·04937548	1·05635211	1·06331018	1·07024991
270	1·03675056	1·04404546	1·05132247	1·05858180	1·06582366	1·07304824
280	1·03813732	1·04571351	1·05327307	1·06081620	1·06834307	1·07585389
290	1·03952595	1·04738423	1·05522729	1·06305531	1·07086845	1·07866687
300	1·04091643	1·04905762	1·05718514	1·06529915	1·07339979	1·08148720
310	1·04230877	1·05073368	1·05914662	1·06754773	1·07593711	1·08431491
320	1·04370297	1·05241242	1·06111174	1·06980105	1·07848044	1·08715002
330	1·04509904	1·05409384	1·06308051	1·07205913	1·08102978	1·08999254
340	1·04649697	1·05577795	1·06505293	1·07432197	1·08358514	1·09284249
350	1·04789678	1·05746475	1·06702901	1·07658959	1·08614654	1·09569988
360	1·04929846	1·05915424	1·06900875	1·07886200	1·08871400	1·09856476
YEARS.						
¼	1·01227220	1·01467385	1·01705852	1·01942655	1·02177818	1·02411369
½	1·02469508	1·02956302	1·03440804	1·03923049	1·04403065	1·04880885
¾	1·03727037	1·04467067	1·05205352	1·05941914	1·06676774	1·07409950
1	1·05000000	1·06000000	1·07000000	1·08000000	1·09000000	1·10000000

TABLE IV.—ENGLISH LIFE TABLE, No. 3. MALES.

Interest 3 per cent.

AGE	NUMBERS.			LOGARITHMS.			AGE
(x).	D_x	N_x	M_x	λD_x	λN_x	λM_x	(x).
0	511745	9800236	226301·22	5·7090538	6·9912366	5·3546869	0
1	415559	9288498	145020·64	5·6186330	6·9679452	5·1614298	1
2	377514	8872932	119079·46	5·5769335	6·9480672	5·0758370	2
3	353510	8495418	106070·72	5·5484015	6·9291847	5·0255955	3
4	335028	8141908	97885·09	5·5250812	6·9107262	4·9907165	4
5	319474	7806880	92089·22	5·5044353	6·8924775	4·9642088	5
6	305953	7487406	87874·16	5·4856554	6·8743314	4·9438612	6
7	293828	7181453	84660·01	5·4680933	6·8562124	4·9276784	7
8	282657	6887625	82047·07	5·4512605	6·8380695	4·9140631	8
9	272329	6604968	79951·69	5·4350945	6·8198707	4·9028276	9
10	262688	6332639	78242·51	5·4194406	6·8015847	4·8934428	10
11	253605	6069951	76809·95	5·4041575	6·7831852	4·8854175	11
12	244972	5816346	75564·30	5·3891169	6·7646503	4·8783166	12
13	236703	5571374	74429·84	5·3742038	6·7459623	4·8717470	13
14	228726	5334671	73347·59	5·3593163	6·7271077	4·8653859	14
15	220987	5105945	72269·90	5·3443659	6·7080761	4·8589575	15
16	213441	4884958	71160·04	5·3292770	6·6888609	4·8522361	16
17	206057	4671517	69993·57	5·3139876	6·6694579	4·8450581	17
18	198815	4465460	68752·99	5·2984485	6·6498662	4·8372917	18
19	191701	4266645	67429·93	5·2826241	6·6300865	4·8288528	19
20	184711	4074944	66023·04	5·2664916	6·6101216	4·8196956	20
21	177845	3890233	64537·25	5·2500418	6·5899756	4·8098104	21
22	171203	3712388	63075·43	5·2335116	6·5696534	4·7998602	22
23	164780	3541185	61638·45	5·2169037	6·5491486	4·7898517	23
24	158570	3376405	60227·58	5·2002199	6·5284546	4·7797954	24
25	152567	3217835	58843·96	5·1834612	6·5075638	4·7697019	25
26	146767	3065268	57487·19	5·1666276	6·4864684	4·7595711	26
27	141162	2918501	56157·33	5·1497186	6·4651599	4·7494064	27
28	135748	2777339	54854·40	5·1327326	6·4436289	4·7392115	28
29	130517	2641591	53577·54	5·1156672	6·4218657	4·7289828	29
30	125464	2511074	52325·92	5·0985192	6·3998595	4·7187169	30
31	120583	2385610	51098·76	5·0812845	6·3775994	4·7084104	31
32	115867	2265027	49894·92	5·0639584	6·3550733	4·6980563	32
33	111310	2149160	48713·32	5·0465351	6·3322687	4·6876478	33
34	106907	2037850	47552·59	5·0290081	6·3091723	4·6771742	34
35	102653	1930943	46411·45	5·0113700	6·2857695	4·6666252	35
36	98540	1828290	45288·71	4·9936125	6·2620451	4·6559899	36
37	94564	1729750	44183·27	4·9757267	6·2379834	4·6452579	37
38	90720	1635186	43093·11	4·9577027	6·2135672	4·6344078	38
39	87002	1544466	42017·65	4·9395297	6·1887784	4·6234317	39
40	83406	1457464	40955·43	4·9211963	6·1635979	4·6123114	40
41	79926	1374058	39905·10	4·9026899	6·1380050	4·6010284	41
42	76559	1294132	38866·00	4·8839975	6·1119786	4·5895698	42
43	73300	1217573	37836·97	4·8651050	6·0854951	4·5779164	43
44	70145	1144273	36816·67	4·8459974	6·0585297	4·5660445	44
45	67090	1074128	35804·93	4·8266591	6·0310560	4·5539429	45
46	64132	1007038	34800·58	4·8070735	6·0030459	4·5415865	46
47	61267	942906	33803·30	4·7872232	5·9744684	4·5289591	47
48	58491	881639	32812·31	4·7670900	5·9452908	4·5160368	48
49	55803	823148	31827·40	4·7466548	5·9154779	4·5028011	49
50	53198	767345	30848·36	4·7258978	5·8849907	4·4892321	50
51	50676	714147	29875·03	4·7047982	5·8537876	4·4753083	51
52	48205	663471	28880·38	4·6830899	5·8218220	4·4606029	52
53	45808	615266	27887·15	4·6609367	5·7890629	4·4454042	53
54	43483	569458	26897·11	4·6383216	5·7554617	4·4297056	54

TABLE IV.—ENGLISH LIFE TABLE, No. 3. MALES.

Interest 3 per cent.

AGE (x).	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
55	41230	525975	25910·72	4·6152171	5·7209651	4·4134795	55
56	39047	484745	24928·03	4·5915855	5·6855134	4·3966879	56
57	36930	445698	23948·55	4·5673789	5·6490407	4·3792792	57
58	34877	408768	22971·13	4·5425388	5·6114769	4·3611824	58
59	32885	373891	21994·91	4·5169966	5·5727450	4·3423222	59
60	30951	341006	21018·61	4·4906732	5·5327620	4·3226040	60
61	29072	310055	20041·57	4·4634793	5·4914387	4·3019318	61
62	27247	280983	19062·75	4·4353151	5·4486801	4·2801855	62
63	25472	253736	18082·00	4·4060708	5·4043821	4·2572465	63
64	23748	228264	17099·50	4·3756260	5·3584374	4·2329834	64
65	22072	204516	16115·60	4·3438500	5·3107273	4·2072465	65
66	20446	182444	15131·79	4·3106018	5·2611296	4·1798903	66
67	18868	161998	14149·86	4·2757301	5·2095096	4·1507521	67
68	17341	143130	13172·14	4·2390733	5·1557307	4·1196563	68
69	15866	125789	12201·96	4·2004505	5·0996427	4·0864296	69
70	14445	109923	11242·98	4·1597062	5·0410886	4·0508815	70
71	13080	95478	10299·43	4·1166210	4·9799033	4·0128132	71
72	11776	82398	9376·10	4·0710008	4·9159167	3·9720222	72
73	10535	70622	8478·04	4·0226325	4·8489400	3·9282954	73
74	9360·4	60086·9	7610·29	3·9712923	4·7787798	3·8814012	74
75	8255·5	50726·5	6778·06	3·9167463	4·7052349	3·8311054	75
76	7223·5	42471·0	5986·57	3·8587504	4·6280925	3·7771780	76
77	6266·9	35247·5	5240·21	3·7970499	4·5471284	3·7193487	77
78	5387·4	28980·6	4543·30	3·7313799	4·4621074	3·6573714	78
79	4586·3	23593·2	3899·12	3·6614651	4·3727869	3·5909666	79
80	3863·8	19006·9	3310·26	3·5870201	4·2789113	3·5198621	80
81	3219·2	15143·1	2778·15	3·5077489	4·1802148	3·4437557	81
82	2650·6	11923·9	2303·26	3·4233452	4·0764184	3·3623430	82
83	2155·2	9273·3	1885·12	3·3334926	3·9672343	3·2753390	83
84	1729·3	7118·1	1521·99	3·2378642	3·8523641	3·1824118	84
85	1368·1	5388·8	1211·18	3·1361228	3·7314921	3·0832087	85
86	1066·4	4020·7	949·25	3·0279208	3·6043017	2·9773806	86
87	818·28	2954·30	732·24	2·9129004	3·4704546	2·8646534	87
88	617·58	2136·02	555·38	2·7906934	3·3296053	2·7445902	88
89	458·06	1518·44	413·85	2·6609214	3·1813976	2·6168430	89
90	333·58	1060·38	302·66	2·5231953	3·0254615	2·4809550	90
91	238·30	726·80	217·12	2·3771162	2·8614149	2·3366998	91
92	166·83	488·50	152·59	2·2222745	2·6888646	2·1835261	92
93	114·35	321·67	104·98	2·0582504	2·5074106	2·0211066	93
94	76·668	207·323	70·62	1·8846137	2·3166475	1·8489277	94
95	50·225	130·655	46·43	1·7009240	2·1161261	1·6667987	95
96	32·117	80·430	29·74	1·5067304	1·9054181	1·4733410	96
97	20·025	48·313	18·60	1·3015719	1·6840640	1·2695129	97
98	12·161	28·288	11·31	1·0849770	1·4516022	1·0534626	98
99	7·1856	16·1271	6·70	0·8564639	1·2075563	0·8260748	99
100	4·1261	8·9415	3·84	0·6155406	0·9514104	0·5843312	100
101	2·2999	4·8154	2·17	0·3617044	0·6826324	0·3364597	101
102	1·2429	2·5155	1·14	0·0944428	0·4006243	0·0569049	102
103	·6505	1·2726	·62	$\bar{1}$·8132326	0·1046919	$\bar{1}$·7923917	103
104	·3293	·6221	·30	$\bar{1}$·5175404	$\bar{1}$·7938602	$\bar{1}$·4771213	104
105	·1610	·2928	·17	$\bar{1}$·2068224	$\bar{1}$·4665711	$\bar{1}$·2304489	105
106	·0759	·1318	·08	$\bar{2}$·8805246	$\bar{1}$·1199154	$\bar{2}$·9030900	106
107	·0345	·0559	·04	$\bar{2}$·5380826	$\bar{2}$·7474118	$\bar{2}$·6020600	107
108	·0151	·0214	……	$\bar{2}$·1789217	2·3304138	……	108
109	0063	·0063	……	$\bar{3}$·8024567	$\bar{3}$·7993405	……	109

TABLE V.—ENGLISH LIFE TABLE, No. 3. MALES.

Interest $3\frac{1}{2}$ per cent.

Age (x).	NUMBERS.			LOGARITHMS.			Age (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
0	511745	8925340	209921·9	5·7090538	6·9506248	5·3220578	0
1	413552	8413595	129034·2	5·6165299	6·9249817	5·1107049	1
2	373876	8000043	103343·1	5·5727272	6·9030924	5·0142815	2
3	348411	7626167	90521·3	5·5420922	6·8823063	4·9567507	3
4	328601	7277756	82493·3	5·5166687	6·8619975	4·9164187	4
5	311831	6949155	76835·5	5·4939197	6·8419320	4·8855619	5
6	297192	6637324	72741·4	5·4730367	6·8219930	4·8617817	6
7	284035	6340132	69634·4	5·4533714	6·8020983	4·8428238	7
8	271916	6056097	67120·5	5·4344355	6·7821928	4·8268552	8
9	260715	5784181	65114 7	5 4161663	6·7622419	4·8136791	9
10	250270	5523466	63486·2	5·3984094	6·7422117	4·8026794	10
11	240449	5273196	62128·3	5·3810232	6·7220739	4·7932895	11
12	231142	5032747	60952·5	5·3638794	6·7018051	4·7849916	12
13	222261	4801605	59887·8	5·3468632	6·6813865	4·7773384	13
14	213733	4579344	58875·9	5·3298726	6·6608033	4·7699375	14
15	205503	4365611	57873·7	5·3128190	6·6400450	4·7624812	15
16	197527	4160108	56847·0	5·2956270	6·6191046	4·7547076	16
17	189773	3962581	55772·7	5·2782345	6·5979781	4·7464217	17
18	182218	3772808	54635·1	5·2605923	6·5766647	4·7374717	18
19	174850	3590590	53429·1	5·2426647	6·5551658	4·7277778	19
20	167660	3415740	52151·8	5·2244291	6·5334848	4·7172693	20
21	160648	3248080	50809·5	5·2058762	6·5116267	4·7059449	21
22	153902	3087432	49496·1	5·1872429	6·4895974	4·6945710	22
23	147412	2933530	48210·5	5·1685318	6·4673905	4·6831415	23
24	141171	2786118	46954·4	5·1497449	6·4449996	4·6716763	24
25	135171	2644947	45728·4	5·1308831	6·4224171	4·6601860	25
26	129404	2509776	44532·34	5·1119464	6·3996349	4·6486755	26
27	123861	2380372	43365·32	5·0929343	6·3766448	4·6371426	27
28	118535	2256511	42227·85	5·0738451	6·3534374	4·6255990	28
29	113417	2137976	41118·28	5·0546765	6·3300028	4·6140349	29
30	108499	2024559	40035·64	5·0354254	6·3063304	4·6026467	30
31	103774	1916060	38979·68	5·0160877	6·2824091	4·5908383	31
32	99233	1812286	37947·96	4·9966584	6·2582267	4·5791884	32
33	94871	1713053	36941·64	4·9771320	6·2337709	4·5675162	33
34	90678	1618182	35956·87	4·9575018	9·2090273	4·5557819	34
35	86648	1527504	34993·27	4·9377606	6·1839823	4·5439846	35
36	82775	1440856	34050·39	4·9179000	6·1586206	4·5321221	36
37	79052	1358081	33126·55	4·8979111	6·1329256	4·5201763	37
38	75472	1279029	32219·81	4·8777839	6·1068804	4·5081229	38
39	72029	1203557	31329·00	4·8575078	6·0804667	4·4959464	39
40	68718	1131528	30453·75	4·8370713	6·0536654	4·4836407	40
41	65533	1062810	29592·56	4·8164618	6·0264556	4·4711826	41
42	62469	997277	28744·65	4·7956663	5·9988158	4·4585571	42
43	59521	934808	27909·13	4·7746706	5·9707224	4·4457464	43
44	56684	875287	27084·92	4·7534599	5·9421505	4·4327235	44
45	53953	818603	26270·77	4·7320185	5·9130733	4·4194728	45
46	51325	764650	25467·26	4·7103298	5·8834627	4·4059822	46
47	48795	713325	24672·90	4·6883763	5·8532875	4·3922200	47
48	46360	664530	23887·97	4·6661400	5·8225146	4·3781792	48
49	44015	618170	33110·70	4·6436017	5·7911079	4·3638131	49
50	41758	574155	22342·12	4·6207416	5·7590292	4·3491244	50
51	39586	532397	21582·23	4·5975388	5·7262356	4·3340963	51
52	37474	492811	20808·90	4·5737274	5·6926804	4·3182489	52
53	35438	455337	20040·12	4·5494711	5·6583330	4·3019003	53
54	33477	419899	19277·51	4·5247528	5·6231449	4·2850509	54

TABLE V.—ENGLISH LIFE TABLE, No. 3. MALES.

Interest 3½ per cent.

AGE (x).	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
55	31590	386422	18522·59	4·4995452	5·5870618	4·2677017	55
56	29772	354832	17772·84	4·4738105	5·5500228	4·2497568	56
57	28022	325060	17029·63	4·4475008	5·5119635	4·2312053	57
58	26336	297038	16291·23	4·4205576	5·4728120	4·2119539	58
59	24712	270702	15557·82	4·3929122	5·4324915	4·1919488	59
60	23147	245990	14828·49	4·3644857	5·3909175	4·1710969	60
61	21637	222843	14101·24	4·3351886	5·3479990	4·1492573	61
62	20180	201206	13375·93	4·3049214	5·3036410	4·1263240	62
63	18775	181026	12653·35	4·2735739	5·2577410	4·1022055	63
64	17419	162251	11932·25	4·2410260	5·2101874	4·0767223	64
65	16112	144832	11214·30	4·2071468	5.1608645	4·0497724	65
66	14852	128720	10499·15	4·1717955	5·1096460	4·0211542	66
67	13640	113868	9789·39	4·1348207	5·0564017	3·9907556	67
68	12476	100228	9086·65	4·0960608	5·0009890	3·9584038	68
69	11359	87752	8391·54	4·0553438	4·9432570	3·9238416	69
70	10292	76393	7708·66	4·0124875	4·8830536	3·8869789	70
71	9274·7	66100·7	7039·41	3·9672991	4·8202061	3·8475362	71
72	8309·5	56826·0	6387·85	3·9195758	4·7545471	3·8053547	72
73	7397·8	48516·5	5757·14	3·8691043	4·6858895	3·7602068	73
74	6541·3	41118·7	5150·81	3·8156610	4·6140394	3·7118755	74
75	6741·3	34577·4	4572·02	3·7590120	4·5387923	3·6601081	75
76	4999·3	28836·1	4024·17	3·6989129	4·4599365	3·6046764	76
77	4316·3	23836·8	3510·23	3·6351092	4·3772480	3·5453356	77
78	3692·6	19520·5	3032·49	3·5673361	4·2904910	3·4817995	78
79	3128·4	15827·9	2593·16	3·4953182	4·1994233	3·4138293	79
80	2622·8	12699·5	2193·35	3·4187701	4·1037866	3·3411079	80
81	2174·7	10076·7	1834·95	3·3373957	4·0033184	3·2636242	81
82	1781·9	7902·0	1514·69	3·2508890	3·8977370	3·1803237	82
83	1441·9	6120·1	1234·95	3·1589333	3·7867585	3·0916495	83
84	1151·3	4678·2	993·10	3·0612017	3·6700788	2·9969930	84
85	906·48	3526·92	787·21	2·9573572	3·5473956	2·8960906	85
86	703·16	2620·44	615·55	2·8470521	3·4183742	2·7892633	86
87	536·94	1917·28	472·11	2·7299285	3·2826856	2·6740432	87
88	403·29	1380·34	356·62	2·6056184	3·1399168	2·5522057	88
89	297·68	977·05	264·64	2·4737432	2·9899168	2·4226555	89
90	215·73	679·37	192·76	2·3339141	2·8321064	2·2850169	90
91	153·37	463·64	137·70	2·1857318	2·6661809	2·1389339	91
92	106·85	310·27	96·36	2·0287870	2·4917398	1·9838968	92
93	72·889	203·416	66·01	1·8626597	2·3083851	1·8196097	93
94	48·632	130·527	44·22	1·6869199	2·1157003	1·6456187	94
95	31·705	81·895	28·94	1·5011271	1·9132574	1·4614985	95
96	20·176	50·190	18·48	1·3048304	1·7006172	1·2667020	96
97	12·519	30·014	11·50	1·0975688	1·4773239	1·0606978	97
98	7·5661	17·4954	6·97	0·8788708	1·2429239	0·8432328	98
99	4·4489	9·9293	4·11	0·6482546	0·9969186	0·6138418	99
100	2·5423	5·4804	2·36	0·4052280	0·7388123	0·3729120	100
101	1·4102	2·9381	1·31	0·1492888	0·4680666	0·1172713	101
102	·7584	1·5279	0·71	$\bar{1}$·8799241	0·1840949	$\bar{1}$·8512583	102
103	·3950	·7695	0·37	$\bar{1}$·5966107	$\bar{1}$·8862086	$\bar{1}$·5682017	103
104	·1990	·3745	0·19	$\bar{1}$·2988154	$\bar{1}$·5734518	$\bar{1}$·2787536	104
105	·0968	·1755	0·09	$\bar{2}$·9859943	$\bar{1}$·2442771	$\bar{2}$·9542425	105
106	·0455	·0787	0·04	$\bar{2}$·6575934	$\bar{2}$·8959747	$\bar{2}$·6020600	106
107	·0206	·0332	0·02	$\bar{2}$·3130482	$\bar{2}$·5211381	$\bar{2}$·3010300	107
108	·0089	·0126	0·01	$\bar{3}$·9517842	$\bar{2}$·1003705	$\bar{2}$·0000000	108
109	·0037	·0037	0·00	$\bar{3}$·5732161	$\bar{3}$·5682017	$\bar{2}$·0000000	109

Interest 4 per cent.

AGE (x).	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
0	511745	8184833	196943·65	5·7090538	6·9130099	5·2943419	0
1	411563	7673088	116444·61	5·6144369	6·8849703	5·0661194	1
2	370289	7261525	90999·90	5·5685412	6·8610278	4·9590409	2
3	343410	6891236	78362·82	5·5358132	6·8382972	4·8941101	3
4	322327	6547826	70487·51	5·5082967	6·8160971	4·8481122	4
5	304407	6225499	64964·98	5·4834547	6·7941741	4·8126792	5
6	288721	5921092	60987·33	5·4604787	6·7724019	4·7852396	6
7	274613	5632371	57983·38	5·4387205	6·7506913	4·7633036	7
8	261632	5357758	55564·79	5·4176915	6·7289832	4·7447997	8
9	249649	5096126	53643·92	5·3973294	6·7072401	4·7295206	9
10	238495	4846477	52092·15	5·3774795	6·6854261	4·7167722	10
11	228034	4607982	50804·03	5·3580003	6·6635108	4·7058982	11
12	218154	4379948	49694·75	5·3387636	6·6414690	4·6963105	12
13	208763	4161794	48694·19	5·3196543	6·6192806	4·6874772	13
14	199789	3953031	47748·86	5·3005707	6·5969302	4·6789631	14
15	191172	3753242	46816·57	5·2814242	6·5744065	4·6703997	15
16	182869	3562070	45865·68	5·2621392	6·5517024	4·6614879	16
17	174845	3379201	44875·90	5·2426536	6·5288140	4·6520131	17
18	167078	3204356	43833·36	5·2229185	6·5057408	4·6418048	18
19	159550	3037278	42732·19	5·2028979	6·4824845	4·6307551	19
20	152254	2877728	41572·51	5·1825693	6·4590497	4·6188063	20
21	145186	2725474	40359·57	5·1619234	6·4354420	4·6059466	21
22	138419	2580288	39177·67	5·1411971	6·4116681	4·5930387	22
23	131945	2441869	38027·03	5·1203931	6·3877224	4·5800924	23
24	125751	2309924	36908·16	5·0995132	6·3635978	4·5671224	24
25	119828	2184173	35821·45	5·0785583	6·3392870	4·5541431	25
26	114164	2064345	34766·07	5·0575287	6·3147823	4·5411556	26
27	108749	1950181	33741·57	5·0364235	6·2900749	4·5281653	27
28	103572	1841432	32747·47	5·0152414	6·2651557	4·5151777	28
29	98623	1737860	31782·63	4·9939798	6·2400148	4·5021898	29
30	93894	1639237	30845·96	4·9726357	6·2146418	4·4891984	30
31	89373	1545343	29936·42	4·9512050	6·1890248	4·4761999	31
32	85052	1455970	29052·74	4·9296828	6·1631524	4·4631871	32
33	80921	1370918	28193·73	4·9080633	6·1370115	4·4501526	33
34	76973	1289997	27358·01	4·8863402	6·1105887	4·4370845	34
35	73199	1213024	26544·29	4·8645059	6·0838694	4·4239712	35
36	69591	1139825	25751·39	4·8425524	6·0568382	4·4108007	36
37	66141	1070234	24978·21	4·8204705	6·0294787	4·3975614	37
38	62842	1004093	24223·05	4·7982503	6·0017740	4·3842288	38
39	59687	941251	23485·24	4·7758813	5·9737055	4·3707950	39
40	56670	881564	22763·52	4·7533517	5·9452539	4·3572395	40
41	53784	824894	22056·74	4·7306492	5·9163981	4·3435413	41
42	51022	771110	21364·24	4·7077607	5·8871163	4·3296875	42
43	48381	720088	20685·04	4·6846721	5·8573856	4·3156563	43
44	45853	671707	20018·08	4·6613684	5·8271799	4·3014224	44
45	43434	625854	19363·07	4·6378340	5·7964731	4·2869742	45
46	41120	582420	18719·10	4·6140522	5·7652363	4·2722849	46
47	38905	541300	18085·81	4·5900058	5·7334380	4·2573379	47
48	36785	502395	17462·57	4·5656765	5·7010453	4·2421082	48
49	34757	465610	16849·11	4·5410452	5·6680223	4·2265770	49
50	32816	430853	16245·17	4·5160921	5·6343291	4·2107243	50
51	30960	398037	15650·52	4·4907963	5·5999234	4·1945288	51
52	29167	367077	15048·69	4·4648919	5·5647572	4·1774987	52
53	27450	337910	14453·50	4·4385426	5·5288010	4·1599731	53
54	25807	310460	13865·92	4·4117314	5·4920057	4·1419487	54

TABLE VI.—ENGLISH LIFE TABLE, NO. 3. MALES.

Interest 4 per cent.

AGE (x).	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM	
55	24234	284653	13286·14	4·3844308	5·4543158	4·1233988	55
56	22730	260419	12714·09	4·3566031	5·4156726	4·1042853	56
57	21291	237689	12149·39	4·3282004	5·3760091	4·0845545	57
58	19914	216398	11591·30	4·2991642	5·3352533	4·0641322	58
59	18596	196484	11039·25	4·2694258	5·2933272	4·0429396	59
60	17334	177888	10492·46	4·2389063	5·2501466	4·0208774	60
61	16126	160554	9950·52	4·2075163	5·2056211	3·9978458	61
62	14968	144428	9412·82	4·1751560	5·1596514	3·9737197	62
63	13858	129460	8870·23	4·1417156	5·1121356	3·9483753	63
64	12796	115602	8349·83	4·1070746	5·0629653	3·9216777	64
65	11779	102806	7824·78	4·0711025	5·0120185	3·8934721	65
66	10806	91027	7304·82	4·0336582	4·9591702	3·8636095	66
67	9876·2	80220·7	6790·85	3·9945904	4·9042865	3·8319241	67
68	8989·5	70344·5	6284·00	3·9537375	4·8472302	3·7982362	68
69	8145·7	61355·0	5785·89	3·9109276	4·7878500	3·7623702	69
70	7344·8	53209·3	5298·27	3·8659782	4·7259876	3·7241340	70
71	6587·1	45864·5	4823·11	3·8186969	4·6614767	3·6833272	71
72	5873·3	39277·4	4362·60	3·7688806	4·5941427	3·6397454	72
73	5203·7	33404·1	3919·00	3·7163161	4·5237998	3·5931753	73
74	4579·1	28200·4	3494·50	3·6607798	4·4502553	3·5433850	74
75	3999·8	23621·3	3091·29	3·6020377	4·3733038	3·4901398	75
76	3466·1	19621·5	2711·50	3·5398457	4·2927322	3·4332096	76
77	2978·2	16155·4	2356·81	3·4739491	4·2083177	3·3723246	77
78	2535·6	13177·2	2028·81	3·4040829	4·1198231	3·3072413	78
79	2137·8	10641·6	1728·54	3·3299721	4·0270069	3·2376794	79
80	1783·7	8503·8	1456·69	3·2513303	3·9296130	3·1633671	80
81	1471·9	6720·1	1213·40	3·1678636	3·8273757	3·0840040	81
82	1200·2	5248·2	998·36	3·0792638	3·7200104	2·9992872	82
83	966·53	4047·99	810·84	2·9852151	3·6072394	2·9089352	83
84	768·05	3081·46	649·56	2·8853906	3·4887566	2·8126193	84
85	601·80	2313·41	512·84	2·7794531	3·3642526	2·7099819	85
86	464·57	1711·61	398·73	2·6670550	3·2334048	2·6006789	86
87	353·05	1247·04	305·10	2·5478385	3·0958805	2·4844422	87
88	263·90	893·99	229·53	2·4214354	2·9513327	2·3608395	88
89	193·85	630·09	169·63	2·2874672	2·7994029	2·2295027	89
90	139·81	436·24	123·03	2·1455450	2·6397255	2·0900110	90
91	98·917	296·429	87·52	1·9952698	2·4719207	1·9421073	91
92	68·585	197·512	60·99	1·8362319	2·2955935	1·7852586	92
93	46·560	128·927	41·60	1·6680117	2·1103439	1·6190933	93
94	30·916	82·367	27·75	1·4901789	1·9157532	1·4432630	94
95	20·058	51·451	18·09	1·3022931	1·7113938	1·2574386	95
96	12·703	31·393	11·49	1·1039034	1·4968328	1·0603200	96
97	7·8442	18·6904	7·12	0·8945488	1·2716186	0·8524800	97
98	4·7180	10·8462	4·29	0·6737578	1·0352776	0·6324573	98
99	2·7609	6·1282	2·52	0·4410486	0·7873329	0·4014005	99
100	1·5701	3·3673	1·43	0·1959291	0·5272818	0·1553360	100
101	·8668	1·7972	·80	9·9378969	0·2545964	$\bar{1}$·9030900	101
102	·4639	·9304	·42	9·6664391	$\bar{1}$·9686697	$\bar{1}$·6232493	102
103	·2405	·4665	·23	9·3810328	$\bar{1}$·6688516	$\bar{1}$·3617278	103
104	·1205	·2260	·11	9·0811445	$\bar{1}$·3541084	$\bar{1}$·0413927	104
105	·0584	·1055	·06	8·7662304	$\bar{1}$·0232525	$\bar{2}$·7781513	105
106	·0273	·0471	·03	8·4357365	$\bar{2}$·6730209	$\bar{2}$·4771213	106
107	·0123	·0198	·01	8·0890984	$\bar{2}$·2966652	$\bar{2}$·0000000	107
108	·0053	·0075	·00	7·7257413	$\bar{3}$·8750613	...	108
109	·0022	·0022	·00	7·3450802	$\bar{3}$·3424227	...	109

Annuities on Single Lives.

AGE.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.	5 PER CENT.	6 PER CENT.	AGE.
0	18·1506	16·4410	14·9940	13·7589	12·6964	10·9716	0
1	21·3518	19·3447	17·6438	16·1904	14·9388	12·9046	1
2	22·5036	20·3976	18·6104	17·0815	15·7636	13·6188	2
3	23·0316	20·8884	19·0671	17·5068	16·1608	13·9672	3
4	23·3022	21·0991	19·3142	17·7419	16·3835	14·1669	4
5	23·4367	21·2850	19·4512	17·8766	16·5147	14·2894	5
6	23·4724	21·3335	19·5080	17·9384	16·5794	14·3554	6
7	23·4410	21·3217	19·5102	17·9500	16·5988	14·3832	7
8	23·3674	21·2719	19·4782	17·9319	16·5898	14·3871	8
9	23·2536	21·1858	19·4132	17·8830	16·5533	14·3677	9
10	23·1071	21·0700	19·3211	17·8093	16·4941	14·3289	10
11	22·9347	20·9306	19·2074	17·7159	16·4166	14·2744	11
12	22·7429	20·7734	19·0773	17·6072	16·3251	14·2078	12
13	22·5374	20·6035	18·9355	17·4877	16·2235	14·1324	13
14	22·3234	20·4255	18·7861	17·3611	16·1153	14·0512	14
15	22·1052	20·2435	18·6328	17·2309	16·0036	13·9670	15
16	21·8867	20·0609	18·4788	17·0999	15·8911	13·8819	16
17	21·6710	19·8806	18·3268	16·9705	15·7801	13·7982	17
18	21·4604	19·7049	18·1788	16·8449	15·6725	13·7174	18
19	21·2568	19·5353	18·0365	16·7244	15·5697	13·6408	19
20	21·0612	19·3730	17·9008	16·6101	15·4727	13·5693	20
21	20·8743	19·2186	17·7723	16·5026	15·3821	13·5036	21
22	20·6841	19·0611	17·6411	16·3171	15·2891	13·4361	22
23	20·4904	18·9003	17·5067	16·2795	15·1936	13·3664	23
24	20·2929	18·7358	17·3690	16·1631	15·0952	13·2945	24
25	20·0913	18·5674	17·2276	16·0366	14·9937	13·2200	25
26	19·8853	18·3949	17·0823	15·9209	14·8889	13·1427	26
27	19·6748	18·2181	16·9329	15·7941	14·7806	13·0625	27
28	19·4596	18·0367	16·7793	15·6633	14·6686	12·9792	28
29	19·2394	17·8506	16·6212	15·5282	14·5528	12·8925	29
30	19·0143	17·6597	16·4585	15·3889	14·4328	12·8024	30
31	18·7840	17·4638	16·2910	15·2450	14·3087	12·7087	31
32	18·5486	17·2629	16·1187	15·0966	14·1802	12·6111	32
33	18·3078	17·0567	15·9414	14·9434	14·0473	12·4809	33
34	18·0618	16·8454	15·7590	14·7854	13·9098	12·4043	34
35	17·8105	16·6288	15·5716	14·6225	13·7676	12·2947	35
36	17·5538	16·4069	15·3789	14·4546	13·6207	12·1808	36
37	17·2918	16·1797	15·1811	14·2817	13·4690	12·0627	37
38	17·0245	15·9471	14·9780	14·1037	13·3124	11·9400	38
39	16·7521	15·7093	14·7697	13·9205	13·1508	11·8129	39
40	16·4744	15·4662	14·5562	13·7322	12·9842	11·6811	40
41	16·1916	15·2179	14·3373	13·5387	12·8125	11·5446	41
42	15·9037	14·9643	14·1132	13·3400	12·6357	11·4034	42
43	15·6108	14·7055	13·8838	13·1360	12·4538	11·2548	43
44	15·3129	14·4415	13·6491	12·9268	12·2667	11·1064	44
45	15·0102	14·1724	13·4092	12·7122	12·0742	10·9503	45
46	14·7026	13·8982	13·1639	12·4923	11·8765	10.7891	46
47	14·3902	13·6188	12·9134	12·2670	11·6733	10·6226	47
48	14·0730	13·3342	12·6574	12·0362	11·4646	10·4508	48
49	13·7511	13·0445	12·3961	11·7998	11·2503	10·2733	49

Annuities on Single Lives.

AGE	3 PER CENT	3½ PER CENT	4 PER CENT	4½ PER CENT	5 PER CENT	5½ PER CENT	AGE
50	13·4242	12·7495	12·1292	11·5577	11·0302	10·0901	50
51	13·0925	12·4492	11·8566	11·3124	10·8042	9·9008	51
52	12·7636	12·1508	11·5853	11·0626	10·5785	9·7116	52
53	12·4315	11·8488	11·3100	10·8111	10·3482	9·5175	53
54	12·0960	11·5427	11·0302	10·5548	10·1131	9·3183	54
55	11·7570	11·2325	10·7459	10·2937	9·8727	9·1136	55
56	11·4145	10·9188	10·4570	10·0276	9·6272	8·9036	56
57	11·0687	10·6001	10·1637	9·7568	9·3767	8·6881	57
58	10·7203	10·2786	9.8665	9·4815	9·1214	8·4675	58
59	10·3697	9·9542	9·5658	9·2021	8·8618	8·2420	59
60	10·0176	9·6275	9·2622	8·9298	8·5984	8·0121	60
61	9·6650	9·2993	8·9565	8·6344	8·3317	7·7782	61
62	9·3125	8·9705	8·6493	8·3471	8·0626	7·5411	62
63	8·9612	8·6440	8·3416	8·0586	7·7916	7·2997	63
64	8·6119	8·3146	8·0342	7·7696	7·5197	7·0597	64
65	8·2657	7·9891	7·7280	7·4812	7·2476	6·8168	65
66	7·9233	7·6667	7·4239	7·1940	6·9762	6·5735	66
67	7·5858	7·3480	7·1226	6·9090	6·7063	6·3305	67
68	7·2539	7·0339	6·8252	6·6270	6·4386	6·0887	68
69	6·9284	6·7253	6·5322	6·3487	6·1739	5·8487	69
70	6·6100	6·4227	6·2445	6·0748	5·9130	5·6112	70
71	6·2993	6·1270	5·9627	5·8061	5·6565	5·3769	71
72	5·9971	5·8387	5·6875	5·5431	5·4050	5·1464	72
73	5·7036	5·5582	5·4192	5·2863	5·1590	4·9203	73
74	5·4193	5·2860	5·1585	5·0363	4·9192	4·6991	74
75	5·1445	5·0226	4·9056	4·7935	4·6858	4·4831	75
76	4·8795	4·6595	4·6609	4·5581	4·4592	4·2728	76
77	4·6244	4·5225	4·4246	4·3304	4·2398	4·0685	77
78	4·3793	4·2863	4·1969	4·1107	4·0276	3·8413	78
79	4·1442	4·0595	3·9778	3·9080	3·8230	3·6789	79
80	3·9192	3·8419	3·7674	3·6955	3·6260	3·4940	80
81	3·7040	3·6336	3·5657	3·5001	3·4366	3·3158	81
82	3·4986	3·4345	3·3727	3·3128	3·2549	3·1444	82
83	3·3027	3·2445	3·1882	3·1336	3·0808	2·9798	83
84	3·1162	3·0633	3·0120	2·9624	2·9142	2·8220	84
85	2·9388	2·8908	2·8441	2·7989	2·7550	2·6708	85
86	2·7703	2·7267	2·6843	2·6431	2·6031	2·5263	86
87	2·6104	2·5703	2·5322	2·4947	2·4583	2·3883	87
88	2·4587	2·4227	2·3876	2·3535	2·3203	2·2565	88
89	2·3149	2·2822	2·2504	2·2194	2·1892	2·1310	89
90	2·1788	2·1491	2·1202	2·0920	2·0645	2·0114	90
91	2·0500	2·0231	1·9968	1·9711	1·9461	1·8977	91
92	1·9281	1·9037	1·8798	1·8565	1·8337	1·7897	92
93	1·8129	1·7908	1·7691	1·7478	1·7271	1·6870	93
94	1·7042	1·5830	1·6642	1·6450	1·6261	1·5896	94
95	1·6014	1·4876	1·5651	1·5475	1·5304	1·4971	95
96	1·5043	1·3975	1·4713	1·4554	1·4397	1·4094	96
97	1·4126	1·3123	1·3827	1·3682	1·3539	1·3263	97
98	1·3261	1·2318	1·2989	1·2857	1·2727	1·2476	98
99	1·2444	1·1557	1·2196	1·2076	1·1959	1·1729	99
100	1·1671	1·0834	1·1446	1·1336	1·1230	1·1022	100

Interest 3 per cent.

AGE (*x*).	NUMBERS.			LOGARITHMS.			AGE (*x*).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
0	488255	9691956	205969·03	5·6886465	6·9864115	5·3137936	0
1	410175	9203701	142106·77	5 6129697	6·9639626	5·1526148	1
2	373571	8793526	117449·40	5·5723735	6·9441631	5·0698509	2
3	349858	8419955	104616·37	5·5438916	6·9253098	5·0195996	3
4	331456	8070097	96404·08	5·5204254	6·9068788	4·9840955	4
5	316112	7738641	90714·31	5·4998405	6·8886947	4·9576758	5
6	302830	7422529	86639·11	5·4811983	6·8705519	4·9377140	6
7	290907	7119699	83537·17	5·4637539	6·8524617	4·9218798	7
8	279870	6828792	80972·38	5·4469558	6·8343439	4·9083369	8
9	269630	6548922	78884·66	5·4307681	6·8161698	4·8969925	9
10	260044	6279292	77152·41	5·4150471	6·7979106	4·8873495	10
11	250993	6019248	75675·06	5·3996612	6·7795423	4·8789528	11
12	242377	5768255	74369·79	5·3844914	6·7610445	4·8713965	12
13	234116	5525878	73167·91	5·3694305	6·7424013	4·8643207	13
14	226143	5291762	72014·26	5·3543840	6·7236003	4·8574185	14
15	218408	5065619	70865·97	5·3392692	6·7046326	4·8504377	15
16	210870	4847211	69689·43	5·3240158	6·6854919	4·8431669	16
17	203501	4636341	68461·85	5·3085658	6·6661754	4·8354486	17
18	196279	4432840	67166·64	5·2928734	6·6466820	4·8271537	18
19	189193	4236561	65797·95	5·2769048	6·6270135	4·8182123	19
20	182238	4047368	64353·41	5·2606388	6·6071727	4·8085716	20
21	175415	3865130	62838·06	5·2440662	6·5871641	4·7982228	21
22	168809	3689715	61341·79	5·2273961	6·5669929	4·7877564	22
23	162417	3520906	59866·30	5·2106318	6·5466544	4·7771824	23
24	156234	3358489	58414·11	5·1937762	6·5261440	4·7665178	24
25	150256	3202255	56986·55	5·1768319	6·5054560	4·7557724	25
26	144478	3051999	55584·34	5·1598006	6·4845844	4·7449524	26
27	138894	2907521	54209·01	5·1426840	6·4635229	4·7340715	27
28	133501	2768627	52861·07	5·1254831	6·4422944	4·7231360	28
29	128292	2635126	51540·50	5·1081983	6·4208014	4·7121486	29
30	123262	2506834	50247·69	5·0908299	6·3991256	4·7011161	30
31	118407	2383572	48982·53	5·0733773	6·3772283	4·6900413	31
32	113721	2265165	47744·90	5·0558399	6·3550999	4·6789270	32
33	109198	2151444	46535·02	5·0382162	6·3327301	4·6677799	33
34	104835	2042246	45351·60	5·0205045	6·3101081	4·6565926	34
35	100624	1937411	44194·83	5·0027024	6·2872217	4·6453714	35
36	96562	1836787	43063·47	4·9848074	6·2640589	4·6341090	36
37	92644	1740225	41957·69	4·9668161	6·2406055	4·6228115	37
38	88864	1647581	40875·99	4·9487250	6·2168468	4·6114683	38
39	85218	1558717	39818·22	4·9305300	6·1927673	4·6000818	39
40	81701	1473499	38783·28	4·9122263	6·1683499	4·5886445	40
41	78309	1391798	37770·75	4·8938090	6·1435762	4·5771557	41
42	75036	1313489	36779·33	4·8752725	6·1184295	4·5656038	42
43	71880	1238453	35808·93	4·8566108	6·0928796	4·5539914	43
44	68836	1166573	34858·35	4·8378174	6·0669119	4·5423068	44
45	65900	1097737	33927·00	4·8188855	6·0404984	4·5305455	45
46	63068	1031837	33014·30	4·7998075	6·0136111	4·5187021	46
47	60336	968769	32119·21	4·7805757	5·9862203	4·5067648	47
48	57701	908433	31241·48	4·7611817	5·9582929	4·4947316	48
49	55159	850732	30380·39	4·7416166	5·9297927	4·4825934	49
50	52707	795573	29535·25	4·7218713	5·9006801	4·4703406	50
51	50343	742866	28705·65	4·7019358	5·8709105	4·4579675	51
52	48062	692523	27891·18	4·6818002	5.8404342	4·4454670	52
53	45862	644461	27091·25	4·6614535	5·8091967	4·4328290	53
54	43741	598599	26305·70	4·6408847	5·7771360	4·4200499	54

TABLE IV.—ENGLISH LIFE TABLE, No. 3. *Females.*

Interest 3 per cent.

AGE (x).	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
55	41631	554858	25470·23	4·6194190	5·7441818	4·4060328	55
56	39571	513227	24622·22	4·5973737	5·7103095	4·3913272	56
57	37560	473656	23763·86	4·5747229	5·6754630	4·3759170	57
58	35598	436096	22896·46	4·5514296	5·6395821	4·3597683	58
59	33686	400498	22020·76	4·5274461	5·6026003	4·3428323	59
60	31821	366812	21137·13	4·5027135	5·5644436	4·3250460	60
61	30003	334991	20245·78	4·4771622	5·5250331	4·3063245	61
62	28230	304988	19346·80	4·4507115	5.4842828	4·2866091	62
63	26501	276758	18440·45	4·4232697	5·4421002	4·2657715	63
64	24816	250257	17527·02	4·3947345	5·3983862	4·2437081	64
65	23174	225441	16607·25	4·3649922	5·3530329	4·2202977	65
66	21573	202267	15692·00	4·3339185	5·3059251	4·1954014	66
67	20016	180694	14753·06	4·3013780	5·2569437	4·1688821	67
68	18502	160678	13822·24	4·2672244	5·2059564	4·1405785	68
69	17033	142176	12892·25	4·2313006	5·1528263	4·1103287	69
70	15611	125143	11966·24	4·1934382	5·0974066	4·0779578	70
71	14238	109532	11047·95	4·1534583	5·0395410	4·0432818	71
72	12917	95294	10141·64	4·1111707	4·9790656	4·0061082	72
73	11651	82377	9251·91	4·0663745	4·9158060	3·9662314	73
74	10444	70726	8383·83	4·0188578	4·8495791	3·9234425	74
75	9298·2	60281·5	7542·34	3·9683976	4·7801841	3·8775061	75
76	8217·9	50983·3	6732·87	3·9147601	4·7074280	3·8282003	76
77	7206·1	42765·4	5960·53	3·8577006	4·6310925	3·7752849	77
78	6265·6	35559·3	5229·83	3·7969634	4·5509532	3·7184876	78
79	5398·6	29293·7	4545·38	3·7322819	4·4667743	3·6575702	79
80	4606·6	23895·1	3910·56	3·6633785	4·3783088	3·5922390	80
81	3890·1	19288·5	3328·27	3·5899647	4·2852985	3·5222186	81
82	3248·9	15398·4	2800·41	3·5117410	4·1874756	3·4472217	82
83	2681·6	11149·5	2327·74	3·4283971	4·0845585	3·3669345	83
84	2185·8	9467·9	1910·00	3·3396115	3·9762537	3·2810334	84
85	1758·1	7282·1	1546·02	3·2450521	3·8622566	3·1892151	85
86	1394·4	5524·0	1233·41	3·1443756	3·7422537	3·0911074	86
87	1089·5	4129·6	969·18	3·0372279	3·6159080	2·9864044	87
88	838·00	3040·10	749·44	2·9232439	3·4828879	2·8747369	88
89	683·94	2202·10	569·81	2·8020475	3·3428370	2·7557301	89
90	471·25	1568·16	425·55	2·6732518	3·1953904	2·6289506	90
91	343·92	1096·91	311·97	2·5364589	2·0401710	2·4941128	91
92	246·18	752·99	224·24	2·3912599	2·8767892	2·3507131	92
93	172·68	506·81	157·88	2·2372350	2·7048452	2·1983271	93
94	118·56	334·13	108·80	2·0739535	2·5239155	2·0366289	94
95	79·611	215·574	73·33	1·9009738	2·3335964	1·8652817	95
96	52·221	135·963	48·26	1·7178431	2·1334208	1·6835873	96
97	33·427	83·742	30·98	1·5240980	1·9229433	1·4910814	97
98	20·858	50·315	19·39	1·3192640	1·7016975	1·2875778	98
99	12·672	29·457	11·78	1·1028557	1·4691885	1·0711453	99
100	7·4882	16·7848	6·99	0·8743766	1·2249161	0·8444772	100
101	4·2985	9·2966	4·01	0·6333195	0·9683241	0·6031444	101
102	2·3942	4·9981	2·24	0·3791661	0·6988049	0·3502480	102
103	1·2924	2·6039	1·19	0·1113872	0·4156243	0·0755470	103
104	·6752	1·3115	·64	$\bar{1}$·8294427	0·1177683	$\bar{1}$·8061800	104
105	·3410	·6363	·33	$\bar{1}$·5327815	$\bar{1}$·8036619	$\bar{1}$·5185139	105
106	·1663	·2953	·16	$\bar{1}$·2208416	$\bar{1}$·4702634	$\bar{1}$·2041200	106
107	·0782	·1290	·08	$\bar{2}$·8930502	$\bar{1}$·1105897	$\bar{2}$·9030900	107
108	·0354	·0508	·04	$\bar{2}$·5488231	$\bar{2}$·7058637	$\bar{2}$·6020600	108
109	·0154	·0154	·00	$\bar{2}$·1875658	$\bar{2}$·1875207	$\bar{2}$·0000000	109

TABLE IV.—ENGLISH LIFE TABLE, No. 3. *Females.*

Interest 3½ per cent.

AGE (x).	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
0	488255	8819292	190018·0	5·6886465	6·9454337	5·2787947	0
1	408194	8331037	126468·2	5·6108666	6·9206991	5·1019813	1
2	369971	7922843	102048·7	5·5681672	6·8988811	5·0088076	2
3	344812	7552872	89400·9	5·5375823	6·8781121	4·9513418	3
4	325097	7208060	81699·4	5·5120129	6·8578184	4·9122189	4
5	308549	6882963	75791·8	5·4893249	6·8377755	4·8796222	5
6	294157	6574414	71833·7	5·4685795	6·8178571	4·8563282	6
7	281211	6280257	68835·2	5·4490320	6·7979775	4·8378105	7
8	269235	5999046	66368·7	5·4301308	6·7780822	4·8219633	8
9	258131	5729811	64369·2	5·4118400	6·7581404	4·8086781	9
10	247751	5471680	62718·4	5·3940158	6·7381207	4·7973950	10
11	237972	5223929	61317·3	5·3765268	6·7179974	4·7875830	11
12	228694	4985957	60086·8	5·3592539	6·6977486	4·7787791	12
13	219831	4757263	58957·4	5·3420899	6·6773572	4·7705383	13
14	211320	4537432	57880·2	5·3249402	6·6568102	4·7625300	14
15	203106	4326112	56812·4	5·3077223	6·6360978	4·7544431	15
16	195149	4123006	55723·7	5·2903658	6·6152139	4·7460400	16
17	187419	3927857	54582·8	5·2728127	6·5941557	4·7370558	17
18	179894	3740438	53405·7	5·2550171	6·5729224	4·7275876	18
19	172562	3560544	52157·1	5·2369455	6·5515164	4·7173134	19
20	165416	3387982	50846·5	5·2185763	6·5299411	4·7062611	20
21	158453	3222566	49477·3	5·1999006	6·5082018	4·6944060	21
22	151750	3064113	48132·6	5·1811273	6·4863047	4·6824394	22
23	145298	2912363	46812·3	5·1622599	6·4642455	4·6703597	23
24	139092	2767065	45519·72	5·1433012	6.4420194	4·6581996	24
25	133123	2627973	44254·37	5·1242537	6·4196209	4·6459562	25
26	127385	2494850	43018·09	5·1051194	6·3970444	4·6336511	26
27	121871	2367465	41811·79	5·0858996	6·3742835	4·6212988	27
28	116572	2245594	40634·02	5·0665956	6·3513313	4·6088898	28
29	111483	2129022	39487·06	5·0472077	6·3281802	4·5964548	29
30	106595	2017539	38369·04	5·0277361	6·3048219	4·5839810	30
31	101901	1910944	37279·69	5·0081805	6·2812480	4·5714723	31
32	97396	1809043	36220·62	4·9885399	6·2574489	4·5589559	32
33	93071	1711647	35189·19	4·9688131	6·2334143	4·5464092	33
34	88920	1618576	34185·54	4·9489982	6·2091331	4·5338425	34
35	84936	1529656	33208·49	4·9290930	6·1845938	4·5212491	35
36	81114	1444720	32258·72	4·9090949	6·1597837	4·5086472	36
37	77446	1363606	31333·71	4·8890005	6·1346889	4·4960118	37
38	73928	1286160	30434·66	4·8688063	6·1092950	4·4833685	38
39	70552	1212232	29558·64	4·8485081	6·0835858	4·4706844	39
40	67313	1141680	28705·45	4·8281013	6·0575445	4·4579644	40
41	64207	1074367	27875·74	4·8075808	6·0311527	4·4452265	41
42	61227	1010160	27066·98	4·7869412	6·0043903	4·4324398	42
43	58368	948933	26278·48	4·7661764	5·9772356	4·4196002	43
44	55626	890565	25510·27	4·7452799	5·9496657	4·4067151	44
45	52966	834939	24761·34	4·7242448	5·9216548	4·3937742	45
46	50474	781943	24031·48	4·7030638	5·8931751	4·3807805	46
47	48054	731469	23318·33	4·6817289	5.8641959	4·3676975	47
48	45733	683415	22622·34	4·6602317	5·8346845	4·3545376	48
49	43507	637682	21942·87	4·6385635	5·8046042	4·3412934	49
50	41373	594175	21280·12	4·6167150	5·7739144	4·3279741	50
51	39326	552802	20632·22	4·5946765	5·7425696	4·3145459	51
52	37363	513476	19999·08	4·5724376	5·7105201	4·3010100	52
53	35480	476113	19379·57	4·5499879	5·6777100	4·2873442	53
54	33676	440633	18775·37	4·5273160	5·6440770	4·2735884	54

TABLE IV.—ENGLISH LIFE TABLE, No. 3. *Females.*

Interest $3\frac{1}{2}$ *per cent.*

AGE (x.)	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
55	31897	406957	18135·17	4·5037472	5·6095485	4·2585217	55
56	30172	375060	17488·82	4·4795987	5·5741007	4·2427605	56
57	28500	344888	16837·13	4·4548448	5·5376781	4·2262681	57
58	26881	316388	16181·88	4·4294484	5·5002200	4·2090290	58
59	25314	289507	15523·91	4·4033617	5·4616591	4·1910011	59
60	23797	264193	14862·94	4·3765261	5·4219213	4·1720756	60
61	22329	240396	14199·67	4·3488716	5·3809273	4·1522783	61
62	20908	218067	13533·75	4·3203177	5·3385899	4·1314182	62
63	19533	197159	12865·79	4·2907729	5·2948166	4·1094365	63
64	18203	177626	12196·33	4·2601345	5·2495065	4·0862292	64
65	16916	159423	11524·88	4·2282891	5·2025510	4·0616363	65
66	15672	142507	10852·92	4·1951122	5.1538362	4·0355466	66
67	14470	126835	10180·89	4·1604686	5·1032391	4·0077857	67
68	13311	112365	9511·22	4·1242120	5·0506311	3·9782363	68
69	12195	99054	8845·35	4·0861850	4·9958720	3·9467150	69
70	11123	86859	8185·74	4·0462195	4·9388148	3·9130579	70
71	10096	75736	7534·88	4·0041365	4·8793024	3·8770763	71
72	9114·8	65640·2	6895·08	3·9597458	4·8171699	3·8385395	72
73	8181·8	56525·4	6270·31	3·9128464	4·7522437	3·7972992	73
74	7298·4	48343·6	5663·59	3·8632265	4·6843390	3·7530920	74
75	6466·4	41045·2	5078·40	3·8106632	4·6132624	2·7057266	75
76	5687·5	34578·8	4518·17	3·7549226	4·5388099	3·6549625	76
77	4963·2	28891·3	3986·20	3·6957600	4·4607671	3·6005591	77
78	4294·6	23928·1	3485·44	3·6329197	4·3789082	3·5422573	78
79	3682·4	19633·5	3018·47	3·5661351	4·2929978	3·4797717	79
80	3127·0	15951·1	2587·59	3·4951286	4·2027906	3·4128956	80
81	2627·9	12824·1	2194·24	3·4196116	4·1080269	3·3412831	81
82	2184·2	10196·2	1839·40	3·3392848	4·0084383	3·2646764	82
83	1794·1	8012·0	1523·16	3·2538377	3·9037409	3·1827464	83
84	1455·3	6217·9	1245·03	3·1629490	3·7936437	3·0951808	84
85	1164·9	4762·6	1003·85	3·0662865	3·6778441	3·0016888	85
86	919·41	3597·66	797·75	2·9635069	3·5560201	2·9018668	86
87	714·92	2678·25	624·35	2·8542561	3·4278511	2·7954288	87
88	547·23	1963·33	480·84	2·7381689	3·2929933	2·6819982	88
89	411·97	1416·10	364·08	2·6148694	3·1510939	2·5611998	89
90	304·77	1004·13	270·81	2·4839706	3·0017900	2·4326709	90
91	221·35	699·36	197·70	2·3450746	2·8447008	2·2960070	91
92	157·68	478·01	141·52	2·1977724	2·6794370	2·1508037	92
93	110·06	320·33	99·23	2·0416444	2·5055976	1·9966325	93
94	75·207	210·273	68·10	1·8762598	2·3227835	1·8331236	94
95	50·255	135·066	45·69	1·7011769	2·1305460	1·6597979	95
96	32·805	84·811	29·94	1·5159432	1·9284522	1·4762083	96
97	20·898	52·006	19·14	1·3200950	1·7160535	1·2819269	97
98	12·977	31·108	11·93	1·1131578	1·4928721	1·0764599	98
99	7·8460	18·1308	7·23	0·8946463	1·2584170	0·8593114	99
100	4·6139	10·2848	4·27	0·6640641	1·0121959	0·6300315	100
101	2·6357	5·6709	2·44	0·4209039	0·7536520	0·3880887	101
102	1·4610	3·0352	1·36	0·1646473	0.4821873	0·1330149	102
103	·7848	1·5742	·73	$\bar{1}$·8947653	0·1970599	$\bar{1}$·8642537	103
104	·4081	·7894	·38	$\bar{1}$·6107177	$\bar{1}$·8972971	$\bar{1}$·5813869	104
105	·2051	·3813	·19	$\bar{1}$·3119534	$\bar{1}$·5812668	$\bar{1}$·2837665	105
106	·0995	·1762	·06	$\bar{2}$·9979104	$\bar{1}$·2460059	$\bar{2}$·9710046	106
107	·0466	·0767	·04	$\bar{2}$·6680158	$\bar{2}$·8847954	$\bar{2}$·6435149	107
108	·0210	·0301	·02	$\bar{2}$·3216857	$\bar{2}$·4785665	$\bar{2}$·3006412	108
109	·0091	·0091	·01	$\bar{3}$·9583252	$\bar{3}$·9590414	$\bar{3}$·9575690	109

TABLE IV.—ENGLISH LIFE TABLE, No. 3. *Females.*

Interest 4 per cent.

AGE (x).	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
0	488255	8082050	177406·81	5·6886465	6·9075215	5·2489703	0
1	406231	7593795	114162·57	5·6087736	6·8804589	5.0575238	1
2	366422	7187564	89977·11	5.5639812	6.8565817	4·9541321	2
3	339862	6821142	77510·72	5.5313033	6·8338571	4·8893617	3
4	318890	6481280	69609·77	5·5036409	6·8116608	4·8426701	4
5	301203	6162390	64188·34	5·4788599	6·7897492	4·8074561	5
6	285773	5861187	60342·67	5·4560216	6·7679855	4·7806245	6
7	271882	5575414	57443·58	5·4343811	6·7462771	4·7592415	7
8	259052	5303532	55069·57	5·4133869	6·7245653	4·7409118	8
9	247174	5044480	53155·72	5·3930031	6·7028164	4.7255500	9
10	236094	4797306	51583·01	5·3730859	6·6809974	4·7125067	10
11	225686	4561212	50254·62	5·3535040	6·6590803	4·7011760	11
12	215843	4335526	49092·25	5·3341380	6·6370418	4·6910130	12
13	206481	4119683	48032·24	5·3148811	6·6148638	4·6815329	13
14	197532	3913202	47024·55	5·2956384	6·5925322	4·6723247	14
15	188942	3715670	46031·18	5·2763275	6·5700371	4·6630521	15
16	180667	3526728	45023·16	5·2568780	6·5473720	4·6534360	16
17	172676	3346061	43981·53	5·2372319	6·5245338	4·6432704	17
18	164947	3173385	42893·08	5·2173433	6·5015228	4·6323872	18
19	157463	3008438	41753·94	5·1971787	6·4783411	4.6206975	19
20	150216	2850975	40563·23	5·1767166	6·4549934	4·6081325	20
21	143202	2700759	39326·16	5·1559478	6·4314859	4·5946816	21
22	136484	2557557	38116·41	5·1350816	6·4078253	4·5811120	22
23	130053	2421073	36934·93	5·1141211	6·3840078	4·5674373	23
24	123899	2291020	35783·29	5·0930695	6·3600289	4·5536802	24
25	118013	2167121	34662·07	5·0719290	6·3358832	4·5398545	25
26	112383	2049108	33571·35	5·0507016	6·3115649	4·5259689	26
27	107001	1936725	32511·83	5·0293889	6·2870680	4·5120414	27
28	101857	1829724	31483·39	5·0079918	6·2623856	4·4980814	28
29	96942	1727867	30485·52	4·9865110	6·2375104	4·4840936	29
30	92246	1630925	29518·02	4·9649464	6·2124339	4·4700872	30
31	87760	1538679	28580·32	4·9432978	6·1871481	4.4560671	31
32	83477	1450919	27671·84	4·9215642	6·1616432	4·4429380	32
33	79386	1367442	26792·27	4·8997444	6·1359089	4·4280094	33
34	75481	1288056	25940·21	4·8778366	6·1099348	4·4139735	34
35	71753	1212575	25115·34	4·8558384	6.0837086	4·3999391	35
36	68194	1140822	24316·35	4·8337473	6·0572179	4·3858984	36
37	64798	1072628	23542·94	4·8115599	6·0304491	4·3718607	37
38	61556	1007830	22793·64	4.7892727	6·0033873	4·3578137	38
39	58463	946274	22067·96	4·7668815	5·9760169	4·3437622	39
40	55511	887811	21364·78	4·7443817	5·9483205	4·3296984	40
41	52695	832300	20683·43	4·7217683	5·9202799	4·3156225	41
42	50008	779605	20022·71	4·6990356	5·8918746	4·3015228	42
43	47444	729597	19382·21	4·6761778	5·8630830	4·2874033	43
44	44998	682153	18760·83	4·6531884	5·8338818	4·2732521	44
45	42664	637155	18157·87	4·6300603	5·8042451	4·2590649	45
46	40438	594491	17572·67	4·6067863	5·7741453	4·2448377	46
47	38314	554053	17004·28	4·5833583	5·7435513	4·2305582	47
48	36288	515739	16452·27	4·5597682	5.7124300	4·2162259	48
49	34356	479451	15915·93	4·5360070	5·6807442	4·2018320	49
50	32514	445095	15394·59	4·5120655	5·6484527	4·1873681	50
51	30756	412581	14887·75	4·4879340	5·6155093	4·1728291	51
52	29081	381825	14394·94	4·4636022	5·5818644	4·1582099	52
53	27483	352744	13915·58	4·4390594	5·5474596	4·1435013	53
54	25959	325261	13449·37	4·4142945	5·5122320	4.1287020	54

TABLE IV.—ENGLISH LIFE TABLE, No. 3. *Females.*

Interest 4 *per cent.*

AGE (x).	NUMBERS.			LOGARITHMS.			AGE (x).
	D_x	N_x	M_x	λD_x	λN_x	λM_x	
55	24470	299302	12958·30	4·3886327	5·4761096	4·1125481	55
56	23035	274832	12464·65	4·3623913	5·4390673	4·0956801	56
57	21654	251797	11969·78	4·3355443	5·4010505	4·0780862	57
58	20326	230143	11474·51	4·3080550	5·3619978	4·0597342	58
59	19049	209817	10979·31	4·2798753	5·3218407	4·0405751	59
60	17822	190768	10484·43	4·2509466	5·2805055	4·0205449	60
61	16642	172946	9990·02	4·2211992	5·2379106	3·9995664	61
62	15508	156304	9496·17	4·1905524	5·1939701	3·9775485	62
63	14418	140796	9003·06	4·1589145	5·1485903	3·9543902	63
64	13372	126378	8510·88	4·1261831	5·1016715	3·9299745	64
65	12366	113006	8020·05	4·0922447	5·0531015	3·9041771	65
66	11402	100640	7531·04	4·0569749	5·0027706	3·8768549	66
67	10477	89238	7044·81	4·0202383	4·9505498	3·8478693	67
68	9591·5	78761·2	6562·27	3·9818886	4·8963123	3·8170541	68
69	8745·2	69169·7	6084·80	3·9417687	4·8399159	3·7842463	69
70	7938·0	60424·5	5613·94	3·8997102	4·7812131	3·7492678	70
71	7170·2	52486·5	5151·50	3·8555342	4·7200477	3·7119337	71
72	6442·4	45316·3	4699.48	3·8090505	4·6562545	3·6720498	72
73	5755·2	38873·9	4259·99	3·7600582	4·5896581	3·6294086	73
74	5109·1	33118·7	3835·32	3·7083453	4·5200733	3·5838016	74
75	4504·9	28009·6	3427·62	3·6536890	4·4473069	3·5349926	75
76	3943·3	23504·7	3039·20	3·5958554	4·3711548	3·4827593	76
77	3424·5	19561·4	2672·16	3·5345998	4·2914000	3·4268625	77
78	2948·9	16136·9	2328·25	3·4696665	4·2078201	3·3670296	78
79	2516·5	13188·0	2009·21	3·4007889	4·1201789	3·3030254	79
80	2126·6	10671·5	1716·15	3·3276894	4·0282255	3·2345553	80
81	1778·6	8554·9	1449·92	3·2500794	3·9317070	3·1613440	81
82	1471·2	6766·3	1210·90	3·1676596	3·8303512	3·0831083	82
83	1202·6	5295·1	998·93	3·0801195	3·7238742	2·9995351	83
84	970·82	4092·49	813·39	2·9871379	3·6119876	2·9102988	84
85	773·36	3121·67	653·28	2·8883824	3·4943870	2·8150994	85
86	607·45	2348·31	517·09	2·7835098	3·3707555	2·7135661	86
87	470·07	1740·86	403·08	2·6721659	3·2407638	2·6053912	87
88	358·08	1270·79	309.18	2·5539858	3·1040738	2·4902114	88
89	268·28	912·71	233·16	2·4285933	2·9603328	2·3676540	89
90	197·52	644·43	172·70	2·2956015	2·8091757	2·2372923	90
91	142·76	446·91	125·55	2·1546125	2·6502201	2·0988167	91
92	101·21	304·15	89·48	2·0052173	2·4830878	1·9517260	92
93	70·307	202·938	62·46	1·8469964	2·3073634	1·7956020	93
94	47·810	132·631	42·67	1·6795188	2·1226451	1·6301226	94
95	31·794	84·821	28·51	1·5023429	1·9285034	1·4549972	95
96	20·655	53·027	18·60	1·3150161	1·7244971	1·2695129	96
97	13·094	32·372	11·83	1·1170749	1·5101695	1·0729847	97
98	8·0918	19·2778	7·33	0·9080448	1·2850575	0·8651040	98
99	4·8690	11·1860	4·41	0·6874403	1·0486748	0·6444386	99
100	2·8495	6·3170	2·59	0·4547651	0·8005109	0·4132998	100
101	1·6200	3·4675	1·47	0·2095119	0·5400165	0·1673173	101
102	·8936	1·8475	·81	$\bar{1}$·9511624	0·2665844	$\bar{1}$·9084850	102
103	·4777	·9539	·42	$\bar{1}$·6791874	$\bar{1}$·9795028	$\bar{1}$·6232493	103
104	·2472	·4762	·22	$\bar{1}$·3930468	$\bar{1}$·6777894	$\bar{1}$·3424227	104
105	·1236	·2290	·11	$\bar{1}$·0921895	$\bar{1}$·3598355	$\bar{1}$·0413927	105
106	·0597	·1054	·05	$\bar{2}$·7760535	$\bar{1}$·0228406	$\bar{2}$·6989700	106
107	·0278	·0457	·02	$\bar{2}$·4440659	$\bar{2}$.6599162	$\bar{2}$·3010300	107
108	·0125	·0179	·01	$\bar{2}$·0956428	$\bar{2}$·2528530	$\bar{2}$·0000000	108
109	·0054	·0054	……	$\bar{3}$·7301893	$\bar{3}$·7323938	…	109

Annuities on Single Lives.

AGE.	3 PER CENT	3½ PER CENT	4 PER CENT	4½ PER CENT	5 PER CENT	6 PER CENT	AGE.
0	18·8502	17·0629	15·5529	14·3481	13·1608	11·3689	0
1	21·4385	19·4095	17·6933	16·3280	14·9702	12·9272	1
2	22·5391	19·4960	18·6155	17·1890	15·7562	13·6072	2
3	23·0668	20·8740	19.0703	17·6214	16·1508	13·9527	3
4	23·3474	21·1720	19·3245	17·8749	16·3785	14·1563	4
5	23·4807	21·3075	19·4592	18·0111	16·5070	14·2758	5
6	23·5106	21·3500	19·5099	18·0749	16·5656	14·3360	6
7	23·4742	21·3329	19·5067	18·0897	16·5794	14·3582	7
8	23·3999	21·2819	19·4729	18·0770	16·5678	14·3591	8
9	23.2886	21·1973	19·4086	17·8678	16·5310	14·3386	9
10	23·1470	21·0854	19·3194	17·7962	16·4732	14·2002	10
11	22·9818	20·9518	19·2105	17·7065	16·3986	14·2474	11
12	22·7987	20·8019	19·0865	17·6029	16·3113	14·1836	12
13	22·6032	20·6405	18·9518	17·4895	16·2148	14·1117	13
14	22.4000	20·4719	18·8104	17·3697	16·1123	14·0348	14
15	22·1933	20·2998	18·6657	17·2468	16·0069	13·9551	15
16	21·9867	20·1266	18·5206	17·1235	15·9010	13·8748	16
17	21·7829	19·9577	18·3777	17·0020	15·7968	13·7961	17
18	21·5844	19·7924	18·2389	16·8843	15·6961	13·7203	18
19	21·5928	19·6334	18·1057	16·7717	15·6001	13·6488	19
20	21·2093	19·4816	17·9792	16·6654	15·5100	13·5824	20
21	21·0342	19·3377	17·8598	16·5658	15·4261	13·5217	21
22	20·8573	19·1919	17·7380	16·4647	15·3410	13·4601	22
23	20·6782	19·0441	17·6160	16·3619	15·2544	13·3973	23
24	20·4965	18·8938	17·4910	16·2531	15·1660	13·3331	24
25	20·3120	18·7409	17·3634	16·1499	15·0756	13·2674	25
26	20·1244	18·5851	17·2332	16·0409	14·9830	13·1999	26
27	19·9334	18·4260	17·1000	15·9282	14·8879	13·1305	27
28	19·7387	18·2635	16·9636	15·8130	14·7902	13·0589	28
29	19·5401	18·0973	16·8238	15·6947	14·6896	12·9849	29
30	19·3374	17·9272	16·6802	15·5730	14·5858	12·9083	30
31	19·1303	17·7529	16·5328	15·4476	14·4787	12·8290	31
32	18·9187	17·5742	16·3812	15·3184	14·3681	12·7467	32
33	18·7022	17·3908	16·2252	15·1852	14·2537	12·6612	33
34	18·4807	17·1027	16·0647	15·0476	14·1353	12·5724	34
35	18·2539	17·0095	15·8994	14·9056	14·0127	12·4800	35
36	18·0218	16·8110	15·7290	14·7588	13·8857	12·3837	36
37	17·7841	16·6071	15·5535	14·6070	13·7540	12·2835	37
38	17·5405	16·3976	15·3725	14·4498	13·6175	12·1789	38
39	17·2910	16·1822	15·1858	14·2878	13·4759	12·0699	39
40	17·0353	15·9607	14·9933	14·1196	13·3289	11·9562	40
41	16·7733	15·7329	14·7947	13·9459	13.1763	11·8374	41
42	16·5047	15·4987	14·5897	13·7665	13·0179	11·7135	42
43	16·2293	15·2577	14·3782	13·5759	12·8533	11·5839	43
44	15·9471	15·0098	14·1598	13·3870	12·6824	11·4485	44
45	15·6576	14·7887	13·9343	13·1871	12·5047	11·3069	45
46	15·3608	14·4922	13·7014	12·9799	12·3200	11·1588	46
47	15·0563	14·2218	13·4608	12·7652	12·1279	11·0038	47
48	14·7439	13·9435	13·2122	12·5426	11·9280	10·8415	48
49	14·4233	13·6569	12·9552	12·3116	11·7199	10·6714	49

TABLE IV.—ENGLISH LIFE TABLE, NO. 3. *Females.*

Annuities on Single Lives.

AGE.	3 PER CENT.	3½ PER CENT.	4 PER CENT.	4½ PER CENT.	5 PER CENT.	6 PER CENT.	AGE.
50	14·0942	13·3615	12·6895	12·0720	11·5032	10·4931	50
51	13·7562	13·0570	12·4145	11·8231	11·2774	10·3059	51
52	13·4090	12·7430	12·1299	11·5645	11·0419	10·1095	52
53	13·0522	12·4191	11·8351	11·2957	10·7963	9·9030	53
54	12·6852	12·0846	11·5296	11·0155	10·5397	9·6857	54
55	12·3279	11·7585	11·2314	10·7427	10·2887	9·4728	55
56	11·9699	11·4309	10·9310	10·4667	10·0347	9·2564	56
57	11·6107	11·1013	10·6280	10·1871	9·7772	9·0360	57
58	11·2505	10·7699	10·3225	9·9056	9·5162	8·8116	58
59	10·8892	10·4366	10·0145	9·6204	9·2518	8·5831	59
60	10·5274	10·1019	9·7043	9·3325	8·9842	8·3507	60
61	10·1653	9·7660	9·3923	9·0428	8·7136	8·1147	61
62	9·8037	9·4297	9·0790	8·7500	8·4406	7·8755	62
63	9·4431	9·0936	8·7651	8·4564	8·1658	7·6335	63
64	9·0844	8·7562	8·4512	8·1622	7·8896	7·3894	64
65	8·7284	8·4246	8·1381	7·8681	7·6130	7·1436	65
66	8·3758	8·0933	7·8266	7·5748	7·3364	6·8970	66
67	8·0275	7·7636	7·5175	7·2830	7·0608	6·6501	67
68	7·6842	7·4415	7·2115	6·9937	6·7868	6·4038	68
69	7·3469	7·1225	6·9095	6·7074	6·1552	6·1586	69
70	7·0162	6·8090	6·6121	6·4250	6·2467	5·9154	70
71	6·6928	6·5018	6·3200	6·1471	5·9820	5·6747	71
72	6·3773	6·2015	6·0340	5·8744	5·7219	5·4372	72
73	6·0702	5·9087	5·7546	5·6076	5·4668	5·2036	73
74	5·7721	5·6239	5·4823	5·3470	5·2173	4·9743	74
75	5·4832	5·3474	5·2175	5·0933	4·9739	4·7498	75
76	5·2039	5·0798	4·9607	4·8356	4·7370	4·5307	76
77	4·9346	4·8211	4·7122	4·6077	4·5070	4·3174	77
78	4·6753	4·5717	4·4721	4·3766	4·2842	4·1100	78
79	4·4262	4·3317	4·2407	4·1534	4·0687	3·9090	79
80	4·1872	4·1002	4·0181	3·9384	3·8609	3·7146	80
81	3·9583	3·8800	3·8043	3·7316	3·6607	3·5268	81
82	3·7395	3·6682	3·5993	3·5332	3·4683	3·3458	82
83	3·5307	3·4658	3·4031	3·4209	3·2836	3·1717	83
84	3·3315	3·2726	3·2155	3·1610	3·1067	3·0045	84
85	3·1419	3·0884	3·0365	2·9872	2·9374	2·8441	85
86	2·9617	2·9130	2·8659	2·8213	2·7757	2·6906	86
87	2·7904	2·7456	2·7034	2·6575	2·6213	2·5438	87
88	2·6278	2·5878	2·5489	2·5110	2·4742	2·4036	88
89	2·4737	2·4374	2·4020	2·3676	2·3342	2·2698	89
90	2·3277	2·2947	2·2627	2·2314	2·2009	2·1424	90
91	2·1894	2·1595	2·1305	2·1020	2·0743	2·0210	91
92	2·0586	2·0315	2·0052	1·9793	1·9542	1·9056	92
93	1·9350	1·9105	1·8865	1·8630	1·8401	1·7959	93
94	1·8181	1·7959	1·7741	1·7528	1·7320	1·6918	94
95	1·7078	1·6876	1·6678	1·6485	1·6296	1·5929	95
96	1·6036	1·5853	1·5673	1·5497	1·5325	1 4992	96
97	1·5052	1·4886	1·4723	1·4563	1·4407	1·4103	97
98	1·4123	1·3972	1·3824	1·3679	1·3537	1·3261	98
99	1·3245	1·3108	1·2974	1·2842	1·2714	1·2461	99
100	1·2415	1·2291	1·2169	1·2050	1.1934	1·1704	100

TABLE XII.—THE PROBABILITY OF A LIFE LIVING

One or more years, according to the English Life Table.

AGE (x).	MALES.			FEMALES.			AGE (x).
	I. $\lambda\, c_x$.	II. $\lambda\, c_{x(q}$.	III. Co-$\lambda\, c_x$.	I. $\lambda\, c_x$.	II. $\lambda\, c_{x(q}$.	III. Co-$\lambda\, c_x$.	
0	$\bar{1}$·9224164	3·4926604	0·0775836	$\bar{1}$·9371604	3·8981768	0·0628396	0
1	·9711377	·5702440	·0288623	·9722410	·9610164	·0277590	1
2	·9843053	·5991063	·0156947	·9843554	·9887754	·0156446	2
3	·9895169	·6148010	·0104831	·9893710	·0044200	·0106290	3
4	·9921913	·6252841	·0078087	·9922523	·0150490	·0077477	4
5	$\bar{1}$·9940574	3·6330928	0·0059426	$\bar{1}$·9941950	3·0227967	0·0058050	5
6	·9952751	·6390354	·0047249	·9953928	·0286017	·0046072	6
7	·9960044	·6437603	·0039956	·9960392	·0332089	·0039608	7
8	·9966712	·6477559	·0033288	·9966496	·0371697	·0033504	8
9	·9971834	·6510847	·0028166	·9971161	·0405201	·0028839	9
10	$\bar{1}$·9975541	3·6539013	0·0024459	$\bar{1}$·9974514	3·0434040	0·0025486	10
11	·9977966	·6563472	·0022034	·9976674	·0459526	·0023326	11
12	·9979241	·6585506	·0020759	·9977764	·0482852	·0022236	12
13	·9979498	·6606265	·0020502	·9977906	·0505088	·0022094	13
14	·9978867	·6626767	·0021133	·9977224	·0527182	·0022776	14
15	$\bar{1}$·9977484	3·6647900	0·0022516	$\bar{1}$·9975839	3·0549958	0·0024161	15
16	·9975478	·6670416	·0024522	·9973872	·0574119	·0026128	16
17	·9972982	·6694938	·0027018	·9971448	·0600247	·0028552	17
18	·9970127	·6721956	·0029873	·9968687	·0628799	·0031313	18
19	·9967048	·6751829	·0032952	·9965712	·0660112	·0034288	19
20	$\bar{1}$·9963874	3·6784781	0·0036126	$\bar{1}$·9962646	3·0694400	0·0037354	20
21	·9963071	·6820907	·0036929	·9961671	·0731754	·0038329	21
22	·9962293	·6857836	·0037707	·9960729	·0770083	·0039271	22
23	·9961534	·6895543	·0038466	·9959817	·0809354	·0040183	23
24	·9960785	·6934009	·0039215	·9958929	·0849537	·0041071	24
25	$\bar{1}$·9960037	3·6973224	0·0039963	$\bar{1}$·9958060	3·0890608	0·0041940	25
26	·9959282	·7013187	·0040718	·9957206	·0932548	·0042794	26
27	·9958512	·7053905	·0041488	·9956362	·0975342	·0043638	27
28	·9957718	·7095393	·0042282	·9955525	·1018980	·0044475	28
29	·9956892	·7137675	·0043108	·9954688	·1063455	·0045312	29
30	$\bar{1}$ 9956026	3·7180783	0·0043974	$\bar{1}$·9953847	3·1108767	0·0046153	30
31	·9955111	·7224757	·0044889	·9952998	·1154920	·0047002	31
32	·9954139	·7269646	·0045861	·9952135	·1201922	·0047865	32
33	·9953102	·7315507	·0046898	·9951255	·1249787	·0048745	33
34	·9951991	·7362405	·0048009	·9950352	·1298532	·0040648	34
35	$\bar{1}$·9950798	3·7410414	0·0049202	$\bar{1}$·9949422	3·1348180	0·0050578	35
36	·9949514	·7459616	·0050486	·9948460	·1398758	·0051540	36
37	·9948132	·7510102	·0051868	·9947461	·1450298	·0052539	37
38	·9946643	·7561970	·0053357	·9946421	·1502837	·0053579	38
39	·9945038	·7615327	·0054962	·9945336	·1556416	·0054664	39
40	$\bar{1}$·9943309	3·7670289	0·0056691	$\bar{1}$·9944199	3·1611080	0·0055801	40
41	·9941448	·7726980	·0058552	·9943007	·1666881	·0056993	41
42	·9939446	·7785532	·0060554	·9941755	·1723874	·0058245	42
43	·9937297	·7846086	·0062703	·9940439	·1782119	·0059561	43
44	·9934989	·7908789	·0065011	·9939053	·1841680	·0060947	44
45	$\bar{1}$·9932516	3·7973800	0·0067484	$\bar{1}$·9937593	3·1902627	0·0062407	45
46	·9929870	·8041284	·0070130	·9936054	·1965034	·0063946	46
47	·9927040	·8111414	·0072960	·9934432	.2028980	·0065568	47
48	·9924020	·8184374	·0075980	·9932722	·2094548	·0067278	48
49	·9920802	·8260354	·0079198	·9930918	·2161826	·0069082	49
50	$\bar{1}$·9917376	3·8339552	0·0082624	$\bar{1}$·9929018	3·2230908	0·0070982	50
51	·9911290	·8422176	·0088710	·9927015	·2301890	·0072985	51
52	·9906840	·8510886	0093160	·9924906	·2374875	·0075094	52
53	·9902221	·8604046	·0097779	·9922685	·2449969	·0077315	53
54	·9897327	·8701825	·0102673	·9913715	·2527284	·0086285	54

TABLE XII.—THE PROBABILITY OF A LIFE LIVING

One or more years, according to the English Life Table, No. 3.

AGE (x).	MALES.			FEMALES.			AGE (x).
	I. $\lambda\ c_x$.	II. $\lambda\ c_{x\|q}$.	III. Co-$\lambda\ c_x$.	I. $\lambda\ c_x$.	II. $\lambda\ c^{x\|q}$.	III. Co-$\lambda\ c_x$.	
55	$\bar{1}$·9892057	3·8804498	0·0107943	$\bar{1}$·9907919	3·2613569	0·0092081	55
56	·9886306	·8912441	·0113694	·9901864	·2705650	·0098136	56
57	·9879971	·9026135	·0120029	·9895440	·2803786	·0104560	57
58	·9872950	·9146164	·0127050	·9888536	·2908346	·0111464	58
59	·9865139	·9273214	·0134861	·9881047	·3019810	·0118953	59
60	$\bar{1}$·9856433	3·9408075	0·0143567	$\bar{1}$·9872859	3·3138763	0·0127141	60
61	·9846731	·9551642	·0153269	·9863865	·3265904	·0136135	61
62	·9835929	·9704911	·0164071	·9853955	·3402039	·0146045	62
63	·9823924	·9868982	·0176076	·9843019	·3548084	·0156981	63
64	·9810612	4·0045058	·0189388	·9830950	·3705065	·0169050	64
65	$\bar{1}$·9795890	4·0234446	0·0204110	$\bar{1}$·9819635	3·3874115	0·0180365	65
66	·9779656	·0438556	·0220344	·9802967	·4056480	·0197033	66
67	·9761804	·0658900	·0238196	·9786837	·4253513	·0213163	67
68	·9742234	·0897096	·0257766	·9769134	·4466676	·0230866	68
69	·9720840	·1154862	·0279160	·9749749	·4697542	·0250251	69
70	$\bar{1}$·9697520	4·1434022	0·0302480	$\bar{1}$·9728573	3·4947793	0·0271427	70
71	·9672170	·1736502	·0327830	·9705496	·5219220	·0294504	71
72	·9644689	·2064332	·0355311	·9680410	·5513724	·0319590	72
73	·9614970	·2419643	·0385030	·9653205	·5833314	·0346795	73
74	·9582913	·2804673	·0417087	·9623770	·6180109	·0376230	74
75	$\bar{1}$·9548413	4·3221760	0·0451587	$\bar{1}$·9591998	3·6556339	0·0408002	75
76	·9511367	·3673347	·0488633	·9557777	·6964341	·0442223	76
77	·9471672	·4161980	·0528328	·9521001	·7406564	·0478999	77
78	·9429225	·4690308	·0570775	·9481557	·7885563	·0518443	78
79	·9383922	·5261083	·0616078	·9439338	·8404006	·0560662	79
80	$\bar{1}$·9335660	4·5877161	0·0664340	$\bar{1}$·9394234	3·8964668	0·0605766	80
81	·9284336	·6541501	·0715664	·9346135	·9570434	·0653865	81
82	·9229846	·7257165	·0770154	·9294933	4·0224299	·0705067	82
83	·9172088	·8027319	·0827912	·9240517	·0929366	·0759483	83
84	·9110958	·8855231	·0889042	·9182778	·1688849	·0817222	84
85	$\bar{1}$·9046352	4·9744273	0·0953648	$\bar{1}$·9121607	4·2506071	0·0878393	85
86	·8978169	5·0697921	·1021831	·9056896	·3384464	·0943104	86
87	·8906302	·1719752	·1093698	·8988532	·4320568	·1011468	87
88	·8830651	·2813450	·1169349	·8916408	·5339036	·1083592	88
89	·8751113	·3982799	·1248887	·8840415	·6422628	·1159585	89
90	$\bar{1}$·8667581	5·5231686	0·1332419	$\bar{1}$·8760443	4·7582213	0·1239557	90
91	·8579955	·6564105	·1420045	·8676383	·8821770	·1323617	91
92	·8488131	·7984150	·1511869	·8588123	5·0145387	·1411877	92
93	·8392005	·9496019	·1607995	·8495557	·1557264	·1504443	93
94	·8291475	6·1104014	·1708525	·8398575	·3061707	·1601425	94
95	$\bar{1}$·8186437	6·2812539	0·1813563	$\bar{1}$·8297066	5·4663132	0·1702934	95
96	·8076787	·4626102	·1923213	·8190921	·6366066	·1809079	96
97	·7962424	·6549315	·2037576	·8080033	·8175145	·1919967	97
98	·7843241	·8586891	·2156759	·7964288	6·0095112	·2035712	98
99	·7719138	7·0743650	·2280862	·7843582	·2130824	·2156418	99
100	$\bar{1}$·7590011	7·3024512	0·2409989	$\bar{1}$·7717801	6·4287242	0·2282199	100
101	·7455756	·5434501	·2544244	·7586838	·6569441	·2413162	101
102	·7316270	·7978745	·2683730	·7450583	·8982603	·2549417	102
103	·7171451	8·0662475	·2828549	·7308927	7·1532020	·2691073	103
104	·7021192	·3491024	·2978808	·7161761	·4223093	·2838239	104
105	$\bar{1}$·6865395	8·6469832	0·3134605	$\bar{1}$·7008974	7·7061332	0·2991026	105
106	·6703952	·9604437	·3296048	·6850457	8·0052358	·3149543	106
107	·6536762	9·2900485	·3463238	·6686102	·3201901	·3313898	107
108	·6363723	·6363723	·3636277	·6515799	·6515799	·3484201	108

ENGLISH LIFE TABLE, No. 3. *Males.*

Mortality and Expectation of Life.

AGE (x)	d_x	l_x	λl_x	E_x	AGE (x)	d_x	l_x	λl_x	E_x
0	83719	511745	5·7090538	39·91	55	5144	209539	5·3212644	16·45
1	27521	428026	5·6314702	46 65	56	5281	204395	5·3104701	15·86
2	14215	400505	5·6026079	48·83	57	5428	199114	5·2991007	15·26
3	9213	386290	5·5869132	49·61	58	5584	193686	5·2870978	14·68
4	6719	377077	5·5764301	49·81	59	5752	188102	5·2743928	14·10
5	5033	370358	5·5686214	49·71	60	5929	182350	5·2609067	13·53
6	3953	365325	5·5626788	49·39	61	6118	176421	5·2465500	12·96
7	3310	361372	5·5579539	48·92	62	6314	170303	5·2312231	12·41
8	2734	358062	5·5539583	48·37	63	6515	163989	5·2148160	11·87
9	2297	355328	5·5506295	47·74	64	6720	157474	5·1972084	11·34
10	1983	353031	5·5478129	47·05	65	6921	150754	5·1782696	10·82
11	1776	351048	5·5453670	46·31	66	7115	143833	5·1578586	10·32
12	1666	349272	5·5431636	45·54	67	7297	136718	5·1358242	9·83
13	1637	347606	5·5410877	44·76	68	7458	129421	5·1120046	9·36
14	1679	345969	5·5390375	43·97	69	7593	121963	5·0862280	8·90
15	1781	344290	5·5369242	43·18	70	7695	114370	5·0583120	8·45
16	1928	342509	5·5346726	42·40	71	7756	106675	5·0280640	8·03
17	2112	340581	5·5322204	41·64	72	7770	98919	4·9952810	7·62
18	2320	338469	5·5295186	40·90	73	7733	91149	4·9597499	7·22
19	2541	336149	5·5265313	40·17	74	7639	83416	4·9212469	6·85
20	2764	333608	5·5232361	39·48	75	7483	75777	4·8795382	6·49
21	2801	330844	5·5196235	38·80	76	7268	68294	4·8343795	6·15
22	2836	328043	5·5159306	38·13	77	6990	61026	4·7855162	5·82
23	2868	325207	5·5121599	37·46	78	6655	54036	4·7326834	5·51
24	2897	322339	5·5083133	36·79	79	6266	47381	4·6756059	5·21
25	2926	319442	5·5043918	36·12	80	5832	41115	4·6139981	4·93
26	2954	316516	5·5003955	35·44	81	5361	35283	4·5475641	4·66
27	2981	313562	5·4963237	34·77	82	4862	29922	4·4759977	4·41
28	3009	310581	5·4921749	34·10	83	4349	25060	4·3989823	4·17
29	3038	307572	5·4879467	33·43	84	3834	20711	4·3161911	3·95
30	3068	304534	5·4836359	32·76	85	3328	16877	4·2272869	3·73
31	3100	301466	5·4792385	32·09	86	2840	13549	4·1319221	3·53
32	3134	298366	5·4747496	31·42	87	2384	10709	4·0297390	3·34
33	3171	295232	5·4701635	30·74	88	1965	8325	3·9203692	3·16
34	3211	292061	5·4654737	30·07	89	1590	6360	3·8034343	3·00
35	3254	288850	5·4606728	29·40	90	1260	4770	3·6785456	2·84
36	3300	285596	5·4557526	28·73	91	979	3510	3·5453037	2·69
37	3352	282296	5·4507040	28·06	92	744	2531	3·4032992	2·55
38	3406	278944	5·4455172	27·39	93	553	1787	3·2521123	2·41
39	3465	275538	5·4401815	26·72	94	401	1234	3·0913128	2·29
40	3529	272073	5·4346853	26·06	95	285	833	2·9204603	2·17
41	3596	268544	5·4290162	25·39	96	196	548	2·7391040	2·06
42	3668	264948	5·4231610	24·73	97	132	352	2·5467827	1·95
43	3746	261280	5·4171056	24·07	98	86	220	2·3430251	1·85
44	3826	257534	5·4108353	23·41	99	55	134	2·1273492	1·76
45	3912	253708	5·4043342	22·76	100	33	79	1·8992630	1·68
46	4001	249796	5·3975858	22·11	101	21	46	1·6582641	...
47	4095	245795	5·3905728	21·46	102	11	25	1·4038397	...
48	4192	241700	5·3832768	20·82	103	7	14	1·1354667	...
49	4292	237508	5·3756788	20·17	104	3	7	0·8526118	...
50	4395	233216	5·3677590	19·54	105	2	4	0·5547310	...
51	4626	228821	5·3594966	18·90	106	1	2	0·2412705	...
52	4758	224195	5·3506256	18·28	107	1	1	$\bar{1}$·9116657	...
53	4885	219437	5·3413096	17·67	108	...	...	$\bar{1}$·5653419	...
54	5013	214552	5·3315317	17·06	109	...	...	$\bar{1}$·2017142	...

ENGLISH LIFE TABLE, No. 3. *Females.*

Mortality and Expectation of Life.

AGE (x)	d_x	l_x	λl_x	E_x	AGE (x)	d_x	l_x	λl_x	E_x
0	65774	488255	5·6886465	41·85	55	4439	211576	5·3254664	17·43
1	26159	422481	5·6258069	47·31	56	4628	207137	5·3162583	16·79
2	14023	396322	5·5980479	49·40	57	4817	202509	5·3064447	16·17
3	9243	382299	5·5824033	50·20	58	5009	197692	5·2959887	15·55
4	6596	373056	5·5717743	50·43	59	5206	192683	5·2848423	14·94
5	4866	366460	5·5640266	50·33	60	5409	187477	5·2729470	14·34
6	3815	361594	5·5582216	50·00	61	5619	182068	5·2602329	13·75
7	3249	357779	5·5536144	49·53	62	5835	176449	5·2466194	13·17
8	2724	354530	5·5496536	48·98	63	6057	170614	5·2320149	12·60
9	2328	351806	5·5463032	48·35	64	6282	164557	5·2163168	12·05
10	2045	349478	5·5434193	47·67	65	6509	158275	5·1994118	11·51
11	1861	347433	5·5408707	46·95	66	6731	151766	5·1811753	10·98
12	1765	345572	5·5385381	46·20	67	6947	145035	5·1614720	10·47
13	1745	343807	5·5363145	45·44	68	7149	138088	5·1401557	9·97
14	1789	342062	5·5341051	44·66	69	7332	130939	5·1170691	9·48
15	1888	340273	5·5318275	43·90	70	7489	123607	5·0920440	9·02
16	2029	338385	5·5294114	43·14	71	7613	116118	5·0649013	8·57
17	2205	336356	5·5267986	42·40	72	7698	108505	5·0354509	8·13
18	2400	334151	5·5239434	41·67	73	7736	100807	5·0034919	7·71
19	2609	331751	5·5208121	40·97	74	7724	93071	4·9688124	7·31
20	2819	329142	5·5173833	40·29	75	7653	85347	4·9311894	6·93
21	2867	326323	5·5136479	39·63	76	7521	77694	4·8903892	6·56
22	2912	323456	5·5098150	38·98	77	7329	70173	4·8461669	6·21
23	2952	320544	5·5058879	38·33	78	7071	62844	4·7982670	5·88
24	2989	317592	5·5018696	37·68	79	6755	55773	4·7464227	5·56
25	3024	314603	5·4977625	37·04	80	6382	49018	4·6903565	5·26
26	3055	311579	5·4935685	36·39	81	5959	42636	4·6297799	4·98
27	3084	308524	5·4892891	35·75	82	5496	36677	4·5643934	4·71
28	3112	305440	5·4849253	35·10	83	5003	31181	4·4938867	4·45
29	3138	302328	5·4804778	34·46	84	4490	26178	4·4179384	4·21
30	3163	299190	5·4759466	33·81	85	3972	21688	4·3362162	3·98
31	3187	296027	5·4713313	33·17	86	3458	17716	4·2483769	3·76
32	3209	292840	5·4666311	32·53	87	2962	14258	4·1540665	3·56
33	3233	289631	5·4618446	31·88	88	2494	11296	4·0529197	3·36
34	3255	286398	5·4569701	31·23	89	2063	8802	3·9445605	3·18
35	3279	283143	5·4520053	30·59	90	1673	6739	3·8286020	3·01
36	3301	279864	5·4469475	29·94	91	1331	5066	3·7046463	2.85
37	3326	276563	5·4417935	29·29	92	1037	3735	3·5722846	2·70
38	3350	273237	5·4365396	28·64	93	790	2698	3·4310969	2·55
39	3376	269887	5·4311817	27·99	94	588	1908	3·2806526	2·42
40	3402	266511	5·4257153	27·34	95	428	1320	3·1205101	2·29
41	3431	263109	5·4201352	26·69	96	304	892	2·9502167	2·17
42	3459	259678	5·4144359	26·03	97	210	588	2·7693088	2·06
43	3490	256219	5·4086114	25·38	98	142	378	2·5773121	1·96
44	3522	252729	5·4026553	24·72	99	92	236	2.3737409	1·86
45	3555	249207	5·3965606	24·06	100	59	144	2·1580991	1·76
46	3591	245652	5·3903199	23·40	101	36	85	1·9298792	...
47	3627	242061	5·3839253	22·74	102	22	49	1·6885630	...
48	3665	238434	5·3773685	22·08	103	12	27	1·4336213	...
49	3705	234769	5·3706407	21·42	104	7	15	1·1645140	...
50	3746	231064	5·3637325	20·75	105	4	8	0·8806901	...
51	3788	227318	5·3566343	20·09	106	2	4	0·5815875	...
52	3832	223530	5·3493358	19·42	107	1	2	0·2666332	...
53	3876	219698	5·3418264	18·75	108	1	1	9·9352434	...
54	4246	215822	5·3340949	18·08	109	...	...	9·5868233	...

Annuities to cease on the Death of the FIRST *of Two Lives. 3 per cent.*

AGES	5	10	15	20	25	AGES
5	19·5987					5
10	19·5427	19·5201				10
15	18·9081	18·9202	18·3766			15
20	18·2134	18·2578	17·7712	17·1847		20
25	17·5626	17·6363	17·2038	16·7187	16·2728	25
30	16·7991	16·8880	16·5188	16·0945	15·7239	30
35	15·9017	16·0209	15·6934	15·3282	15·0128	35
40	14·8615	14·9955	14·7172	14·4082	14·1542	40
45	13·6781	13·8211	13·5894	13·3321	13·1334	45
50	12·3530	12·4992	12·3111	12·1010	11·9501	50
55	10·9200	10·8984	10·9260	10·7480	10·6370	55
60	9.3861	9·5215	9·4105	9·2799	9·2016	60
65	7·8067	8·0370	7·8489	7·7509	7·6964	65
70	6·2875	6·3897	6·3757	6·2662	6·2312	70
75	4·9241	5·0058	4·9734	4·9233	4·9010	75
80	3·7716	3·8335	3·8149	3·7805	3·7666	80
85	2·8414	2·8864	2·8763	2·8533	2·8447	85
90	2·1197	2·1463	2·1413	2·1261	2·1208	90
95	1·5575	1·5790	1·5778	1·5669	1·5636	95
100	1·0619	1·1238	1·1330	1·1171	1·1695	100

	30	35	40	45	50	
30	15·2284					30
35	14·6052	14·0699				35
40	13·8235	13·3808	12·7963			40
45	12·8743	12·5224	12·0471	11·4207		45
50	11·7542	11·4856	11·1163	10·6171	9·9558	50
55	10·4942	10·2972	10·0229	9·6447	9·3541	55
60	9·1014	8·9633	8·7694	8·9780	8·2856	60
65	7·6305	7·5381	7·4077	7·2231	7·0768	65
70	6·1881	6·1289	6·0452	5·9252	5·8293	70
75	4·8742	4·8374	4·7852	4·7104	4·6413	75
80	3·7502	3·7290	3·6961	3·6506	3·6136	80
85	2·8351	2·8214	2·8023	2·7749	2·7526	85
90	2·1148	2·1068	2·0953	2·0789	2·0656	90
95	1·5600	1·5552	1·5483	1·5385	1·5306	95
100	1·1133	1·1106	1·1068	1·1014	1·0971	100

	55	60	65	70	75	
55	8·4606					55
60	7·6119	6·9412				60
65	6·6005	6·1053	5·4581			65
70	5·5115	5·1689	4·6993	4·1223		70
75	4·4483	4·2226	3·9012	3·4874	3.0105	75
80	3·4878	3·3464	3·1363	2·8537	2·4890	80
85	2·6759	2·5895	2·3770	2·2696	2·0360	85
90	2·0190	1·9661	1·8831	1·7634	1·6078	90
95	1·5024	1·4705	1·4193	1·3436	1·2440	95
100	1·0813	1·0636	1·0345	0·9901	0·9284	100

	80	85	90	95	100	
80	2·1421					80
85	1 7706	1·4936				85
90	1·1440	1·2245	1·0232			90
95	1·1178	0·9782	0·8320	0·6879		95
100	0·8498	0·8404	0·6570	0·5521	0·4365	100

TABLE XIV.—ENGLISH LIFE TABLE No. 3. *Females.*

Annuities to cease on the Death of the LAST *of Two Lives.* 3 *per cent.*

OLDER AGES	5	10	15	20	25	OLDER AGES
5	19·5647					5
10	19·4936	19·4530				10
15	18·8913	18·8827	18·3636			15
20	18·2420	18·2629	17·7953	17·2820		20
25	17·6512	17·6986	17·2790	16·8192	16·4122	25
30	16·9796	17·0496	16·6764	16·2701	15·9216	30
35	16·1972	16·2861	15·9572	15·6025	15·3119	35
40	15·2771	15·3811	15·0949	14·7885	14·5514	40
45	14·1919	14·3073	14·0630	13·8017	13·6125	45
50	12·9097	13·0323	12·8299	12·6114	12·4638	50
55	11·4065	11·5303	11·3695	11·1923	11·0803	55
60	9·8328	9·9526	9·8302	9·6903	9·6075	60
65	8·2231	8·3332	8·2451	8·1386	8·0796	65
70	6·6612	6·7566	6·6971	6·6191	6·5787	70
75	5·2409	5·3188	5·2813	5·2262	5·1995	75
80	4·0255	4 0857	4·0636	4·0259	4·0087	80
85	3·0357	3·0799	3·0678	3·0426	3·0318	85
90	2·2581	2·2894	2·2832	2·2666	2·2599	90
95	1·6598	1·6813	1·6783	1·6677	1·6655	95
100	1·1755	1·1892	1·1884	1·1820	1·1796	100
	30	**35**	**40**	**45**	**50**	
30	15·4970					30
35	14·9568	14·4962				35
40	14·2650	13·8889	13·3788			40
45	13·3890	13·0954	12·6913	12·1165		45
50	12·2948	12·0757	11·7683	11·3222	10·6745	50
55	10·9566	10·8002	10·5814	10·2561	9·7612	55
60	9·5191	9·4109	9·2620	9·0382	8·6830	60
65	8·0182	7·9455	7·8484	7·7032	7·4646	65
70	6·5375	6·4900	6·4287	6·3391	6·1887	70
75	5·1728	5·1426	5·1047	5·0513	4·9613	75
80	3·9919	3·9731	3·9499	3·9186	3·8668	80
85	3·0213	3·0098	2·9958	2·9774	2·9482	85
90	2·2535	2·2465	2·2380	2·2271	2·2107	90
95	1·6596	1·6553	1·6503	1·6438	1·6346	95
100	1·1774	1·1751	1·1723	1·1687	1·1641	100
	55	**60**	**65**	**70**	**75**	
55	9·0247					55
60	8·1222	7·4081				60
65	7·0635	6·5339	5·8558			65
70	5·9182	5·5499	5·0574	4·4494		70
75	4·7877	4·5454	4·2085	3·7728	3·2647	75
80	3·7593	3·6065	3·3868	3·0904	2 7287	80
85	2·8831	2·7892	2·6507	2·4568	2·2102	85
90	2·1718	2·1149	2·0293	1·9057	1·7427	90
95	1·6116	1·5774	1·5253	1·4480	1·3428	95
100	1·1518	1·1331	1·1042	1·0600	0·9976	100
	80	**85**	**90**	**95**	**100**	
80	2·3294					80
85	1·9259	1·6259				85
90	1·5473	1·3326	1·1140			90
95	1·2121	1 0630	0·9053	0·7492		95
100	0·9166	0·8198	0·7121	0·6014	0·5033	100

Annuities to cease on the Death of the LAST *of Two Lives.* 3 *per cent.*

AGES	5	10	15	20	25	AGES
5	27.2747					5
10	27·0011	26·6941				10
15	26·6338	26·2921	25·8338			15
20	26·2845	25·9105	25·3952	24·9377		20
25	25·9654	25·5621	24·9927	24·3338	23·9098	25
30	25·6519	25·2334	24·6007	23·9810	23·3817	30
35	25·3455	24·8967	24·2223	23·5435	22·8890	35
40	25·0496	24·5860	23·8624	23·1274	22·4115	40
45	24·7688	24·2962	23·5260	22·7393	21·9681	45
50	24·5079	24·0321	23·2183	22·3844	21·5654	50
55	24·2737	23·9657	22·9362	22·0702	21·2113	55
60	24·0682	23·6032	22·7123	21·7989	20·9073	60
65	23·8957	23·3358	22·5220	21·5760	20·6606	65
70	23·7592	23·3274	22·3395	21·4050	20·4701	70
75	23·6571	23·2458	22·2763	21·2824	20·3348	75
80	23·5843	23·1928	22·2095	21·1999	20·2439	80
85	23·5341	23·1695	22·1677	21·1467	20·1854	85
90	23·4958	25·1396	22·1427	21·1139	20·1493	90
95	23·4806	23·1295	22·1288	21·0957	20·1291	95
100	23·5419	23·1504	22·1393	21·1112	20·0889	100

	30	35	40	45	50	
30	22·8002					30
35	22·2196	21·5511				35
40	21·6652	20·9041	20·1525			40
45	21·1402	20·2983	19·4375	18·5997		45
50	20·6843	19·7491	18·7823	17·8173	16·8926	50
55	20·2771	19·2703	18·2085	17·1225	15·8271	55
60	19·9305	18·8648	17·7226	16·0498	15·1562	60
65	19·6495	18·5381	17·3324	17·0518	14·6131	65
70	19·4362	18·2916	17·0392	15·6950	14·1049	70
75	19·2846	18·1176	16·8337	15·4443	13·9274	75
80	19·1833	18·0007	16·6975	15·2788	13·7298	80
85	19·1180	17·9279	16·6109	15·1741	13·6104	85
90	19·0783	17·8825	16·5579	15·1091	13·5374	90
95	19·0557	17·9367	16·5275	15·0731	13·4950	95
100	19·0681	17·8670	16·5347	15·0759	13·4942	100

	55	60	65	70	75	
55	15·0534					55
60	14·1627	13·0940				60
65	13·4222	12·1770	11·0733			65
70	12·8555	11·4587	10·1764	9·0977		70
75	12·3532	11·9395	9·5090	8·2671	7·2885	75
80	12·1884	10·5904	9·0486	7·6755	6·5747	80
85	12·0199	10·3669	8·8275	7·2792	6·0473	85
90	11·9168	10·2303	8·5614	7·0254	5·7155	90
95	11·8560	10·1475	8·4478	6·8678	5·5039	95
100	11·8428	10·1211	8·3882	6·7870	5·3832	100

	80	85	90	95	100	
80	6·6963					80
85	5·0874	4.3840				85
90	4·6540	3·8931	3.3344			90
95	4·4028	3·5620	2·9482	2·5149		95
100	4·2365	3·2655	2·6889	2·2164	1·8977	100

TABLE XV.—ENGLISH LIFE TABLE, No. 3. *Females.*

Annuities to cease on the Death of the LAST *of Two Lives.* *3 per cent.*

OLDER AGES	5	10	15	20	25	OLDER AGES
5	27·3967					5
10	27·1341	26·8410				10
15	26·7827	26·4576	26·0230			15
20	26·4480	26·0934	25·6073	25·1366		20
25	26·1415	25·7604	25·2263	24·7021	24·2118	25
30	25·8385	25·4348	24·8543	24·2766	23·7278	30
35	25·5374	25·1148	24·4900	23·8607	23·2540	35
40	25·2389	24·8012	24·1337	23·4561	22·7959	40
45	24·9464	24·4973	23·7879	23·0652	22·3571	45
50	24·6652	24·2089	23·4576	22·6921	21·9424	50
55	24·4021	23·9446	23·1517	22·3449	21·5596	55
60	24·1753	23·7218	22·8905	27·0464	21·2319	60
65	23·9860	23·5422	22·6766	21·7991	20·9608	65
70	23·8357	23·4066	22·5124	21·6064	20·7495	70
75	23·7230	23·3114	22·3952	21·4663	20·5957	75
80	23·6424	23·2485	22·3169	21·3706	20·4905	80
85	23·5869	23·2090	22·2674	21·3086	20·4221	85
90	23·5503	23·1853	22·2378	21·2704	20·3778	90
95	23·5287	23·1735	22·2228	21·2494	20·3543	95
100	23·5467	23·1993	22·2464	21·2688	20·3739	100
	30	85	40	45	50	
30	23·1778					30
35	22·6345	22·0116				35
40	22·1077	21·4003	20·6918			40
45	21·6060	20·8161	20·0016	19·1987		45
50	21·1368	20·2724	19·3612	18·4296	17·5139	50
55	20·7087	19·7816	18·7818	17·7294	16·6609	55
60	20·3457	19·3704	18·3007	17·1468	15·9386	60
65	20·0476	19·0368	17·9153	16·6828	15·3580	65
70	19·8161	18·7801	17·6228	16·3347	14·8317	70
75	19·6478	18·5945	17·4138	16·0895	14·6161	75
80	19·5327	18·4680	17·2726	15·9262	14·4146	80
85	19·4580	18·3860	17·1814	15·8221	14·2879	85
90	19·4116	18·3351	17·1250	15·7582	14·2112	90
95	19·3856	18·3064	17.0928	15·7216	14·1674	95
100	19·4015	18·3203	17·1045	15·7304	14·1716	100
	55	60	65	70	75	
55	15·6311					55
60	14·7331	13·6467				60
65	13·9928	12·7219	11·6010			65
70	13·4259	11·9937	10·6872	9·5830		70
75	13·0234	11·4652	10·0031	8·7266	7·7017	75
80	12·7558	11·1081	9·5288	8·1130	6·9417	80
85	12·5867	10·8801	9·2196	7·7013	6·4149	85
90	12·4838	10·7402	9·0268	7·4382	6·0682	90
95	12·4241	10·6578	8·9109	7·2760	5·8482	95
100	12·4176	10·6358	8·8657	7·1977	5·7271	100
	80	85	90	95	100	
80	6·0450					80
85	5·4032	4·6579				85
90	4·9676	4·1370	3·5414			90
95	4·6829	3·7867	3.1302	3·6664		95
100	4·5121	3·5636	2·8571	2·3479	1·9797	100

TABLE XVI.—ENGLISH LIFE TABLE, No. 3. *Males.*

For calculating Temporary and Deferred Annuities. 3 *per cent.*

AGE	I. λc_x	II. $\lambda c_x + \lambda v$	III. $\lambda c_x + \lambda v (109-x)$	AGE	I. λc_x	II. $\lambda c_x + \lambda v$	III. $\lambda c_x + \lambda v (109-x$
0	$\bar{1}$·9224164	$\bar{1}$·9095792	2·0934038	55	$\bar{1}$·9892057	$\bar{1}$·9763684	3·1872415
1	·9711377	·9583005	·1838246	56	·9886306	·9757934	·2108731
2	·9843053	·9714680	·2255241	57	·9879971	·9751599	·2350797
3	·9895169	·9766797	·2540561	58	·9872950	·9744578	·2599198
4	·9921913	·9793541	·2773764	59	·9865139	·9736766	·2854620
5	$\bar{1}$·9940574	$\bar{1}$·9812201	2·2980223	60	$\bar{1}$·9856433	$\bar{1}$·9728061	3·3117854
6	·9952751	·9824379	·3168022	61	·9846731	·9718358	·3389793
7	·9960044	·9831672	·3343653	62	·9835929	·9707557	·3671435
8	·9966712	·9838340	·3511981	63	·9823924	·9695552	·3963878
9	·9971834	·9843461	·3673641	64	·9810612	·9682240	·4268326
10	$\bar{1}$·9975541	$\bar{1}$·9847169	2·3830180	65	$\bar{1}$·9795890	$\bar{1}$·9667518	3·4586086
11	·9977966	·9849594	·3983011	66	·9779656	·9651283	·4918568
12	·9979241	·9850869	·4133417	67	·9761804	·9633432	·5267285
13	·9979498	·9851125	·4282548	68	·9742244	·9613872	·5633853
14	·9978867	·9850496	·4431423	69	·9720840	·9592467	·6019981
15	$\bar{1}$·9977484	$\bar{1}$·9849111	2·4580927	70	$\bar{1}$·9697520	$\bar{1}$·9569148	3·6427514
16	·9975478	·9847106	·4731816	71	·9672170	·9543798	·6858366
17	·9972982	·9844609	·4884710	72	·9644689	·9516317	·7314568
18	·9970127	·9841756	·5040101	73	·9614970	·9486598	·7798251
19	·9967048	·9838675	·5190345	74	·9582913	·9454540	·8311653
20	$\bar{1}$·9963874	$\bar{1}$·9835502	2·5359670	75	$\bar{1}$·9548413	$\bar{1}$·9420041	3·8857113
21	·9963071	·9834698	·5524168	76	·9511367	·9382995	·9437072
22	·9962293	·9833921	·5689470	77	·9471672	·9343300	4·0054077
23	·9961534	·9833162	·5855549	78	·9429225	·9300852	·0710777
24	·9960785	·9832413	·6022387	79	·9383922	·9255550	·1409925
25	$\bar{1}$·9960037	$\bar{1}$·9831664	2·6189974	80	$\bar{1}$·9335660	$\bar{1}$·9207288	4·2154375
26	·9959282	·9830910	·6358310	81	·9284336	·9155963	·2947087
27	·9958512	·9830140	·6527400	82	·9229846	·9101474	·3791119
28	·9957718	·9829346	·6697260	83	·9172088	·9043716	·4689645
29	·9956892	·9828520	·6867914	84	·9110958	·8982586	·5645929
30	$\bar{1}$·9956026	$\bar{1}$·9827653	2·7039394	85	$\bar{1}$·9046352	$\bar{1}$·8917980	4·6663343
31	·9955111	·9826739	·7211741	86	·8978169	·8849796	·7745363
32	·9954139	·9825767	·7385002	87	·8906302	·8777930	·8895567
33	·9953102	·9824730	·7559235	88	·8830651	·8702280	5·0117637
34	·9951991	·9823619	·7734505	89	·8751113	·8622739	·1415357
35	$\bar{1}$·9950798	$\bar{1}$·9822425	2·7910886	90	$\bar{1}$·8667581	$\bar{1}$·8539209	5·2792618
36	·9949514	·9821142	·8088461	91	·8579955	·8451583	·4253409
37	·9948132	·9819760	·8267319	92	·8488131	·8359759	·5801826
38	·9946643	·9818270	·8447559	93	·8392005	·8263633	·7442067
39	·9945038	·9816666	·8629289	94	·8291475	·8163103	·9178434
40	$\bar{1}$·9943309	$\bar{1}$·9814936	2·8812623	95	$\bar{1}$·8186437	$\bar{1}$·8058064	6·1015331
41	·9941448	.9813076	·8997687	96	·8076787	·7948415	·2957267
42	·9939446	·9811075	·9184611	97	·7962424	·7834051	·5008852
43	·9937297	·9808924	·9373536	98	·7843241	·7714869	·7174801
44	·9934989	·9806617	·9564612	99	·7719138	·7590767	·9459932
45	$\bar{1}$·9932516	$\bar{1}$·9804144	2.9757995	100	$\bar{1}$·7590011	$\bar{1}$·7461638	7·1869165
46	·9929870	·9801497	·9953851	101	·7455756	·7327384	·4407527
47	·9927040	·9798668	3·0152354	102	·7316270	·7187898	·7080143
48	·9924020	·9795648	·0353686	103	·7171451	·7043080	·9892245
49	·9920802	·9792430	·0558038	104	·7021192	·6892822	8·2849165
50	$\bar{1}$·9917376	$\bar{1}$·9789004	3·0765608	105	$\bar{1}$·6865395	$\bar{1}$·6737022	·5956343
51	·9911290	·9782917	·0976604	106	·6703952	·6575580	·9219321
52	·9906840	·9778468	·1193687	107	·6536762	·6408391	9·2643741
53	·9902221	·9773849	·1415219	108	·6363723	·6235350	·6235350
54	·9897327	·9768955	·1641370				

TABLE XVI.—ENGLISH LIFE TABLE, No. 3. *Females.*

For calculating Temporary and Deferred Annuities. 3 per cent.

AGE.	I. λc_x	II. $\lambda c_x + \lambda v$	III. $\lambda c_x + \lambda v$ (109-x)
0	$\bar{1}$·9371604	$\bar{1}$·9243232	2·4989220
1	·9722410	·9594038	·5745988
2	·9843554	·9715182	·6151950
3	·9893710	·9765338	·6436768
4	·9922523	·9794151	·6671430
5	$\bar{1}$·9941950	$\bar{1}$·9813578	2·6877279
6	·9953928	·9825556	·7063701
7	·9960392	·9832020	·7238145
8	·9966496	·9838124	·7406125
9	·9971161	·9842789	.7568001
10	$\bar{1}$·9974514	$\bar{1}$·9846142	2·7725212
11	·9976674	·9848302	·7879070
12	·9977764	·9849392	·8030768
13	·9977906	·9849534	·8181376
14	·9977224	·9848852	·8331842
15	$\bar{1}$·9975839	$\bar{1}$·9847467	2·8482990
16	·9973872	·9845500	·8635523
17	·9971448	·9843076	·8790023
18	·9968687	·9840315	·8946947
19	·9965712	·9837340	·9106632
20	$\bar{1}$·9962646	$\bar{1}$·9834274	2·9269292
21	·9961671	·9833299	·9435018
22	·9960729	·9832357	·9601719
23	·9959817	·9831445	·9769362
24	·9958929	·9830557	·9937917
25	$\bar{1}$·9958060	$\bar{1}$·9829688	3·0107360
26	·9957206	·9828834	·0277672
27	·9956362	·9827990	·0448838
28	·9955525	·9827153	·0620848
29	·9954688	·9826316	·0793695
30	$\bar{1}$·9953847	$\bar{1}$·9825475	3·0967379
31	·9952998	·9824626	·1141904
32	·9952135	·9823763	·1317278
33	·9951255	·9822883	·1493515
34	·9950352	·9821980	·1670632
35	$\bar{1}$·9949422	$\bar{1}$·9821050	3·1848652
36	·9948460	·9820088	·2027602
37	·9947461	·9819089	·2207514
38	·9945791	·9817419	·2388425
39	·9945966	·9817594	·2571006
40	$\bar{1}$·9944199	$\bar{1}$·9815827	3·2753412
41	·9943007	·9814635	·2937585
42	·9941755	·9813383	·3122950
43	·9940439	·9812067	·3309567
44	·9939053	·9810681	·3497500
45	$\bar{1}$·9937593	$\bar{1}$·9809221	3·3686819
46	·9936054	·9807682	·3877598
47	·9934432	·9806060	·4069916
48	·9932722	·9804350	·4263856
49	·9930918	·9802546	·4459506
50	$\bar{1}$·9929018	$\bar{1}$·9800646	3·4656960
51	·9927015	·9798643	·4856314
52	·9924906	·9796534	·5057671
53	·9922685	·9794313	·5261137
54	·9913715	·9785343	·5466824
55	$\bar{1}$·9907919	$\bar{1}$·9779547	3·5681481
56	·9901864	·9773492	·5901934
57	·9895440	·9767068	·6128442
58	·9888536	·9760164	·6361374
59	·9881047	·9752675	·6601210
60	$\bar{1}$·9872859	$\bar{1}$·9744487	3·6848535
61	·9863865	·9735493	·7104048
62	·9853955	·9725583	·7368555
63	·9843019	·9714647	·7642972
64	·9830950	·9702578	·7928325
65	$\bar{1}$·9817635	$\bar{1}$·9689263	3·8225747
66	·9802967	·9674595	·8536484
67	·9786837	·9658465	·8861889
68	·9769134	·9640762	·9203424
69	·9749749	·9621377	·9562662
70	$\bar{1}$·9728573	$\bar{1}$·9600201	3·9941285
71	·9705496	·9577124	4·0341084
72	·9680410	·9552038	·0763960
73	·9653205	·9524833	.1211922
74	·9623770	·9495398	·1687089
75	$\bar{1}$·9591998	$\bar{1}$·9463626	4·2191691
76	·9557777	·9429405	·2728065
77	·9521001	·9392629	·3298660
78	·9481557	·9353185	·3906031
79	·9439338	·9310966	·4552846
80	$\bar{1}$·9394234	$\bar{1}$·9265862	4·5241880
81	·9346135	·9217763	·5976018
82	·9294933	·9166561	·6758255
83	·9240517	·9112145	·7591694
84	·9182778	·9054406	·8479549
85	$\bar{1}$·9121607	$\bar{1}$·8993235	4·9425143
86	·9063896	·8935524	5·0431908
87	·8981532	·8853160	·1496384
88	·8916408	·8788036	·2643224
89	·8840415	·8712043	·3855188
90	$\bar{1}$·8760443	$\bar{1}$·8632071	5·5143145
91	·8676383	·8548011	·6511074
92	·8588123	·8459751	·7963063
93	·8495557	·8367185	·9503312
94	·8398575	·8270203	6·1136127
95	$\bar{1}$·8297066	$\bar{1}$·8168694	6·2865924
96	·8190921	·8062549	·4697230
97	·8080033	·7951661	·6634681
98	·7964288	·7835916	·8683020
99	·7843582	·7715210	7·0847104
100	$\bar{1}$·7717801	$\bar{1}$·7589429	·3131894
101	·7586838	·7458466	·5542465
102	·7450583	·7322211	·8083999
103	·7308927	·7180555	8·0761788
104	·7161761	·7033389	·3581233
105	$\bar{1}$·7008974	$\bar{1}$·6880602	·6547844
106	·6850457	·6722085	·9667242
107	·6686102	·6557730	9·2945157
108	·6515799	·6387427	·6387427

Showing the Age at which an ordinary Assurance may become payable during life-time in consideration of Bonuses. (See Introduction.)

Age at Valuation.	AGE AT ENTRY.										Age at Valuation.
	20	21	22	23	24	25	26	27	28	29	
21	36922										21
22	35543	35208									22
23	34209	33887	33571								23
24	32920	32610	32306	31980							24
25	31674	31375	31083	30769	30462						25
26	30470	30183	29901	29600	29304	28991					26
27	29306	29030	28759	28470	28185	27884	27588				27
28	28182	27916	27656	27377	27104	26814	26530	26201			28
29	27096	26841	26591	26323	26060	25781	25508	25220	24940		29
30	26047	25802	25561	25303	25051	24783	24520	24244	23974	23691	30
31	25033	24798	24566	24319	24076	23818	23566	23300	23041	22769	31
32	24055	23828	23606	23368	23134	22887	22644	22389	22140	21879	32
33	23109	22891	22678	22449	22205	21987	21754	21509	21270	21018	33
34	22195	21986	21781	21561	21346	21117	20893	20658	20428	20187	34
35	21311	21111	20914	20703	20496	20277	20062	19836	19615	19383	35
36	20457	20265	20076	19873	19675	19464	19258	19041	18829	18607	36
37	19632	19447	19266	19072	18881	18679	18481	18273	18070	17856	37
38	18834	18656	18482	18296	18113	17920	17730	17530	17335	17130	38
39	18062	17892	17725	17546	17371	17185	17003	16812	16625	16428	39
40	17316	17153	16993	16821	16653	16475	16300	16117	15937	15749	40
41	16593	16437	16284	16119	15958	15788	15620	15444	15272	15092	41
42	15894	15745	15598	15440	15286	15123	14962	14794	14629	14456	42
43	15217	15074	14933	14783	14635	14479	14325	14164	14006	13841	43
44	14563	11425	14291	14147	14006	13856	13709	13554	13404	13245	44
45	13928	13797	13668	13531	13396	13252	13112	12964	12820	12668	45
46	13314	13189	13066	12934	12805	12668	12533	12392	12254	12110	46
47	12719	12599	12482	12356	12233	12102	11973	11839	11707	11569	47
48	12143	12029	11917	11796	11679	11554	11431	11302	11173	11045	48
49	11585	11476	11369	11254	11142	11023	10906	10783	10663	10537	49
50	11044	10940	10838	10729	10622	10508	10397	10280	10165	10045	50
51	10521	10428	10324	10220	10118	10010	9903·8	9792·2	9683·2	9568·9	51
52	10008	9913·3	9820·9	9721·9	9624·8	9521·8	9420·8	9314·7	9211·1	9102·3	52
53	9510·0	9420·4	9332·5	9238·5	9146·2	9048·4	8952·4	8851·6	8753·1	8649·7	53
54	9027·3	8942·3	8858·9	8769·6	8682·0	8589·1	8498·1	8402·3	8308·8	8210·7	54
55	8559·7	8479·1	8400·0	8315·3	8232·3	8144·2	8057·9	7967·1	7878·4	7785·4	55
56	8106·4	8030·1	7955·2	7875·0	7796·3	7712·9	7631·2	7545·2	7461·2	7373·1	56
57	7666·9	7594·9	7523·8	7448·0	7373·6	7294·7	7217·4	7136·1	7056·7	6973·3	57
58	7240·7	7172·5	7105·6	7034·0	6963·7	6889·2	6816·2	6739·4	6664·4	6585·7	58
59	6827·2	6762·9	6699·8	6632·2	6566·0	6495·8	6426·9	6354·5	6283·8	6209·5	59
60	6425·5	6365·0	6305·7	6242·1	6179·7	6113·6	6048·5	5980·7	5914·1	5844·3	60
61	6035·6	5978·7	5923·0	5999·8	5804·7	5742·6	5681·7	5617·7	5555·2	5489·6	61
62	5656·7	5603·4	5551·0	5495·2	5440·3	5382·1	5325·0	5265·0	5206·4	5144·9	62
63	5288·2	5238·4	5189·6	5137·2	5085·9	5031·5	4978·2	4922·1	4867·3	4809·8	63
64	4930·3	4883·8	4838·3	4789·5	4741·7	4690·9	4641·2	4588·9	4537·8	4484·3	64
65	4582·4	4539·2	4496·9	4451·5	4407·1	4359·9	4313·7	4265·1	4217·6	4167·8	65
66	4244·6	4204·7	4165·4	4123·4	4082·3	4038·6	3995·8	3950·8	3906·8	3860·6	66
67	3917·1	3880·3	3844·1	3805·3	3767·3	3727·0	3687·5	3645·9	3605·4	3562·8	66
68	3600·1	3566·2	3532·9	3497·3	3462·3	3425·3	3389·0	3350·8	3313·5	3274·4	68
69	3293·7	3262·7	3232·3	3199·7	3167.7	3133·9	3100·6	3065·7	3031·6	2995·8	69
70	2998·8	2970·6	2942·9	2913·2	2884·1	2853.3	2823·0	2791·2	2750·1	2727·5	70

TABLE XVII.—ENGLISH LIFE TABLE, No. 3. *Males.*

Showing the Age at which an ordinary Assurance may become payable during life-time in consideration of Bonuses. (See Introduction.)

Age at Valuation.	AGE AT ENTRY.										Age at Valuation.
	30	31	32	33	34	35	36	37	38	39	
31	22485										31
32	21606	21341									32
33	20757	20502	20238								33
34	19936	19691	19437	19176							34
35	19142	18907	18664	18413	18155						35
36	18375	18150	17916	17675	17427	17174					36
37	17634	17418	17193	16962	16724	16482	16246				37
38	16917	16709	16494	16272	16044	15811	15585	15343			38
39	16224	16025	15818	15606	15387	15164	14947	14715	14480		39
40	15553	15362	15165	14961	14751	14537	14329	14107	13882	13654	40
41	14904	14721	14532	14336	14135	13930	13731	13518	13302	13085	41
42	14276	14101	13920	13733	13540	13344	13152	12949	12742	12533	42
43	13668	13501	13327	13148	12963	12775	12592	12397	12199	12000	43
44	13080	12920	12754	12582	12407	12226	12051	11864	11675	11483	44
45	12511	12357	12198	12034	11865	11693	11526	11347	11166	10983	45
46	11959	11812	11660	11503	11342	11177	11017	10847	10674	10499	46
47	11425	11284	11139	10989	10835	10678	10525	10362	10197	10030	47
48	10907	10773	10635	10492	10345	10194	10048	9892·6	9734·9	9575·5	48
49	10406	10278	10146	10009	9868·9	9725·7	9586·5	9437·8	9287·3	9135·2	49
50	9920·2	9798·5	9672·3	9542·2	9408·5	9271·9	9139·2	8997·5	8854·0	8709·0	50
51	9442·7	9333·8	9213·6	9089·7	8962·3	8832·2	8705·8	8570·8	8434·1	6296·0	51
52	8989·0	8878·7	8764·4	8646·5	8525·3	8401·6	8281·3	8152·9	8022·9	7891·5	52
53	8542·0	8437·2	8328·6	8916·6	8101·4	7983·8	7869·6	7747·5	7623·9	7499·1	53
54	8108·5	8009·0	7905·9	7799·6	7690·2	7578·6	7470·2	7354·3	7237·0	7118·4	54
55	7688·5	7594·2	7496·4	7398·5	7291·9	7186·0	7083·2	6973·3	6862·1	6749·8	55
56	7281·3	7192·0	7099·4	7003·9	6905·7	6805·5	6708·1	6604·0	6498·8	6392·3	56
57	6886·5	6802·1	6714·4	6624·1	6531·3	6436·5	6344·4	6246·0	6146·4	6045·7	57
58	6503·7	6423·9	6341·2	6255·9	6168·2	6078·7	5991·7	5898·8	5804·7	5709·7	58
59	6132·3	6057·0	5979·0	5898·6	5815·9	5731·5	5649·5	5561·9	5473·2	5383·6	59
60	5771·5	5700·7	5627·3	5551·6	5473·8	5394·4	5317·2	5234·7	5151·2	5066·9	60
61	5421·3	5354·8	5285·8	5214·7	5141·6	5067·0	4994·5	4917·0	4838·6	4759·4	61
62	5080·9	5018·6	4953·9	4887·3	4818·8	4748·9	4680·9	4608·3	4534·8	4460·6	62
63	4750·0	4691·7	4631·3	4569·0	4505·0	4439·6	4376·0	4308·1	4239.5	4160·0	63
64	4428·4	4374·1	4317·8	4259·7	4200·0	4139.0	4079·8	4016·5	3952·5	3887·8	64
65	4116·0	4065·5	4013·1	3959·1	3903·6	3847·0	3791·9	3733·1	3673·6	3613·4	65
66	3812·6	3765·8	3717·3	3667·3	3615·9	3563·4	3512·4	3458·0	3402·8	3347·1	66
67	3518·4	3475·3	3430·5	3384·4	3337·0	3288·5	3241·5	3191·2	3140·3	3088·9	67
68	3233·6	3194·0	3152·8	3110·4	3066·8	3022·3	2979·1	2932·9	2886·1	2838·8	68
69	2958·5	2922·2	2884·6	2845·8	2805·9	2765·2	2725·6	2683·3	2640·5	2597·3	69
70	2693·6	2660·5	2626·3	2591·0	2554·6	2517·6	2481·5	2443·0	2404·1	2364·7	70

Showing the Age at which an ordinary Assurance may become payable during life time in consideration of Bonuses. (See Introduction.)

Age at Valuation.	AGE AT ENTRY.										Age at Valuation.
	40	41	42	43	44	45	46	47	48	49	
41	12865										41
42	12323	12112									42
43	11799	11596	11386								43
44	11291	11097	10896	10695							44
45	10799	10614	10422	10230	11038						45
46	10323	10146	9962·1	9778·5	9595·6	9402·2					46
47	9861·7	9692·6	9738·7	9341·6	9166·8	8982·1	8799·3				47
48	9415·0	9253·6	9085·9	8918·5	8751·7	8575·3	8400·8	8223·6			48
49	8982·1	8828·2	8668·2	8508·4	8349·3	8181·1	8014·6	7845·5	7670·3		49
50	8563·1	8416·3	8263·8	8111·5	7959·8	7799·4	7640·6	7479·5	7312·4	7148·6	50
51	8157·0	8017·2	7871·9	7726·8	7582·3	7429·4	7278·3	7124·8	6965·6	6809·6	51
52	7759·3	7626·2	7488·1	7350·0	7212·6	7067·2	6923·4	6777·8	6626·0	6477·5	52
53	7373·4	7247·0	7115·7	6984·6	6853·9	6715·8	6579·2	6440·4	6296·5	6155·5	53
54	6999·2	6879·2	6754·6	6630·1	6506·1	6375·0	6245·3	6113·5	5977·0	5843·1	54
55	6636·7	6522·9	6404·7	6287·0	6169·1	6044·8	5921·7	5796·8	5667·3	5540·4	55
56	6285·2	6177·5	6065·5	5953·7	5842·4	5724·7	5608·2	5489·8	5367·2	5247·0	56
57	5944·4	5842·5	5736·7	5630·9	5525·6	5414·3	5304·1	5192·2	5076·2	4962·5	57
58	5614·0	5517·7	5417·8	5317·9	5218·4	5113·3	5009·2	4903·5	4794·0	4686·6	58
59	5292·1	5202·6	5108·3	5014·2	4920·4	4821·3	4723·1	4623.5	4520·2	4419·0	59
60	4982·0	4896·5	4807·8	4719·2	4630·9	4537·6	4445·3	4351·5	4254·3	4159·0	60
61	4679·6	4599·4	4516·1	4432·8	4349·9	4262·3	4175.5	4087·4	3996·1	3906·6	61
62	4385·8	4310·6	4232·5	4154·8	4076·8	3994·7	3913·4	3830·8	3745·2	3661·3	62
63	4100·2	4029·9	3956·9	3883·9	3811·3	3734·5	3658·5	3581·3	3501·3	3422·9	63
64	3822·6	3757·1	3689·0	3621·0	3553·3	3481·7	3410·8	3338·9	3264·3	3191·2	64
65	3552·9	3492·0	3428·7	3365·5	3302·6	3236·0	3170·2	3103·3	3034·0	2979·7	65
66	3291·0	3234·6	3176·0	3117·5	3059·1	2997·5	2936·5	2874·6	2810·3	2747·4	66
67	3037·1	2985·0	2931·0	2876·9	2823·1	2766·2	2709·9	2652·8	2593·5	2535·4	67
68	2791·3	2743·4	2693·7	2644·1	2594·6	2542·3	2490·6	2438·0	2383·6	2330·2	68
69	2553·8	2510·0	2464·5	2419·1	2373·8	2326·0	2278·7	2230·6	2180·8	2131·9	69
70	2325·1	2285·2	2243·8	2202·5	2161·3	2117·7	2074·6	2030·9	1985·5	1941·0	70

	50	51	52	53	54	55	56	57	58	59	
51	6646·1										51
52	6322·1	6166·8									52
53	6007·7	5860·2	5711·1								53
54	5702·8	5562·8	5421·3	5275·9							54
55	5407·4	5274·6	5140·4	5002·6	4865·5						55
56	5121·1	4995·3	4868·2	4737·7	4607·9	4473·1					56
57	4843·4	4724·4	4604·3	4480·8	4358·0	4230·6	4101·4				57
58	4574·1	4461·8	4348·3	4231·7	4115·8	3995·4	3873·4	3750·2			58
59	4312·9	4207·0	4100·0	3990·1	3880·7	3767·2	3652·2	3536·0	3422·6		59
60	4059·2	3959·5	3858·8	3755·3	3652·4	3545·6	3437·3	3328·0	3221·2	3115·8	60
61	3812·9	3719·2	3624·6	3527·4	3430·8	3330·4	3228·7	3126·0	3025·7	2926·7	61
62	3573·5	3485·7	3397·0	3306·0	3215·4	3118·5	3026·0	2929·7	2835·8	2743·0	62
63	3340·7	3258·7	3175·8	3090·7	3005·9	2918·0	2828·9	2738·9	2651·1	2564·3	63
64	3114·6	3038·1	2960·8	2881·4	2802·5	2720·5	2637·4	2553·5	2471·6	2390·7	64
65	2894·8	2823·7	2751·9	2678·1	2604·7	2528·5	2451·3	2373·3	2297·2	2222·0	65
66	2681·5	2615·6	2549·1	2480·7	2413·7	2342·2	2270·6	2198·4	2127·9	2058·3	66
67	2474·6	2413·8	2352·4	2289·3	2226·6	2161·5	2095·5	2028·8	1963·7	1899·5	67
68	2274·3	2218·4	2162·0	2104·0	2046·3	1986·5	1925·8	1864·6	1804·8	1745·7	68
69	2080·7	2029·6	1978·0	1925·0	1872·2	1817·5	1762·0	1705·9	1651·2	1597·2	69
70	1894·4	1847·9	1800·9	1752·6	1704·6	1654·7	1604·2	1553·2	1503·3	1454·2	70

END.

Fig. 1.

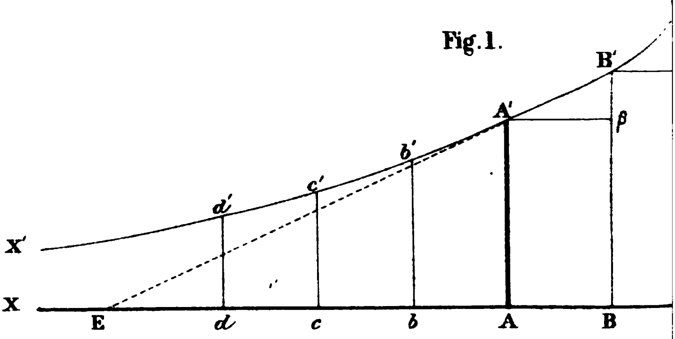
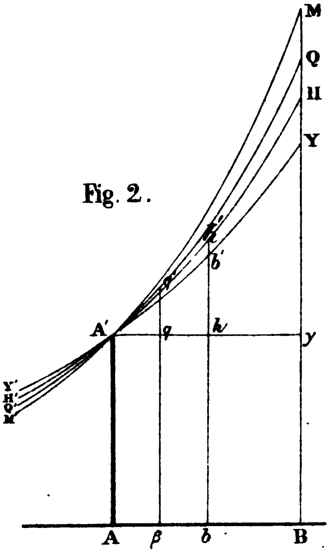
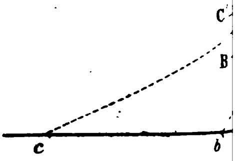
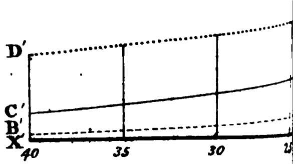


Fig. 2.







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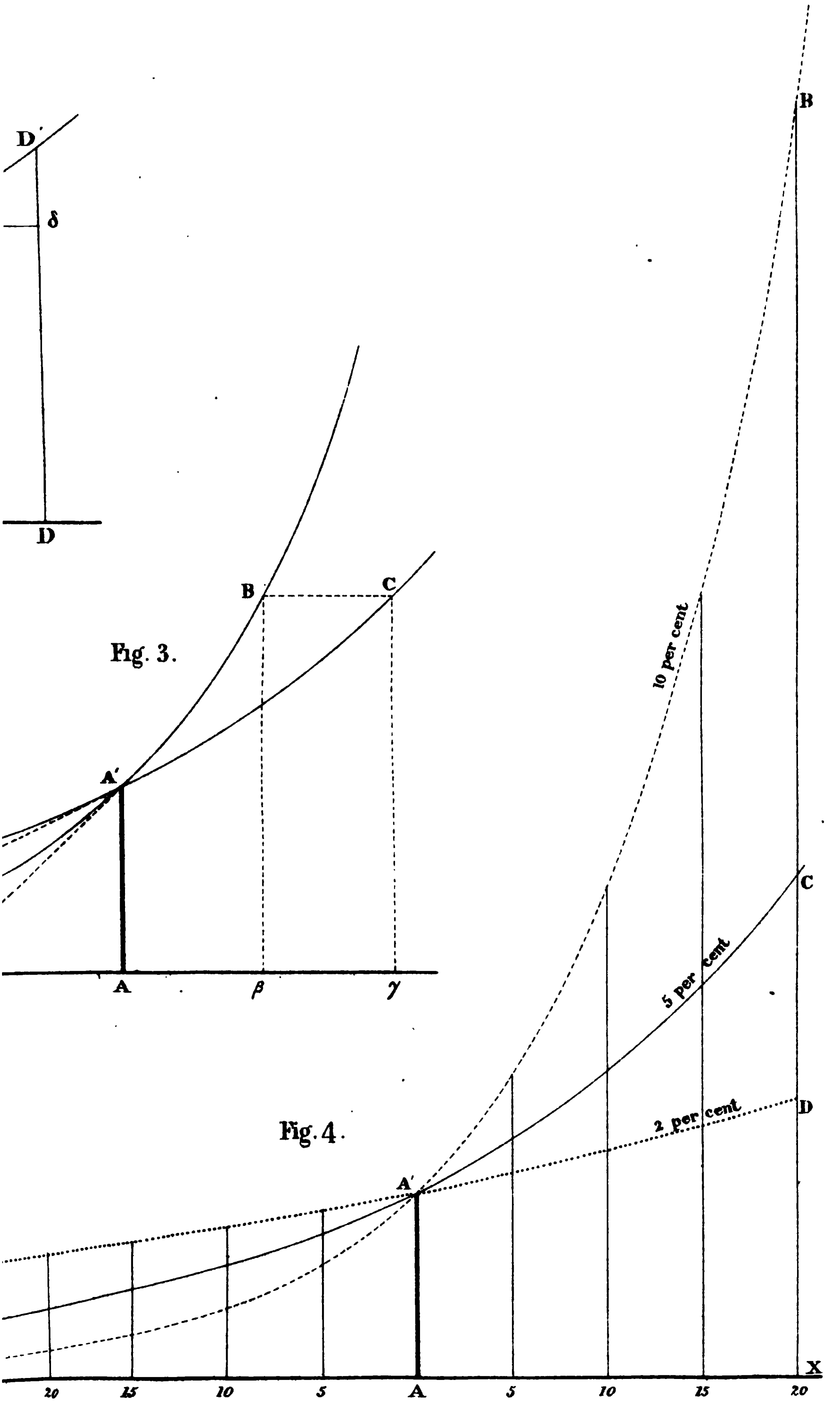

Fig. 3.

Fig. 4.

; Doctrine of Interest & Annuities

KEY TO PREMIUMS

FOR

LIFE ASSURANCE,

AS CHARGED BY ALL

Life Assurance Companies in Great Britain,

ESTABLISHED PREVIOUS TO 1865,

FOR

ALL AGES BETWEEN 20 AND 60,

To insure £100, payable at Death,

EITHER WITH PROFITS OR WITHOUT PROFITS.

ERRATA IN "KEY TO PREMIUMS."

Office 37, established in 1832.
„ 79, „ 1808.
„ 110, „ 1840.
Page 10, line 1, for 97, read 98.
„ 11, „ 3 „ 73, „ 74.
„ 25, „ 49 „ 38, „ 95.
37, Office 86, for £6, read £5.

KEY TO PREMIUMS

FOR

LIFE ASSURANCE,

AS CHARGED BY ALL

LIFE ASSURANCE COMPANIES IN GREAT BRITAIN,

FOR

ALL AGES BETWEEN 20 AND 60,

TO INSURE £100, PAYABLE AT DEATH,

EITHER WITH PROFITS OR WITHOUT PROFITS.

◆

PUBLISHED BY
DEAN & SON, 65, LUDGATE HILL, LONDON.
1865.

M'CORQUODALE AND CO., PRINTERS, LONDON—
WORKS, NEWTON.

USE AND APPLICATION OF THE TABLES.

The following Tables are divided into two parts. The one contains the Annual Premiums for an Assurance of £100 payable at death WITH PROFITS, and the second for a similar Assurance WITHOUT PROFITS, for all Ages from 21 to 60 inclusive, as charged by all Offices of Great Britain respectively, both Tables being headed accordingly.

In page 5 is given the list of all Life Offices, tabulated alphabetically, and numbered from 1 to 120. These numbers refer to the Table all through. The Premiums are equally tabulated from the lowest to the highest, each of which is accompanied, on its left, by a number referring to the list of Offices in page 5. The arrangement is so simple that an explanation is scarcely required. For instance, in column headed "Age 25" of Table "WITH PROFITS," it will be observed that the lowest Premium chargeable for the Assurance of £100 is £1 18s. 0d., and the highest £2 13s. 6d., thus making a difference of 15s. 6d. per cent.; the former is charged by Office 98, which (according to page 5) is "*The Scottish Provident Institution*,"* and the latter by Office 64, viz., "*The London Life Association*."† It will be observed that the first-named Office charges the lowest Premiums for all Ages from 25 to 47 inclusive, and continues to be *second* in Rank up to Age 54, and *third* in Rank at Age 55, etc.

The next in Rank of the Low Premium Offices is "*The General Annuity Endowment Association*."‡ It is next to No. 98 from Age 21 to 36 inclusive, *fourth* up to Age 42, and somewhat higher in the higher Ages.

"*The Economic*" § stands *second* in rank from Age 21 to 23 inclusive, *third* from Age 24 to 44, and then rises gradually, so that at Age 60 it stands *sixth* in order.

The above few examples, it is considered, will be sufficient for the use of the reader.

In the Table for "Without Profits" it will be found that the Offices most prominent for low Premiums are Nos. 5, 9, 17, 42, 119, etc., representing respectively "*The Argus*," "*The Britannia*," "*The Caledonian*," "*The General*," "*The Whittington*," *etc.*

The advantage of the present Table will be manifest both to the Public and the Actuary. The Public can have a complete knowledge of all chargeable Premiums at one view, and of course can use their own discretion as to which Office they should prefer for insuring their lives with; whilst the Actuary can discover in it many interesting items as regards the construction and nature of all Tables, on comparing them to one another. He will also find the present Tables particularly useful in cases of re-Assurances, or in framing New Tables for New Companies.

On comparing (in the With-Profit Table) the collective Premiums with one another, it appears that some Offices have adopted uniform rates; for instance, "*The Alliance*," "*The Sun*," and "*The Victoria*." They keep together from Age 21 to 45; but from Age 46 and above, the last-named office departs by reducing its rates gradually, so that at Age 60 (the extreme of this Table) it is 55 Offices below the other two, whilst the latter still agree together to the end.

* Chief Office—St. Andrew Square, Edinburgh; London Office—Gracechurch street.

† Chief Office—King William Street, City, London.

‡ Cheapside, London.

§ New Bridge Street, Blackfriars, London.

Another instance, "The Equitable," "The Hand in Hand," "The Law Life," and "The Rock." The rates of these four Companies agree closely to the end, except that No. 34—namely, "The Equitable"—differs occasionally with one penny from the others. But No. 46, viz. "The Hand in Hand," from Age 50 and onwards, branches off by raising its rates, so that at age 60 it is 63 Offices ahead of the other three.

In fact, to particularize all the points to be derived from the present Tables would cause a great deal of labour, and such that any Actuary will be best able to do it himself.

There is one point, however, that deserves attention. It will show how now-a-days certain tables are cooked by some promoters or managers of new Companies. They care nought about the rate of mortality, the rate of interest for the use of money, the rate of office expenses, or the like items, but are satisfied with the work of others, which they readily make use of without going to the expense of employing a professional man to calculate original tables for their new establishments. I refer the reader to page 38 for three instances there given, and let him judge of their merit.

The blanks observable in the odd pages show that those Companies whose numbers are to be found opposite, have not published their rates. Nos. 8 and 103 did not send in their prospectuses, and consequently are left blank altogether.

THE AUTHOR.

LIFE INSURANCE COMPANIES IN GREAT BRITAIN IN 1865.

No. of Office.	Name of Office.	Established.	No. of Office.	Name of Office.	Established.
1	Albert	1838	62	London and Southwark	1864
2	Albion	1864	63	London Assurance	1720
3	Alliance	1824	64	London Life Association	1806
4	Amicable	1706			
5	Argus	1833	65	Marine Life and Casualty	1852
6	Atlas	1808	66	Mercantile, Professional, and General	1864
7	Birmingham Alliance	1862	67	Metropolitan	1835
8	Brighton and Sussex Provident	1846	68	Midland Counties	1851
			69	Mutual	1834
9	Britannia	1837			
10	British Empire Mutual	1846	70	National Assurance Company of Ireland	1822
11	British Equitable	1854			
12	British Nation	1854	71	National	1830
13	British Mutual	1854	72	National Mercantile	1837
14	British Prudential	1848	73	National Provident	1835
15	British Union	1853	74	National Union	1863
16	Briton, Medical and General	1853	75	New National	1853
			76	North British and Mercantile	1823
17	Caledonian	1833	77	Northern	1836
18	Church of England	1840	78	Norwich Provident	1808
19	City	1862	79	Norwich Union	1835
20	City and County	1863	80	Nottingham and Derbyshire	1860
21	City of Glasgow	1838			
22	Clergy Mutual	1829	81	Patriotic	1860
23	Clerical, Medical, & General	1824	82	Pelican	1797
24	Colonial	1846	83	Provident Clerks	1840
25	Commercial Union	1862	84	Provincial	1852
26	Consolidated	1846	85	Provident	1806
27	Crown	1825	86	Queen	1857
28	Eagle	1807	87	Reliance	1840
29	Economic	1823	88	Rock	1806
30	Edinburgh	1823	89	Royal	1845
31	Emperor	1853	90	Royal Exchange	1720
32	English and Scottish	1839	91	Royal Farmers	1840
33	Equity and Law	1844	92	Royal Naval, Military, &c.	1837
34	Equitable Society	1762			
35	European	1853	93	Sceptre	1864
			94	Scottish Amicable	1826
36	Financial	1864	95	Scottish Equitable	1831
37	Friends' Provident Institution	1862	96	Scottish Friendly	1863
38	Friend in Need	1829	97	Scottish National	1843
			98	Scottish Provident Institution	1837
39	General Annuity and Life	1862	99	Scottish Provincial	1825
40	General Provident	1832	100	Scottish Widows' Fund	1815
41	General Provincial	1864	101	Scottish Union	1824
42	General	1837	102	Solicitors and General	1846
43	Great Britain	1844	103	Southampton Mutual	1862
44	Gresham	1848	104	Sovereign	1845
45	Guardian	1821	105	Standard	1825
			106	Star	1843
46	Hand-in-Hand	1836	107	Sun	1810
47	Hercules	1863			
48	Home and Colonial	1864	108	Union	1714
			109	United Kent	1824
49	Imperial	1820	110	United Kingdom Temperance	1850
50	International	1837	111	Universal	1834
51	Indisputable of Scotland	1860	112	University	1825
52	Lancashire	1852	113	Victoria	1838
53	Law Life	1823	114	Volunteer	1862
54	Law Property	1850			
55	Law Union	1854	115	West of England	1807
56	Legal and General	1836	116	Western, Manchester, &c.	1842
57	Life Association of Scotland	1838	117	Western Counties	1861
58	Life Investment	1864	118	Westminster and General	1836
59	Liverpool, London, and Globe	1836	119	Whittington	1855
60	London and Lancashire	1862			
61	London and Provincial Law	1845	120	Yorkshire	1824

Age 21.				Age 22.				Age 23.				Age 24.				Age 25.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
39	1	14	8	39	1	15	6	39	1	16	5	39	1	17	4	98	1	18	0	1
29	1	15	5	29	1	16	3	29	1	17	2	98	1	17	7	39	1	18	3	2
22	1	16	0	98	1	16	9	98	1	17	2	29	1	18	1	29	1	19	0	3
47	1	16	3	22	1	17	0	47	1	18	0	47	1	19	0	47	2	0	0	4
98	1	16	3	47	1	17	1	22	1	18	0	22	1	19	0	5	2	0	1	5
80	1	16	5	5	1	17	5	5	1	18	4	5	1	19	2	83	2	0	1	6
86	1	16	6	80	1	17	5	83	1	18	4	83	1	19	2	22	2	0	2	7
119	1	16	6	86	1	17	5	86	1	18	4	86	1	19	4	86	2	0	4	8
5	1	16	7	119	1	17	6	80	1	18	5	80	1	19	6	80	2	0	6	9
60	1	16	9	83	1	17	7	60	1	18	8	13	1	19	9	60	2	0	10	10
13	1	16	10	13	1	17	9	119	1	18	8	60	1	19	9	13	2	0	11	11
27	1	16	10	60	1	17	9	13	1	18	9	41	1	19	10	53	2	1	0	12
41	1	16	10	27	1	17	10	41	1	18	10	119	1	19	10	119	2	1	0	13
83	1	16	10	41	1	18	0	27	1	18	11	27	2	0	1	40	2	1	1	14
99	1	17	4	99	1	18	4	52	1	19	3	52	2	0	1	27	2	1	2	15
15	1	17	7	91	1	18	4	91	1	19	3	91	2	0	2	91	2	1	2	16
20	1	17	7	52	1	18	5	15	1	19	5	40	2	0	3	37	2	1	3	17
91	1	17	7	15	1	18	6	20	1	19	5	15	2	0	5	41	2	1	3	18
117	1	17	7	20	1	18	6	40	1	19	5	20	2	0	5	15	2	1	5	19
52	1	17	8	117	1	18	6	99	1	19	5	37	2	0	6	20	2	1	5	20
77	1	17	8	40	1	18	7	117	1	19	7	99	2	0	6	18	2	1	6	21
40	1	17	9	77	1	18	8	37	1	19	8	18	2	0	7	99	2	1	8	22
3	1	17	11	3	1	18	11	77	1	19	8	117	2	0	8	117	2	1	9	23
107	1	17	11	18	1	18	11	18	1	19	9	77	2	0	9	77	2	1	11	24
113	1	17	11	37	1	18	11	30	2	0	0	30	2	1	0	30	2	2	0	25
21	1	18	0	107	1	18	11	21	2	0	0	21	2	1	1	21	2	2	3	26
18	1	18	1	113	1	18	11	93	2	0	0	75	2	1	1	75	2	2	3	27
30	1	18	2	21	1	19	0	75	2	0	0	93	2	1	2	10	2	2	4	28
37	1	18	2	75	1	19	0	3	2	0	1	3	2	1	3	93	2	2	4	29
51	1	18	2	93	1	19	1	107	2	0	1	107	2	1	3	3	2	2	6	30
93	1	18	2	30	1	19	1	113	2	0	1	113	2	1	3	23	2	2	6	31
23	1	18	3	23	1	19	3	23	2	0	3	118	2	1	4	107	2	2	6	32
36	1	18	3	36	1	19	3	81	2	0	3	10	2	1	5	113	2	2	6	33
81	1	18	3	51	1	19	3	10	2	0	4	36	2	1	5	36	2	2	7	34
106	1	18	3	81	1	19	3	36	2	0	4	51	2	1	5	51	2	2	7	35
118	1	18	3	106	1	19	3	51	2	0	4	58	2	1	5	58	2	2	7	36
58	1	18	4	118	1	19	3	58	2	0	4	81	2	1	5	81	2	2	7	37
50	1	18	4	10	1	19	4	107	2	0	4	106	2	1	5	106	2	2	7	38
110	1	18	4	58	1	19	4	110	2	0	4	110	2	1	5	110	2	2	7	39
10	1	18	5	50	1	19	4	118	2	0	4	23	2	1	6	116	2	2	7	40
97	1	18	6	110	1	19	4	50	2	0	5	97	2	1	6	118	2	2	7	41
87	1	18	7	97	1	19	6	97	2	0	6	116	2	1	6	97	2	2	8	42
116	1	18	7	87	1	19	7	116	2	0	6	50	2	1	7	7	2	2	9	43
2	1	18	10	116	1	19	7	87	2	0	8	42	2	1	9	35	2	2	9	44
48	1	18	10	48	1	19	9	48	2	0	9	87	2	1	9	50	2	2	9	45
26	1	18	10	62	1	19	9	62	2	0	9	35	2	1	9	12	2	2	9	46
62	1	18	10	26	1	19	10	35	2	0	10	7	2	1	10	42	2	2	10	47
35	1	18	11	42	1	19	10	42	2	0	10	48	2	1	10	87	2	2	11	48
78	1	18	11	2	1	19	11	74	2	0	10	62	2	1	10	48	2	2	11	49
114	1	18	11	11	1	19	11	2	2	0	11	11	2	1	11	78	2	2	11	50
11	1	19	0	35	1	19	11	11	2	0	11	74	2	1	11	114	2	2	11	51
42	1	19	0	74	1	19	11	26	2	0	11	12	2	1	11	74	2	2	11	52
74	1	19	0	78	1	19	11	78	2	0	11	78	2	1	11	44	2	2	11	53
76	1	19	1	114	1	19	11	114	2	0	11	114	2	1	11	11	2	3	0	54
31	1	19	2	33	2	0	1	7	2	0	1	2	2	2	0	62	2	3	0	55
33	1	19	2	76	2	0	1	33	2	1	1	26	2	2	0	19	2	3	2	56
66	1	19	2	66	2	0	2	12	2	1	1	44	2	2	0	26	2	3	2	57
120	1	19	2	120	2	0	2	44	2	1	1	33	2	2	2	33	2	3	2	58
16	1	19	4	7	2	0	3	24	2	1	2	66	2	2	2	2	2	3	3	59
24	1	19	4	24	2	0	3	66	2	1	2	14	2	2	3	14	2	3	3	60

Age 21.				Age 22.				Age 23.				Age 24.				Age 25.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
12	1	19	5	31	2	0	3	76	2	1	2	19	2	2	3	24	2	3	3	61
44	1	19	5	12	2	0	3	31	2	1	3	24	2	2	3	66	2	3	3	62
7	1	19	6	44	2	0	3	120	2	1	3	31	2	2	3	111	2	3	3	63
14	1	19	6	16	2	0	4	14	2	1	4	76	2	2	3	31	2	3	4	64
19	1	19	6	14	2	0	5	19	2	1	4	111	2	2	3	76	2	3	5	65
57	1	19	6	19	2	0	5	111	2	1	4	120	2	2	5	120	2	3	7	66
111	1	19	6	111	2	0	5	16	2	1	5	55	2	2	6	55	2	3	7	67
101	1	19	7	57	2	0	6	25	2	1	6	16	2	2	7	25	2	3	8	68
25	1	19	8	55	2	0	6	55	2	1	7	25	2	2	7	79	2	3	8	69
55	1	19	9	25	2	0	7	57	2	1	8	57	2	2	8	16	2	3	10	70
96	1	19	9	101	2	0	9	96	2	1	9	79	2	2	9	54	2	3	10	71
61	1	19	10	96	2	0	9	54	2	1	10	54	2	2	10	57	2	3	10	72
54	2	0	0	61	2	0	10	61	2	1	10	102	2	2	10	59	2	3	10	73
59	2	0	0	54	2	0	11	59	2	1	10	104	2	2	10	72	2	3	10	74
102	2	0	0	59	2	0	11	102	2	1	10	96	2	2	10	102	2	3	10	75
82	2	0	2	102	2	0	11	104	2	1	11	59	2	2	10	104	2	3	10	76
104	2	0	2	104	2	1	0	101	2	1	11	72	2	2	11	96	2	3	11	77
6	2	0	3	6	2	1	2	72	2	2	0	61	2	2	11	67	2	4	0	78
65	2	0	3	65	2	1	2	79	2	2	0	67	2	3	0	61	2	4	0	79
70	2	0	3	70	2	1	2	1	2	2	1	101	2	3	1	1	2	4	1	80
73	2	0	3	72	2	1	2	67	2	2	1	1	2	3	2	4	2	4	2	81
85	2	0	3	73	2	1	2	6	2	2	2	6	2	3	2	89	2	4	2	82
89	2	0	3	82	2	1	2	65	2	2	2	65	2	3	2	17	2	4	2	83
17	2	0	3	85	2	1	2	70	2	2	2	70	2	3	2	6	2	4	3	84
72	2	0	4	89	2	1	2	73	2	2	2	73	2	3	2	65	2	4	3	85
92	2	0	4	17	2	1	2	82	2	2	2	85	2	3	2	70	2	4	3	86
1	2	0	5	67	2	1	3	85	2	2	2	89	2	3	2	73	2	4	3	87
32	2	0	5	79	2	1	3	89	2	2	2	17	2	3	2	85	2	4	3	88
67	2	0	5	1	2	1	4	17	2	2	2	4	2	3	3	92	2	4	3	89
4	2	0	6	32	2	1	4	92	2	2	3	82	2	3	3	101	2	4	3	90
79	2	0	6	92	2	1	4	4	2	2	4	92	2	3	3	109	2	4	3	91
109	2	0	9	4	2	1	5	32	2	2	4	109	2	3	4	82	2	4	4	92
69	2	0	10	109	2	1	7	109	2	2	5	32	2	3	5	32	2	4	6	93
63	2	1	1	69	2	1	8	69	2	2	7	69	2	3	6	69	2	4	7	94
56	2	1	2	63	2	2	0	63	2	3	0	63	2	4	0	68	2	4	10	95
71	2	1	6	56	2	2	1	56	2	3	1	56	2	4	1	63	2	5	0	96
49	2	1	7	71	2	2	4	49	2	3	3	49	2	4	1	49	2	5	0	97
45	2	1	10	49	2	2	5	71	2	3	3	71	2	4	1	71	2	5	0	98
84	2	1	10	45	2	2	8	45	2	3	6	68	2	4	4	56	2	5	1	99
115	2	1	10	84	2	2	8	84	2	3	6	45	2	4	5	45	2	5	4	100
90	2	1	11	105	2	2	9	105	2	3	7	84	2	4	5	84	2	5	4	101
105	2	1	11	115	2	2	9	68	2	3	8	105	2	4	6	105	2	5	4	102
108	2	2	3	90	2	2	10	115	2	3	9	115	2	4	8	28	2	5	7	103
28	2	2	4	68	2	3	0	90	2	3	10	28	2	4	9	115	2	5	8	104
68	2	2	4	28	2	3	1	108	2	3	11	108	2	4	9	108	2	5	8	105
112	2	2	4	108	2	3	1	28	2	3	11	112	2	4	9	112	2	5	9	106
43	2	2	6	112	2	3	1	112	2	4	0	90	2	4	10	90	2	5	11	107
9	2	3	0	43	2	3	4	43	2	4	2	43	2	5	1	43	2	6	0	108
94	2	3	0	94	2	3	10	94	2	4	8	94	2	5	6	94	2	6	5	109
95	2	3	1	9	2	3	11	95	2	4	9	95	2	5	7	95	2	6	6	110
100	2	3	1	95	2	3	11	100	2	4	9	100	2	5	7	100	2	6	6	111
88	2	4	6	100	2	3	11	9	2	4	10	9	2	5	9	9	2	6	9	112
34	2	4	6	88	2	5	4	88	2	6	3	88	2	7	1	88	2	8	1	113
46	2	4	6	34	2	5	4	34	2	6	3	34	2	7	1	34	2	8	1	114
53	2	4	6	46	2	5	4	53	2	6	3	46	2	7	1	46	2	8	1	115
38	2	5	8	53	2	5	4	46	2	6	3	53	2	7	1	53	2	8	1	116
64	2	10	0	38	2	6	9	38	2	7	6	38	2	8	9	38	2	9	9	117
8				64	2	10	9	64	2	11	6	64	2	12	6	64	2	13	6	118
75				8				8				8				8				119
103				103				103				103				103				120

Age 26.				Age 27.				Age 28.				Age 29.				Age 30.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
98	1	18	6	98	1	19	2	98	1	19	11	98	2	0	8	98	2	1	6	1
39	1	19	2	39	2	0	4	39	2	1	6	39	2	2	8	39	2	3	10	2
29	2	0	0	29	2	1	0	29	2	2	0	29	2	3	1	29	2	4	3	3
47	2	1	0	5	2	2	1	5	2	3	2	5	2	4	3	5	2	5	5	4
5	2	1	1	47	2	2	2	47	2	3	4	47	2	4	6	37	2	5	9	5
83	2	1	3	22	2	2	6	86	2	3	8	37	2	4	9	47	2	5	9	6
22	2	1	4	86	2	2	6	37	2	3	10	86	2	4	10	52	2	6	0	7
86	2	1	4	83	2	2	7	22	2	3	10	52	2	4	11	86	2	6	1	8
80	2	1	7	80	2	2	8	52	2	3	11	22	2	5	0	80	2	6	2	9
40	2	1	11	40	2	2	11	80	2	3	11	80	2	5	0	83	2	6	4	10
52	2	1	11	52	2	2	11	83	2	3	11	83	2	5	2	22	2	6	4	11
13	2	2	0	37	2	3	0	40	2	4	1	40	2	5	4	40	2	6	8	12
60	2	2	0	13	2	3	3	60	2	4	5	18	2	5	8	18	2	6	10	13
37	2	2	1	60	2	3	3	13	2	4	6	60	2	5	8	60	2	6	10	14
119	2	2	2	119	2	3	4	18	2	4	7	13	2	5	9	13	2	6	11	15
41	2	2	4	18	2	3	6	15	2	4	8	41	2	5	10	15	2	6	11	16
91	2	2	4	15	2	3	7	20	2	4	8	15	2	5	11	20	2	6	11	17
27	2	2	5	20	2	3	7	119	2	4	8	20	2	5	11	41	2	7	2	18
15	2	2	6	91	2	3	7	41	2	4	9	119	2	6	0	27	2	7	4	19
18	2	2	6	27	2	3	8	27	2	4	10	27	2	6	1	119	2	7	4	20
20	2	2	6	41	2	3	8	91	2	4	10	91	2	6	1	91	2	7	5	21
99	2	2	11	30	2	4	1	30	2	5	3	30	2	6	5	30	2	7	7	22
117	2	2	11	117	2	4	2	117	2	5	5	117	2	6	8	117	2	7	11	23
12	2	3	0	99	2	4	3	77	2	5	7	77	2	6	10	77	2	8	0	24
30	2	3	0	77	2	4	4	99	2	5	7	99	2	6	10	99	2	8	1	25
77	2	3	1	10	2	4	8	10	2	5	10	10	2	7	0	12	2	8	1	26
10	2	3	5	21	2	4	8	12	2	5	11	12	2	7	0	35	2	8	2	27
21	2	3	5	75	2	4	9	35	2	5	11	35	2	7	1	10	2	8	3	28
93	2	3	6	35	2	4	10	21	2	6	0	44	2	7	1	44	2	8	3	29
75	2	3	6	116	2	4	10	116	2	6	0	19	2	7	2	42	2	8	5	30
3	2	3	9	12	2	4	10	44	2	6	0	21	2	7	2	19	2	8	5	31
23	2	3	9	93	2	4	10	42	2	6	1	42	2	7	3	21	2	8	5	32
36	2	3	9	42	2	4	11	19	2	6	1	116	2	7	4	116	2	8	6	33
51	2	3	9	44	2	4	11	93	2	6	2	75	2	7	4	75	2	8	6	34
81	2	3	9	7	2	5	0	75	2	6	2	93	2	7	5	93	2	8	8	35
106	2	3	9	23	2	5	0	23	2	6	3	23	2	7	6	7	2	8	9	36
107	2	3	9	36	2	5	0	7	2	6	4	7	2	7	7	23	2	8	9	37
113	2	3	9	81	2	5	0	36	2	6	4	48	2	7	7	48	2	8	9	38
116	2	3	9	106	2	5	0	51	2	6	4	36	2	7	7	36	2	8	9	39
118	2	3	9	110	2	5	0	58	2	6	4	51	2	7	7	81	2	8	9	40
7	2	3	10	118	2	5	0	81	2	6	4	78	2	7	7	106	2	8	9	41
35	2	3	10	19	2	5	1	106	2	6	4	81	2	7	7	14	2	8	10	42
42	2	3	10	51	2	5	1	110	2	6	4	106	2	7	7	33	2	8	10	43
58	2	3	10	58	2	5	1	118	2	6	4	110	2	7	7	51	2	8	10	44
97	2	3	10	3	2	5	2	48	2	6	5	118	2	7	7	58	2	8	10	45
110	2	3	10	48	2	5	2	11	2	6	5	114	2	7	7	110	2	8	10	46
44	2	3	11	74	2	5	2	33	2	6	5	11	2	7	8	111	2	8	10	47
48	2	4	0	97	2	5	2	74	2	6	5	14	2	7	8	79	2	8	10	48
50	2	4	0	107	2	5	2	78	2	6	5	33	2	7	8	118	2	8	10	49
74	2	4	0	113	2	5	2	114	2	6	5	58	2	7	8	74	2	8	11	50
78	2	4	0	11	2	5	3	50	2	6	6	74	2	7	8	78	2	8	11	51
114	2	4	0	33	2	5	3	97	2	6	6	111	2	7	8	114	2	8	11	52
11	2	4	1	50	2	5	3	3	2	6	7	97	2	7	9	11	2	9	0	53
19	2	4	1	78	2	5	3	14	2	6	7	79	2	7	9	97	2	9	0	54
87	2	4	1	114	2	5	3	107	2	6	7	50	2	7	10	3	2	9	2	55
33	2	4	3	87	2	5	4	111	2	6	7	3	2	7	11	72	2	9	2	56
62	2	4	3	14	2	5	5	113	2	6	7	87	2	7	11	107	2	9	2	57
14	2	4	4	111	2	5	5	87	2	6	8	107	2	7	11	113	2	9	2	58
111	2	4	4	62	2	5	6	79	2	6	8	113	2	7	11	31	2	9	3	59
2	2	4	5	24	2	5	7	31	2	6	9	24	2	8	0	50	2	9	3	60

Age 26.				Age 27.				Age 28.				Age 29.				Age 30.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
24	2	4	5	31	2	5	7	62	2	6	9	31	2	8	0	24	2	9	4	61
26	2	4	5	66	2	5	7	66	2	6	10	72	2	8	0	62	2	9	4	62
31	2	4	5	26	2	5	8	24	2	6	10	62	2	8	1	87	2	9	4	63
66	2	4	5	79	2	5	8	72	2	6	11	66	2	8	1	104	2	9	4	64
120	2	4	6	2	2	5	9	2	2	7	0	104	2	8	2	55	2	9	4	65
76	2	4	8	120	2	5	9	26	2	7	0	55	2	8	2	25	2	9	5	66
79	2	4	8	72	2	5	10	104	2	7	0	25	2	8	3	54	2	9	5	67
25	2	4	9	55	2	5	10	120	2	7	0	54	2	8	3	59	2	9	5	68
55	2	4	9	25	2	5	10	55	2	7	0	59	2	8	3	66	2	9	5	69
72	2	4	10	54	2	5	11	25	2	7	1	102	2	8	3	102	2	9	5	70
104	2	4	10	102	2	5	11	54	2	7	1	2	2	8	4	2	2	9	6	71
54	2	4	11	104	2	5	11	59	2	7	1	26	2	8	4	26	2	9	6	72
59	2	4	11	76	2	6	0	102	2	7	1	120	2	8	4	120	2	9	7	73
102	2	4	11	59	2	6	0	67	2	7	3	67	2	8	5	109	2	9	8	74
16	2	5	1	67	2	6	1	1	2	7	4	4	2	8	6	1	2	9	9	75
67	2	5	1	1	2	6	2	4	2	7	4	109	2	8	6	4	2	9	9	76
61	2	5	1	4	2	6	3	76	2	7	4	1	2	8	7	32	2	9	9	77
96	2	5	1	61	2	6	3	109	2	7	4	76	2	8	7	67	2	9	9	78
1	2	5	2	109	2	6	3	61	2	7	5	89	2	8	7	89	2	9	9	79
4	2	5	2	16	2	6	4	89	2	7	6	32	2	8	8	92	2	9	9	80
57	2	5	2	89	2	6	4	92	2	7	6	61	2	8	8	76	2	9	10	81
89	2	5	3	96	2	6	4	68	2	7	6	68	2	8	8	61	2	9	10	82
92	2	5	3	92	2	6	5	32	2	7	7	92	2	8	8	17	2	9	10	83
17	2	5	3	68	2	6	5	17	2	7	7	17	2	8	9	68	2	9	11	84
109	2	5	3	17	2	6	5	101	2	7	7	101	2	8	9	101	2	9	11	85
6	2	5	4	101	2	6	5	96	2	7	7	16	2	8	10	57	2	10	0	86
65	2	5	4	6	2	6	6	6	2	7	8	96	2	8	10	96	2	10	0	87
70	2	5	4	32	2	6	6	16	2	7	8	6	2	8	11	16	2	10	1	88
73	2	5	4	57	2	6	6	65	2	7	8	65	2	8	11	6	2	10	2	89
85	2	5	4	65	2	6	6	70	2	7	8	69	2	8	11	65	2	10	2	90
101	2	5	4	70	2	6	6	73	2	7	8	70	2	8	11	70	2	10	2	91
82	2	5	5	73	2	6	6	85	2	7	8	73	2	8	11	69	2	10	2	92
32	2	5	6	85	2	6	6	82	2	7	9	85	2	8	11	73	2	10	2	93
68	2	5	6	82	2	6	7	57	2	7	10	82	2	9	0	85	2	10	2	94
69	2	5	7	69	2	6	9	69	2	7	10	49	2	9	1	49	2	10	3	95
49	2	6	0	49	2	7	0	49	2	8	0	57	2	9	2	82	2	10	4	96
71	2	6	0	71	2	7	0	71	2	8	1	71	2	9	2	71	2	10	4	97
63	2	6	1	63	2	7	2	45	2	8	4	105	2	9	5	45	2	10	7	98
56	2	6	2	56	2	7	3	56	2	8	4	45	2	9	6	84	2	10	7	99
45	2	6	4	45	2	7	4	63	2	8	4	56	2	9	6	105	2	10	7	100
84	2	6	4	84	2	7	4	84	2	8	4	63	2	9	6	28	2	10	8	101
105	2	6	5	105	2	7	4	105	2	8	5	84	2	9	6	63	2	10	8	102
28	2	6	6	28	2	7	6	28	2	8	6	28	2	9	7	108	2	10	8	103
108	2	6	7	108	2	7	7	108	2	8	7	108	2	9	8	56	2	10	9	104
112	2	6	8	112	2	7	7	112	2	8	7	112	2	9	8	112	2	10	9	105
115	2	6	9	115	2	7	9	115	2	8	10	115	2	10	0	115	2	11	3	106
43	2	6	11	43	2	7	11	43	2	9	0	43	2	10	1	43	2	11	4	107
90	2	7	0	90	2	8	1	90	2	9	3	90	2	10	5	90	2	11	7	108
94	2	7	5	94	2	8	5	94	2	9	6	94	2	10	7	95	2	11	9	109
95	2	7	6	95	2	8	6	95	2	9	7	95	2	10	8	100	2	11	9	110
100	2	7	6	100	2	8	6	100	2	9	7	100	2	10	8	94	2	11	9	111
9	2	7	9	9	2	8	10	9	2	10	0	9	2	11	3	9	2	12	6	112
34	2	9	0	34	2	10	0	88	2	11	1	88	2	12	3	34	2	13	4	113
88	2	9	1	88	2	10	1	34	2	11	1	34	2	12	3	88	2	13	5	114
46	2	9	1	46	2	10	1	46	2	11	1	46	2	12	3	46	2	13	5	115
53	2	9	1	53	2	10	1	53	2	11	1	53	2	12	3	53	2	13	5	116
38	2	10	9	38	2	11	6	38	2	12	6	38	2	13	9	38	2	14	9	117
64	2	14	6	64	2	15	6	64	2	16	6	64	2	17	9	64	2	19	3	118
8				8				8				8				8				119
103				103				103				103				103				120

Age 31.				Age 32.				Age 33.				Age 34.				Age 35.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
98	2	2	6	98	2	3	5	98	2	4	6	98	2	5	7	97	2	6	10	1
39	2	5	1	39	2	6	5	39	2	7	10	39	2	9	4	39	2	10	10	2
29	2	5	5	29	2	6	8	29	2	8	0	29	2	9	5	29	2	10	11	3
5	2	6	8	37	2	7	9	37	2	8	10	37	2	10	0	37	2	11	2	4
37	2	6	9	5	2	7	11	5	2	9	3	5	2	10	7	5	2	12	1	5
52	2	7	1	52	2	8	3	52	2	9	6	52	2	10	10	52	2	12	3	6
47	2	7	1	47	2	8	6	22	2	10	0	22	2	11	6	80	2	12	11	7
86	2	7	5	22	2	8	8	47	2	10	0	47	2	11	6	22	2	13	0	8
22	2	7	6	80	2	8	9	80	2	10	1	80	2	11	6	47	2	13	1	9
80	2	7	6	83	2	8	10	83	2	10	3	83	2	11	9	83	2	13	5	10
83	2	7	7	86	2	8	10	86	2	10	4	86	2	11	10	40	2	13	6	11
40	2	7	11	40	2	9	2	40	2	10	6	40	2	12	0	86	2	13	6	12
60	2	8	0	60	2	9	3	13	2	10	8	13	2	12	1	13	2	13	7	13
13	2	8	1	13	2	9	4	60	2	10	8	60	2	12	1	60	2	13	7	14
18	2	8	1	15	2	9	6	41	2	10	9	18	2	12	4	18	2	13	11	15
15	2	8	2	18	2	9	6	18	2	10	10	41	2	12	5	41	2	14	0	16
20	2	8	2	20	2	9	6	15	2	11	0	15	2	12	8	15	2	14	4	17
41	2	8	6	41	2	9	11	20	2	11	0	20	2	12	8	20	2	14	4	18
119	2	8	8	119	2	10	0	27	2	11	6	27	2	13	0	27	2	14	6	19
27	2	8	9	27	2	10	1	30	2	11	6	30	2	13	0	30	2	14	6	20
91	2	8	9	91	2	10	1	91	2	11	6	91	2	13	0	91	2	14	6	21
30	2	8	10	30	2	10	2	119	2	11	6	119	2	13	0	119	2	14	8	22
117	2	9	1	117	2	10	4	117	2	11	9	117	2	13	3	117	2	14	9	23
77	2	9	3	77	2	10	6	77	2	11	11	77	2	13	5	79	2	14	10	24
99	2	9	4	35	2	10	8	44	2	12	0	44	2	13	5	14	2	14	11	25
12	2	9	5	44	2	10	8	35	2	12	1	35	2	13	6	77	2	14	11	26
35	2	9	5	99	2	10	8	99	2	12	1	79	2	13	6	44	2	14	11	27
44	2	9	5	12	2	10	10	14	2	12	3	14	2	13	7	111	2	14	11	28
10	2	9	7	10	2	10	11	79	2	12	3	99	2	13	7	35	2	15	0	29
19	2	9	8	21	2	10	11	12	2	12	3	111	2	13	7	42	2	15	1	30
21	2	9	8	14	2	11	0	111	2	12	3	12	2	13	8	99	2	15	2	31
42	2	9	8	42	2	11	0	21	2	12	4	42	2	13	8	12	2	15	2	32
75	2	9	9	93	2	11	0	42	2	12	4	48	2	13	9	48	2	15	3	33
93	2	9	10	75	2	11	0	10	2	12	5	10	2	13	10	10	2	15	4	34
116	2	9	10	111	2	11	0	48	2	12	5	21	2	13	10	21	2	15	5	35
48	2	9	11	48	2	11	1	75	2	12	5	75	2	13	11	75	2	15	6	36
14	2	9	11	79	2	11	1	93	2	12	6	93	2	14	1	110	2	15	7	37
111	2	9	11	19	2	11	2	19	2	12	8	19	2	14	2	19	2	15	8	38
7	2	10	0	23	2	11	3	51	2	12	8	7	2	14	3	93	2	15	8	39
23	2	10	0	7	2	11	4	118	2	12	8	23	2	14	3	23	2	15	9	40
36	2	10	0	36	2	11	4	7	2	12	9	33	2	14	3	25	2	15	9	41
51	2	10	0	51	2	11	4	23	2	12	9	36	2	14	3	7	2	15	10	42
79	2	10	0	58	2	11	4	36	2	12	9	51	2	14	3	33	2	15	10	43
81	2	10	0	81	2	11	4	58	2	12	9	58	2	14	3	51	2	15	10	44
106	2	10	0	106	2	11	4	81	2	12	9	81	2	14	3	58	2	15	10	45
110	2	10	0	110	2	11	4	106	2	12	9	106	2	14	3	81	2	15	10	46
118	2	10	0	116	2	11	4	110	2	12	9	110	2	14	3	97	2	15	10	47
33	2	10	1	118	2	11	4	116	2	12	10	118	2	14	3	118	2	15	10	48
58	2	10	1	33	2	11	6	33	2	12	11	25	2	14	4	36	2	15	11	49
97	2	10	2	97	2	11	6	25	2	13	0	116	2	14	5	106	2	15	11	50
74	2	10	3	11	2	11	9	97	2	13	0	97	2	14	6	116	2	15	11	51
11	2	10	4	25	2	11	9	72	2	13	1	72	2	14	7	72	2	16	0	52
78	2	10	4	78	2	11	9	120	2	13	2	120	2	14	8	17	2	16	1	53
114	2	10	4	72	2	11	9	11	2	13	3	89	2	14	9	89	2	16	2	54
55	2	10	5	74	2	11	9	55	2	13	3	17	2	14	9	68	2	16	2	55
72	2	10	5	55	2	11	9	74	2	13	3	55	2	14	9	104	2	16	3	56
120	2	10	5	114	2	11	9	78	2	13	3	68	2	14	9	109	2	16	3	57
2	2	10	6	120	2	11	9	114	2	13	3	104	2	14	9	120	2	16	3	58
3	2	10	6	3	2	11	10	3	2	13	4	11	2	14	10	54	2	16	5	59
107	2	10	6	107	2	11	10	62	2	13	4	54	2	14	10	59	2	16	5	60

Age 36.				Age 37.				Age 38.				Age 39.				Age 40.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
54	2	18	0	49	2	19	8	49	3	1	4	104	3	3	2	58	3	5	0	61
61	2	18	0	61	2	19	8	57	3	1	4	105	3	3	2	81	3	5	0	62
92	2	18	0	54	2	19	9	104	3	1	4	120	3	5	2	84	3	5	0	63
50	2	18	1	59	2	19	9	112	3	1	4	45	3	3	3	118	3	5	0	64
62	2	18	1	50	2	19	9	116	3	1	4	84	3	3	3	101	3	5	0	65
11	2	18	2	102	2	19	9	54	3	1	6	101	3	3	3	120	3	5	0	66
31	2	18	2	108	2	19	9	59	3	1	6	118	3	3	3	63	3	5	1	67
49	2	18	2	112	2	19	9	50	3	1	6	4	3	3	4	57	3	5	2	68
57	2	18	2	31	2	19	10	101	3	1	6	57	3	3	4	104	3	5	2	69
74	2	18	2	62	2	19	10	102	3	1	6	50	3	3	4	50	3	5	3	70
78	2	18	2	101	2	19	10	105	3	1	6	63	3	3	4	55	3	5	3	71
114	2	18	2	105	2	19	10	31	3	1	7	59	3	3	5	28	3	5	5	72
24	2	18	3	24	2	19	11	45	3	1	7	116	3	3	5	59	3	5	5	73
101	2	18	3	4	2	0	0	63	3	1	7	54	3	3	6	24	3	5	6	74
102	2	18	3	11	3	0	0	84	3	1	7	102	3	3	6	54	3	5	6	75
108	2	18	3	45	3	0	0	4	3	1	8	24	3	3	7	96	3	5	6	76
2	2	18	4	57	3	0	0	24	3	1	8	28	3	3	7	102	3	5	6	77
26	2	18	4	63	3	0	0	62	3	1	9	31	3	3	7	116	3	5	6	78
87	2	18	4	74	3	0	0	96	3	1	9	95	3	3	7	31	3	5	7	79
96	2	18	4	78	3	0	0	28	3	1	10	62	3	3	8	62	3	5	8	80
105	2	18	4	84	3	0	0	11	3	1	11	71	3	3	10	71	3	5	9	81
112	2	18	4	96	3	0	0	74	3	1	11	74	3	3	10	87	3	5	10	82
3	2	18	5	114	3	0	0	78	3	1	11	78	3	3	10	2	3	5	11	83
4	2	18	5	2	3	0	1	87	3	1	11	114	3	3	10	26	3	5	11	84
63	2	18	5	87	3	0	1	114	3	1	11	1	3	3	11	74	3	5	11	85
107	2	18	5	26	3	0	2	2	3	2	0	2	3	3	11	56	3	5	11	86
113	2	18	5	28	3	0	2	26	3	2	0	11	3	3	11	78	3	5	11	87
32	2	18	6	76	3	0	2	71	3	2	0	26	3	3	11	114	3	5	11	88
45	2	18	6	1	3	0	3	76	3	2	0	87	3	3	11	1	3	6	0	89
76	2	18	6	32	3	0	3	1	3	2	2	56	3	4	1	11	3	6	0	90
84	2	18	6	71	3	0	3	32	3	2	3	76	3	4	1	76	3	6	1	91
1	2	18	7	3	3	0	4	3	3	2	4	32	3	4	3	115	3	6	1	92
28	2	18	7	107	3	0	4	56	3	2	4	6	3	4	4	6	3	6	3	93
71	2	18	7	113	3	0	4	107	3	2	4	65	3	4	4	65	3	6	3	94
6	2	19	0	56	3	0	8	113	3	2	4	70	3	4	4	70	3	6	3	95
65	2	19	0	6	3	0	9	6	3	2	6	73	3	4	4	73	3	6	3	96
70	2	19	0	65	3	0	9	65	3	2	6	85	3	4	4	85	3	6	3	97
73	2	19	0	67	3	0	9	70	3	2	6	115	3	4	4	90	3	6	3	98
85	2	19	0	70	3	0	9	73	3	2	6	3	3	4	5	95	3	6	3	99
56	2	19	1	73	3	0	9	85	3	2	6	67	3	4	5	94	3	6	3	100
66	2	19	1	85	3	0	9	67	3	2	7	94	3	4	5	100	3	6	3	101
67	2	19	1	82	3	0	11	82	3	2	8	107	3	4	5	67	3	6	4	102
16	2	19	3	66	3	1	0	115	3	2	8	113	3	4	5	82	3	6	5	103
82	2	19	3	16	3	1	1	90	3	2	9	82	3	4	6	3	3	6	6	104
69	2	19	4	115	3	1	1	94	3	2	11	90	3	4	6	32	3	6	6	105
115	2	19	5	69	3	1	2	16	3	3	0	95	3	4	6	107	3	6	6	106
90	2	19	8	90	3	1	2	66	3	3	0	100	3	4	6	113	3	6	6	107
94	2	19	8	94	3	1	2	95	3	3	0	16	3	5	0	16	3	6	9	108
95	2	19	9	95	3	1	3	100	3	3	0	66	3	5	1	66	3	7	4	109
100	2	19	9	100	3	1	3	69	3	3	3	69	3	5	4	69	3	7	6	110
43	3	0	0	43	3	1	10	43	3	3	9	43	3	5	9	43	3	7	10	111
34	3	1	4	34	3	2	10	34	3	4	6	34	3	6	2	34	3	7	11	112
46	3	1	4	46	3	2	10	46	3	4	6	46	3	6	2	46	3	7	11	113
53	3	1	4	53	3	2	10	53	3	4	6	53	3	6	2	53	3	7	11	114
88	3	1	4	88	3	2	10	88	3	4	6	88	3	6	2	88	3	7	11	115
9	3	1	10	9	3	3	8	9	3	5	9	9	3	7	10	9	3	10	0	116
38	3	3	0	38	3	4	10	38	3	6	10	38	3	10	4	38	3	12	4	117
64	3	8	9	64	3	10	9	64	3	12	9	64	3	14	9	64	3	17	0	118
8				8				8				8				8				119
103				103				103				103				103				120

Age 36.				Age 37.				Age 38.				Age 39.				Age 40.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
98	2	8	2	98	2	9	8	98	2	11	3	98	2	12	11	98	2	14	9	1
39	2	12	5	37	2	13	8	37	2	15	1	37	2	16	6	37	2	18	1	2
37	2	12	5	29	2	14	2	29	2	15	11	29	2	17	9	29	2	19	9	3
29	2	12	6	39	2	14	2	39	2	16	0	39	2	17	11	39	2	19	11	4
5	2	13	7	5	2	15	2	5	2	16	11	5	2	18	8	52	3	0	6	5
52	2	13	9	52	2	15	3	52	2	16	11	52	2	18	8	5	3	0	7	6
80	2	14	4	80	2	15	9	80	2	17	5	80	2	19	1	80	3	0	11	7
22	2	14	8	22	2	16	6	22	2	18	4	22	3	0	2	79	3	2	0	8
47	2	14	9	47	2	16	6	40	2	18	5	40	3	0	4	22	3	2	2	9
40	2	15	0	40	2	16	8	47	2	18	5	47	3	0	5	40	3	2	4	10
83	2	15	1	83	2	16	10	13	2	18	8	13	3	0	6	60	3	2	4	11
60	2	15	2	60	2	16	11	60	2	18	8	60	3	0	6	13	3	2	5	12
13	2	15	3	13	2	17	0	83	2	18	9	79	3	0	6	47	3	2	6	13
86	2	15	3	86	2	17	0	86	2	18	11	83	3	0	8	83	3	2	8	14
18	2	15	8	18	2	17	5	41	2	19	0	86	3	0	11	14	3	3	0	15
41	2	15	10	41	2	17	6	79	2	19	0	41	3	1	1	86	3	3	0	16
15	2	16	1	79	2	17	6	18	2	19	4	14	3	1	3	111	3	3	0	17
20	2	16	1	30	2	17	9	27	2	19	6	30	3	1	3	30	3	3	2	18
27	2	16	1	91	2	17	9	30	2	19	6	111	3	1	3	15	3	3	3	19
30	2	16	1	15	2	17	10	91	2	19	6	18	3	1	4	20	3	3	3	20
91	2	16	1	20	2	17	10	14	2	19	7	27	3	1	4	27	3	3	4	21
79	2	16	2	27	2	17	10	15	2	19	7	91	3	1	4	91	3	3	4	22
119	2	16	4	14	2	18	0	20	2	19	7	15	3	1	5	18	3	3	6	23
14	2	16	5	111	2	18	0	111	2	19	7	20	3	1	5	42	3	3	7	24
111	2	16	5	117	2	18	2	117	2	19	11	42	3	1	9	41	3	3	8	25
117	2	16	5	119	2	18	2	44	2	19	11	44	3	1	9	44	3	3	8	26
10	2	16	6	44	2	18	2	35	3	0	0	35	3	1	10	48	3	3	8	27
44	2	16	6	35	2	18	3	42	3	0	0	117	3	1	10	12	3	3	9	28
35	2	16	7	42	2	18	3	48	3	0	1	48	3	1	11	35	3	3	9	29
42	2	16	7	77	2	18	4	77	3	0	1	77	3	2	0	117	3	3	9	30
77	2	16	7	48	2	18	5	119	3	0	2	12	3	2	0	77	3	3	11	31
12	2	16	9	12	2	18	6	12	3	0	3	119	3	2	2	89	3	4	1	32
48	2	16	10	10	2	18	7	10	3	0	4	10	3	2	3	10	3	4	2	33
99	2	16	11	99	2	18	8	21	3	0	7	89	3	2	4	17	3	4	2	34
21	2	17	0	21	2	18	0	99	3	0	7	109	3	2	4	25	3	4	2	35
75	2	17	2	25	2	18	11	109	3	0	7	25	3	2	5	119	3	4	2	36
93	2	17	2	75	2	18	11	25	3	0	8	21	3	2	6	109	3	4	3	37
25	2	17	3	19	2	19	0	89	3	0	8	99	3	2	6	92	3	4	5	38
19	2	17	4	33	2	19	0	33	3	0	9	17	3	2	6	19	3	4	6	39
33	2	17	5	93	2	19	0	75	3	0	9	33	3	2	7	21	3	4	6	40
7	2	17	6	109	2	19	0	17	3	0	9	19	3	2	8	33	3	4	6	41
23	2	17	6	89	2	19	1	19	3	0	10	75	3	2	8	97	3	4	6	42
36	2	17	6	17	2	19	2	93	3	0	10	97	3	2	8	99	3	4	6	43
51	2	17	6	23	2	19	3	68	3	0	10	68	3	2	8	112	3	4	7	44
58	2	17	6	36	2	19	3	23	3	1	0	93	3	2	9	108	3	4	7	45
81	2	17	6	51	2	19	3	72	3	1	0	72	3	2	10	75	3	4	8	46
97	2	17	6	58	2	19	3	97	3	1	0	92	3	2	10	68	3	4	8	47
106	2	17	6	68	2	19	3	36	3	1	1	108	3	2	10	93	3	4	8	48
110	2	17	6	72	2	19	3	51	3	1	1	112	3	2	11	72	3	4	9	49
118	2	17	6	81	2	19	3	58	3	1	1	23	3	3	0	61	3	4	10	50
120	2	17	6	106	2	19	3	81	3	1	1	36	3	3	0	36	3	4	11	51
72	2	17	7	110	2	19	3	106	3	1	1	51	3	3	0	49	3	4	11	52
89	2	17	7	118	2	19	3	110	3	1	1	58	3	3	0	105	3	4	11	53
109	2	17	7	120	2	19	3	120		1	1	61	3	3	0	106	3	4	11	54
116	2	17	8	7	2	19	4	7	3	1	2	81	3	3	0	110	3	4	11	55
17	2	17	8	55	2	19	4	92	3	1	2	106	3	3	0	4	3	5	0	56
55	2	17	8	97	2	19	4	55	3	1	2	110	3	3	0	7	3	5	0	57
68	2	17	8	116	2	19	5	118	3	1	2	7	3	3	1	23	3	5	0	58
104	2	17	11	92	2	19	7	108	3	1	3	49	3	3	1	45	3	5	0	59
59	2	18	0	104	2	19	7	61	3	1	3	55	3	3	2	51	3	5	0	60

Age 36.				Age 37.				Age 38.				Age 39.				Age 40.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
54	2	18	0	49	2	19	8	49	3	1	4	104	3	3	2	58	3	5	0	61
61	2	18	0	61	2	19	8	57	3	1	4	105	3	3	2	81	3	5	0	62
92	2	18	0	54	2	19	9	104	3	1	4	120	3	5	2	84	3	5	0	63
50	2	18	1	59	2	19	9	112	3	1	4	45	3	3	3	118	3	5	0	64
62	2	18	1	50	2	19	9	116	3	1	4	84	3	3	3	101	3	5	0	65
11	2	18	2	102	2	19	9	54	3	1	6	101	3	3	3	120	3	5	0	66
31	2	18	2	108	2	19	9	59	3	1	6	118	3	3	3	63	3	5	1	67
49	2	18	2	112	2	19	9	50	3	1	6	4	3	3	4	57	3	5	2	68
57	2	18	2	31	2	19	10	101	3	1	6	57	3	3	4	104	3	5	2	69
74	2	18	2	62	2	19	10	102	3	1	6	50	3	3	4	50	3	5	3	70
78	2	18	2	101	2	19	10	105	3	1	6	63	3	3	4	55	3	5	3	71
114	2	18	2	105	2	19	10	31	3	1	7	59	3	3	5	28	3	5	5	72
24	2	18	3	24	2	19	11	45	3	1	7	116	3	3	5	59	3	5	5	73
101	2	18	3	4	2	0	0	63	3	1	7	54	3	3	6	24	3	5	6	74
102	2	18	3	11	3	0	0	84	3	1	7	102	3	3	6	54	3	5	6	75
108	2	18	3	45	3	0	0	4	3	1	8	24	3	3	7	96	3	5	6	76
2	2	18	4	57	3	0	0	24	3	1	8	28	3	3	7	102	3	5	6	77
26	2	18	4	63	3	0	0	62	3	1	9	31	3	3	7	116	3	5	6	78
87	2	18	4	74	3	0	0	96	3	1	9	95	3	3	7	31	3	5	7	79
96	2	18	4	78	3	0	0	28	3	1	10	62	3	3	8	62	3	5	8	80
105	2	18	4	84	3	0	0	11	3	1	11	71	3	3	10	71	3	5	9	81
112	2	18	4	96	3	0	0	74	3	1	11	74	3	3	10	87	3	5	10	82
3	2	18	5	114	3	0	0	78	3	1	11	78	3	3	10	2	3	5	11	83
4	2	18	5	2	3	0	1	87	3	1	11	114	3	3	10	26	3	5	11	84
63	2	18	5	87	3	0	1	114	3	1	11	1	3	3	11	74	3	5	11	85
107	2	18	5	26	3	0	2	2	3	2	0	2	3	3	11	56	3	5	11	86
113	2	18	5	28	3	0	2	26	3	2	0	11	3	3	11	78	3	5	11	87
32	2	18	6	76	3	0	2	71	3	2	0	26	3	3	11	114	3	5	11	88
45	2	18	6	1	3	0	3	76	3	2	0	87	3	3	11	1	3	6	0	89
76	2	18	6	32	3	0	3	1	3	2	2	56	3	4	1	11	3	6	0	90
84	2	18	6	71	3	0	3	32	3	2	3	76	3	4	1	76	3	6	1	91
1	2	18	7	3	3	0	4	3	3	2	4	32	3	4	3	115	3	6	1	92
28	2	18	7	107	3	0	4	56	3	2	4	6	3	4	4	6	3	6	3	93
71	2	18	7	113	3	0	4	107	3	2	4	65	3	4	4	65	3	6	3	94
6	2	19	0	56	3	0	8	113	3	2	4	70	3	4	4	70	3	6	3	95
65	2	19	0	6	3	0	9	6	3	2	6	73	3	4	4	73	3	6	3	96
70	2	19	0	65	3	0	9	65	3	2	6	85	3	4	4	85	3	6	3	97
73	2	19	0	67	3	0	9	70	3	2	6	115	3	4	4	90	3	6	3	98
85	2	19	0	70	3	0	9	73	3	2	6	3	3	4	5	95	3	6	3	99
56	2	19	1	73	3	0	9	85	3	2	6	67	3	4	5	94	3	6	3	100
66	2	19	1	85	3	0	9	67	3	2	7	94	3	4	5	100	3	6	3	101
67	2	19	1	82	3	0	11	82	3	2	8	107	3	4	5	67	3	6	4	102
16	2	19	3	66	3	1	0	115	3	2	8	113	3	4	5	82	3	6	5	103
82	2	19	3	16	3	1	1	90	3	2	9	82	3	4	6	3	3	6	6	104
69	2	19	4	115	3	1	1	94	3	2	11	90	3	4	6	32	3	6	6	105
115	2	19	5	69	3	1	2	16	3	3	0	95	3	4	6	107	3	6	6	106
90	2	19	8	90	3	1	2	66	3	3	0	100	3	4	6	113	3	6	6	107
94	2	19	8	94	3	1	2	95	3	3	0	16	3	5	0	16	3	6	9	108
95	2	19	9	95	3	1	3	100	3	3	0	66	3	5	1	66	3	7	4	109
100	2	19	9	100	3	1	3	69	3	3	3	69	3	5	4	69	3	7	6	110
43	3	0	0	43	3	1	10	43	3	3	9	43	3	5	9	43	3	7	10	111
34	3	1	4	34	3	2	10	34	3	4	6	34	3	6	2	34	3	7	11	112
46	3	1	4	46	3	2	10	46	3	4	6	46	3	6	2	46	3	7	11	113
53	3	1	4	53	3	2	10	53	3	4	6	53	3	6	2	53	3	7	11	114
88	3	1	4	88	3	2	10	88	3	4	6	88	3	6	2	88	3	7	11	115
9	3	1	10	9	3	3	8	9	3	5	9	9	3	7	10	9	3	10	0	116
38	3	3	0	38	3	4	10	38	3	6	10	38	3	10	4	38	3	12	4	117
64	3	8	9	64	3	10	9	64	3	12	9	64	3	14	9	64	3	17	0	118
8				8				8				8				8				119
103				103				103				103				103				120

Age 41.				Age 42.				Age 43.				Age 44.				Age 45.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
98	2	16	8	98	2	18	8	98	3	0	11	98	3	3	3	98	3	5	9	1
37	2	19	8	37	3	1	4	37	3	3	2	29	3	5	0	37	3	7	0	2
29	3	1	10	29	3	4	1	29	3	6	6	37	3	9	0	79	3	11	0	3
39	3	2	0	39	3	4	2	39	3	6	6	39	3	9	0	80	3	11	7	4
52	3	2	6	80	3	4	5	5	3	6	11	79	3	9	0	29	3	11	9	5
5	3	2	7	52	3	4	8	52	3	6	11	80	3	9	2	39	3	11	9	6
80	3	2	7	5	3	4	9	80	3	6	11	52	3	9	3	52	3	11	9	7
79	3	3	6	79	3	5	2	22	3	7	0	5	3	9	4	5	3	11	10	8
22	3	4	0	22	3	6	0	79	3	8	0	22	3	10	0	14	3	12	2	9
13	3	4	3	13	3	6	2	60	3	8	1	14	3	10	2	111	3	12	2	10
60	3	4	3	60	3	6	2	13	3	8	2	48	3	11	2	40	3	12	3	11
40	3	4	4	14	3	6	6	14	3	8	3	60	3	10	2	22	3	12	4	12
14	3	4	9	40	3	6	6	40	3	8	3	111	3	10	2	60	3	12	5	13
111	3	4	9	111	3	6	6	111	3	8	3	13	3	10	3	13	3	12	8	14
47	3	4	10	15	3	7	2	48	3	9	2	40	3	10	3	17	3	13	4	15
15	3	5	2	20	3	7	2	15	3	9	4	17	3	11	4	48	3	13	4	16
20	3	5	2	47	3	7	2	20	3	9	4	15	3	11	8	75	3	13	8	17
30	3	5	2	30	3	7	3	17	3	9	5	20	3	11	8	99	3	13	9	18
83	3	5	2	48	3	7	4	30	3	9	5	75	3	11	8	25	3	13	10	19
86	3	5	4	42	3	7	6	117	3	9	7	25	3	11	9	58	3	14	0	20
27	3	5	5	91	3	7	7	25	3	9	9	30	3	11	9	97	3	14	0	21
91	3	5	5	117	3	7	7	42	3	9	9	99	3	11	9	117	3	14	0	22
42	3	5	6	17	3	7	8	47	3	9	9	117	3	11	9	112	3	14	1	23
48	3	5	6	27	3	7	8	77	3	9	10	58	3	12	0	15	3	14	2	24
12	3	5	8	83	3	7	8	99	3	9	10	77	3	12	0	20	3	14	2	25
41	3	5	8	86	3	7	9	75	3	9	11	112	3	12	0	30	3	14	2	26
117	3	5	8	25	3	7	10	91	3	9	11	42	3	12	2	77	3	14	3	27
18	3	5	9	77	3	7	10	27	3	10	0	89	3	12	2	89	3	14	6	28
44	3	5	9	44	3	7	11	86	3	10	0	86	3	12	3	86	3	14	9	29
77	3	5	10	89	3	7	11	89	3	10	0	41	3	12	4	105	3	14	9	30
17	3	5	11	12	3	8	0	112	3	10	0	47	3	12	4	21	3	14	10	31
35	3	5	11	35	3	8	0	35	3	10	3	97	3	12	4	41	3	14	10	32
89	3	5	11	41	3	8	0	58	3	10	3	27	3	12	5	42	3	14	10	33
25	3	6	0	99	3	8	0	44	3	10	3	91	3	12	5	120	3	14	10	34
10	3	6	2	18	3	8	1	68	3	10	3	68	3	12	6	45	3	14	11	35
119	3	6	2	112	3	8	1	12	3	10	4	120	3	12	6	84	3	14	11	36
92	3	6	3	75	3	8	2	41	3	10	4	21	3	12	7	12	3	15	0	37
99	3	6	3	68	3	8	3	83	3	10	4	105	3	12	7	27	3	15	0	38
68	3	6	4	10	3	8	4	21	3	10	5	12	3	12	8	68	3	15	0	39
109	3	6	4	21	3	8	4	119	3	10	5	45	3	12	8	91	3	15	0	40
112	3	6	4	119	3	8	4	120	3	10	5	84	3	12	8	119	3	15	0	41
21	3	6	5	120	3	8	4	97	3	10	6	119	3	12	8	101	3	15	1	42
75	3	6	5	92	3	8	5	10	3	10	7	35	3	12	9	35	3	15	3	43
120	3	6	5	58	3	8	6	105	3	10	7	44	3	12	9	47	3	15	3	44
33	3	6	6	97	3	8	6	18	3	10	8	101	3	12	11	19	3	15	4	45
97	3	6	6	109	3	8	6	45	3	10	8	93	3	13	0	44	3	15	4	46
19	3	6	7	105	3	8	7	84	3	10	8	10	3	13	1	81	3	15	4	47
93	3	6	8	19	3	8	8	92	3	10	8	83	3	13	1	93	3	15	4	48
108	3	6	8	33	3	8	8	93	3	10	10	110	3	13	1	7	3	15	5	49
4	3	6	9	45	3	8	8	101	3	10	10	7	3	13	2	36	3	15	5	50
45	3	6	9	84	3	8	8	109	3	10	10	33	3	13	2	51	3	15	5	51
58	3	6	9	93	3	8	8	19	3	10	11	36	2	13	2	96	3	15	5	52
61	3	6	9	4	3	8	9	33	3	10	11	51	3	13	2	106	3	15	5	53
84	3	6	9	101	3	8	9	51	3	10	11	81	3	13	2	110	3	15	5	54
105	3	6	9	61	3	8	10	110	3	10	11	92	3	13	2	118	3	15	5	55
49	3	6	10	108	3	8	10	118	3	10	11	106	3	13	2	23	3	15	6	56
101	3	6	10	7	3	8	11	4	3	11	0	118	3	13	2	10	3	15	7	57
7	3	6	11	36	3	8	11	7	3	11	0	23	3	13	3	33	3	15	7	58
36	3	6	11	51	3	8	11	23	3	11	0	96	3	13	3	92	3	15	7	59
51	3	6	11	106	3	8	11	36	3	11	0	4	3	13	4	4	3	15	9	60

Age 41.				Age 42.				Age 43.				Age 44.				Age 45.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
72	3	6	11	110	3	8	11	106	3	11	0	18	3	13	4	61	3	15	9	61
106	3	6	11	49	3	8	11	49	3	11	1	19	3	13	4	49	3	15	10	62
110	3	6	11	23	3	9	0	61	3	11	1	109	3	13	4	63	3	15	10	63
23	3	7	0	63	3	9	0	63	3	11	1	49	3	13	5	109	3	15	11	64
63	3	7	0	118	3	9	0	108	3	11	1	61	3	13	5	108	3	16	0	65
118	3	7	0	72	3	9	1	81	3	11	2	63	3	13	5	83	3	16	1	66
81	3	7	1	81	3	9	1	96	3	11	2	108	3	13	6	16	3	16	2	67
55	3	7	2	96	3	9	3	72	3	11	6	72	3	13	11	115	3	16	2	68
104	3	7	2	28	3	9	5	28	3	11	8	16	3	14	0	18	3	16	3	69
50	3	7	3	50	3	9	5	62	3	11	9	28	3	14	0	62	3	16	3	70
28	3	7	4	104	3	9	5	104	3	11	9	62	3	14	0	94	3	16	3	71
57	3	7	4	57	3	9	6	50	3	11	10	115	3	14	0	95	3	16	4	72
96	3	7	4	55	3	9	8	115	3	11	11	94	3	14	1	100	3	16	4	73
31	3	7	7	62	3	9	8	2	3	12	0	95	3	14	1	26	3	16	6	74
54	3	7	7	24	3	9	10	16	3	12	0	100	3	14	1	28	3	16	6	75
59	3	7	7	31	3	9	10	26	3	12	0	116	3	14	1	2	3	16	7	76
102	3	7	7	54	3	9	10	55	3	12	0	2	3	14	3	56	3	16	7	77
24	3	7	8	56	3	9	10	56	3	12	0	26	3	14	3	72	3	16	7	78
62	3	7	8	59	3	9	10	57	3	12	0	56	3	14	3	76	3	16	7	79
116	3	7	8	102	3	9	10	76	3	12	0	104	3	14	3	104	3	16	10	80
71	3	7	9	26	3	9	11	94	3	12	0	50	3	14	4	50	3	17	0	81
56	3	7	10	71	3	9	11	95	3	12	0	76	3	14	4	55	3	17	0	82
2	3	7	11	76	3	9	11	100	3	12	0	55	3	14	6	90	3	17	0	83
26	3	7	11	115	3	9	11	24	3	12	1	57	3	14	6	31	3	17	2	84
76	3	7	11	116	3	9	11	31	3	12	1	24	3	14	7	54	3	17	2	85
115	3	7	11	2	3	10	0	54	3	12	1	31	3	14	7	57	3	17	2	86
1	3	8	0	95	3	10	0	59	3	12	1	54	3	14	7	59	3	17	2	87
87	3	8	0	94	3	10	0	71	3	12	1	59	3	14	7	71	3	17	2	88
74	3	8	1	100	3	10	0	102	3	12	3	71	3	14	7	102	3	17	2	89
78	3	8	1	1	3	10	2	116	3	12	3	102	3	14	7	116	3	17	2	90
114	3	8	1	16	3	10	3	1	3	12	5	90	3	14	8	24	3	17	3	91
11	3	8	2	87	3	10	3	90	3	12	5	6	3	14	11	6	3	17	4	92
90	3	8	2	90	3	10	3	6	3	12	7	65	3	14	11	65	3	17	4	93
94	3	8	2	6	3	10	5	65	3	12	7	70	3	14	11	70	3	17	4	94
95	3	8	2	65	3	10	5	70	3	12	7	73	3	14	11	73	3	17	4	95
100	3	8	2	73	3	10	5	73	3	12	7	85	3	14	11	82	3	17	4	96
6	3	8	4	74	3	10	5	85	3	12	7	82	3	15	0	85	3	17	4	97
65	3	8	4	70	3	10	5	82	3	12	8	1	3	15	1	1	3	17	7	98
70	3	8	4	78	3	10	5	32	3	12	9	3	3	15	3	3	3	17	8	99
73	3	8	4	85	3	10	5	74	3	12	10	32	3	15	3	107	3	17	8	100
85	3	8	4	114	3	10	5	87	3	12	10	107	3	15	3	113	3	17	8	101
67	3	8	5	11	3	10	6	3	3	12	11	113	3	15	3	32	3	17	9	102
82	3	8	5	82	3	10	6	11	3	12	11	87	3	15	4	34	3	17	11	103
16	3	8	6	3	3	10	9	78	3	12	11	11	3	15	5	46	3	17	11	104
3	3	8	7	32	3	10	9	107	3	12	11	74	3	15	5	53	3	17	11	105
107	3	8	7	67	3	10	9	113	3	12	11	78	3	15	5	69	3	17	11	106
113	3	8	7	107	3	10	9	114	3	12	11	114	3	15	5	88	3	17	11	107
32	3	8	9	113	3	10	9	67	3	13	4	34	3	15	9	87	3	18	0	108
66	3	9	8	34	3	11	8	34	3	13	8	46	3	15	9	78	3	18	1	109
34	3	9	9	46	3	11	8	46	3	13	8	53	3	15	9	114	3	18	1	110
46	3	9	9	53	3	11	8	53	3	13	8	69	3	15	9	11	3	18	2	111
53	3	9	9	69	3	11	8	66	3	13	8	88	3	15	9	74	3	18	2	112
69	3	9	9	88	3	11	8	69	3	13	8	67	3	16	2	67	3	18	11	113
88	3	9	9	66	3	12	1	88	3	14	8	66	3	17	5	66	4	0	3	114
43	3	10	1	43	3	12	6	43	3	15	1	43	3	17	9	43	4	0	7	115
9	3	12	5	9	3	14	10	9	3	17	6	9	4	0	3	38	4	2	10	116
38	3	14	10	38	3	17	2	38	3	19	2	38	4	1	0	9	4	3	2	117
64	3	19	3	64	4	1	6	64	4	4	0	64	4	6	9	64	4	9	9	118
8				8				8				8				8				119
103				103				103				103				103				120

Age 46.				Age 47.				Age 48.				Age 49.				Age 50.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
98	3	8	5	37	3	11	5	37	3	13	10	37	3	16	6	37	3	19	3	1
37	3	9	2	98	3	11	5	98	3	14	8	98	3	18	1	98	4	1	7	2
79	3	13	8	79	3	16	3	111	3	19	4	111	4	2	3	80	4	5	2	3
1	3	14	0	80	3	16	8	40	3	19	6	80	4	2	4	111	4	5	6	4
80	3	14	1	111	3	16	9	79	3	19	6	40	4	2	6	14	4	5	11	5
52	3	14	5	1	3	16	10	80	3	19	6	14	4	2	7	40	4	6	0	6
111	3	14	5	14	3	16	11	14	3	19	7	79	4	2	9	79	4	6	0	7
5	3	14	6	40	3	17	0	1	3	19	11	1	4	3	2	13	4	6	3	8
14	3	14	6	52	3	17	3	5	4	0	3	5	4	3	4	1	4	6	7	9
40	3	14	6	5	3	17	4	13	4	0	3	13	4	3	5	5	4	6	8	10
29	3	14	7	13	3	17	5	52	4	0	3	52	4	3	5	75	4	6	8	11
39	3	14	7	60	3	17	5	60	4	0	3	60	4	3	5	52	4	6	9	12
13	3	14	10	22	3	17	6	22	4	0	4	75	4	3	6	60	4	6	10	13
22	3	14	10	29	3	17	8	17	4	0	7	17	4	3	7	17	4	7	0	14
60	3	14	10	39	3	17	8	75	4	0	7	22	4	3	8	48	4	7	0	15
48	3	15	6	17	3	17	11	48	4	0	8	48	4	3	8	58	4	7	0	16
17	3	15	7	48	3	18	0	29	4	0	11	58	4	3	10	99	4	7	3	17
75	3	15	10	75	3	18	1	39	4	0	11	99	4	3	11	22	4	7	4	18
99	3	15	11	99	3	18	4	58	4	0	11	77	4	4	1	77	4	7	4	19
25	3	16	2	58	3	18	5	99	4	0	11	25	4	4	4	29	4	7	6	20
58	3	16	2	112	3	18	6	77	4	1	3	29	4	4	4	39	4	7	6	21
112	3	16	2	25	3	18	8	25	4	1	4	39	4	4	4	112	4	7	6	22
77	3	16	4	77	3	18	8	112	4	1	4	112	4	4	4	25	4	7	8	23
117	3	16	5	117	3	19	1	117	4	1	10	101	4	5	0	101	4	7	9	24
97	3	16	8	97	3	19	4	97	4	2	2	45	4	5	1	45	4	8	0	25
120	3	16	8	120	3	19	4	89	4	2	3	84	4	5	1	84	4	8	0	26
30	3	16	9	30	3	19	6	120	4	2	3	89	4	5	2	89	4	8	3	27
15	3	16	10	89	3	19	6	45	4	2	4	117	4	5	2	108	4	8	4	28
20	3	16	10	105	3	19	7	84	4	2	4	105	4	5	4	97	4	8	6	29
89	3	16	11	45	3	19	8	101	4	2	4	97	4	5	6	105	4	8	6	30
105	3	17	1	84	3	19	8	105	4	2	5	120	4	5	6	117	4	8	9	31
45	3	17	3	15	3	19	10	30	4	2	6	30	4	5	8	81	4	8	11	32
84	3	17	3	20	3	19	10	82	4	2	10	108	4	5	8	30	4	9	0	33
21	3	17	4	101	3	19	10	21	4	2	11	81	4	5	9	94	4	9	1	34
86	3	17	5	21	4	0	0	86	4	3	0	86	4	6	0	95	4	9	1	35
101	3	17	5	86	4	0	0	108	4	3	0	21	4	6	2	115	4	9	1	36
41	3	17	6	81	4	0	2	15	4	3	2	41	4	6	2	120	4	9	1	37
119	3	17	6	119	4	0	4	20	4	3	2	94	4	6	3	100	4	9	2	38
12	3	17	7	23	4	0	6	41	4	3	4	115	4	6	3	68	4	9	6	39
68	3	17	8	93	4	0	6	23	4	3	6	95	4	6	4	86	4	9	6	40
81	3	17	8	108	4	0	6	68	4	3	6	100	4	6	4	41	4	9	8	41
42	3	17	9	7	4	0	7	92	4	3	6	68	4	6	5	4	4	9	9	42
91	3	17	9	36	4	0	7	94	4	3	6	92	4	6	7	92	4	9	9	43
7	3	17	10	51	4	0	7	115	4	3	6	4	4	6	9	21	4	9	10	44
93	3	17	10	96	4	0	7	119	4	3	6	15	4	6	10	49	4	10	2	45
35	3	17	11	106	4	0	7	7	4	3	7	20	4	6	10	61	4	10	2	46
36	3	17	11	118	4	0	7	36	4	3	7	93	4	6	10	35	4	10	3	47
51	3	17	11	41	4	0	8	51	4	3	7	7	4	6	11	42	4	10	3	48
96	3	17	11	92	4	0	8	93	4	3	7	35	4	6	11	109	4	10	5	49
106	3	17	11	110	4	0	8	95	4	3	7	36	4	6	11	7	4	10	6	50
110	3	17	11	35	4	0	9	96	4	3	7	49	4	6	11	36	4	10	6	51
118	3	17	11	42	4	0	9	100	4	3	7	51	4	6	11	93	4	10	6	52
23	3	18	0	68	4	0	9	106	4	3	7	96	4	6	11	106	4	10	6	53
92	3	18	0	91	4	0	9	110	4	3	7	106	4	6	11	110	4	10	6	54
27	3	18	1	12	4	0	10	118	4	3	8	110	4	6	11	118	4	10	6	55
44	3	18	1	16	4	0	11	35	4	3	9	118	4	6	11	12	4	10	7	56
10	3	18	2	115	4	0	11	61	4	3	9	16	4	7	0	34	4	10	7	57
108	3	18	2	4	4	1	0	4	4	3	10	23	4	7	0	51	4	10	7	58
4	3	18	3	94	4	1	0	16	4	3	10	42	4	7	0	63	4	10	7	59
33	3	18	3	33	4	1	1	42	4	3	10	61	4	7	0	82	4	10	7	60

Age 46.				Age 47.				Age 48.				Age 49.				Age 50.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
47	3	18	3	44	4	1	1	91	4	3	10	119	4	7	0	91	4	10	7	61
61	3	18	4	49	4	1	1	49	4	3	11	91	4	7	1	96	4	10	7	62
63	3	18	4	61	4	1	1	12	4	4	1	109	4	7	2	15	4	10	8	63
16	3	18	5	95	4	1	1	63	4	4	1	63	4	7	3	20	4	10	8	64
49	3	18	5	100	4	1	1	33	4	4	2	12	4	7	4	53	4	10	8	65
94	3	18	6	10	4	1	2	44	4	4	2	33	4	7	4	88	4	10	8	66
115	3	18	6	63	4	1	2	109	4	4	2	44	4	7	5	119	4	10	8	67
19	3	18	7	109	4	1	4	10	4	4	3	32	4	7	6	23	4	10	9	68
95	3	18	7	19	4	1	5	32	4	4	6	10	4	7	7	32	4	10	9	69
100	3	18	7	27	4	1	5	62	4	4	6	56	4	7	7	33	4	10	9	70
109	3	18	7	47	4	1	5	56	4	4	7	34	4	7	10	56	4	10	9	71
83	3	18	8	62	4	1	6	19	4	4	8	46	4	7	10	16	4	10	10	72
62	3	18	9	83	4	1	6	83	4	4	8	53	4	7	10	44	4	10	11	73
32	3	19	0	32	4	1	9	2	4	4	9	82	4	7	10	6	4	11	1	74
2	3	19	1	56	4	1	9	47	4	4	9	88	4	7	10	10	4	11	1	75
26	3	19	1	26	4	1	10	26	4	4	10	6	4	8	0	65	4	11	1	76
56	3	19	1	2	4	1	10	27	4	4	10	28	4	8	0	70	4	11	1	77
76	3	19	1	28	4	1	11	28	4	4	10	62	4	8	0	73	4	11	1	78
28	3	19	2	76	4	1	11	76	4	4	10	65	4	8	0	85	4	11	1	79
18	3	19	3	72	4	2	4	34	4	5	1	70	4	8	0	90	4	11	3	80
72	3	19	4	90	4	2	4	46	4	5	1	73	4	8	0	28	4	11	4	81
90	3	19	7	6	4	2	5	53	4	5	1	85	4	8	0	46	4	11	6	82
104	3	19	7	65	4	2	5	69	4	5	1	90	4	8	1	19	4	11	9	83
55	3	19	8	70	4	2	5	82	4	5	1	19	4	8	2	113	4	11	10	84
6	3	19	10	73	4	2	5	88	4	5	1	26	4	8	2	2	4	11	11	85
50	3	19	10	82	4	2	5	90	4	5	1	83	4	8	2	26	4	11	11	86
65	3	19	10	85	4	2	5	6	4	5	2	2	4	8	3	76	4	11	11	87
70	3	19	10	18	4	2	6	65	4	5	2	76	4	8	3	55	4	12	0	88
71	3	19	10	104	4	2	6	70	4	5	2	47	4	8	4	67	4	12	0	89
73	3	19	10	34	4	2	7	73	4	5	2	113	4	8	4	116	4	12	0	90
82	3	19	10	46	4	2	7	85	4	5	2	27	4	8	6	47	4	12	1	91
85	3	19	10	53	4	2	7	113	4	5	2	69	4	8	9	62	4	12	2	92
116	3	19	10	55	4	2	7	72	4	5	5	72	4	8	9	83	4	12	2	93
24	3	19	11	69	4	2	7	104	4	5	7	104	4	8	10	104	4	12	2	94
31	3	19	11	88	4	2	7	55	4	5	9	55	4	9	0	72	4	12	3	95
54	4	0	0	113	4	2	7	71	4	5	10	116	4	9	1	27	4	12	4	96
59	4	0	0	71	4	2	9	18	4	5	11	54	4	9	2	54	4	12	7	97
102	4	0	0	31	4	2	10	54	4	5	11	59	4	9	2	59	4	12	7	98
34	4	0	2	54	4	2	10	59	4	5	11	67	4	9	2	69	4	12	7	99
46	4	0	2	59	4	2	10	102	4	5	11	71	4	9	2	102	4	12	7	100
53	4	0	2	102	4	2	10	24	4	6	0	102	4	9	2	71	4	12	8	101
57	4	0	2	116	4	2	10	31	4	6	0	31	4	9	4	24	4	13	0	102
69	4	0	2	50	4	2	11	50	4	6	2	24	4	9	5	31	4	13	0	103
88	4	0	2	24	4	3	0	116	4	6	3	18	4	9	6	18	4	13	4	104
113	4	0	2	57	4	3	2	57	4	6	4	50	4	9	8	57	4	13	4	105
3	4	0	5	3	4	3	3	67	4	6	5	57	4	9	8	50	4	13	6	106
107	4	0	5	107	4	3	3	3	4	6	6	3	4	10	2	3	4	14	2	107
87	4	0	11	67	4	3	10	107	4	6	6	107	4	10	2	74	4	14	2	108
11	4	1	0	11	4	4	0	74	4	7	2	11	4	10	7	78	4	14	2	109
74	4	1	0	74	4	4	0	78	4	7	2	74	4	10	7	87	4	14	2	110
78	4	1	0	78	4	4	0	87	4	7	2	78	4	10	7	107	4	14	2	111
114	4	1	0	87	4	4	0	114	4	7	2	87	4	10	7	114	4	14	2	112
66	4	1	3	114	4	4	0	11	4	7	3	114	4	10	7	11	4	14	3	113
67	4	3	3	66	4	6	6	66	4	9	10	66	4	13	6	66	4	17	4	114
43	4	3	8	43	4	6	10	43	4	10	3	43	4	13	10	43	4	17	8	115
9	4	5	4	38	4	8	6	9	4	12	0	9	4	16	8	9	5	0	6	116
38	4	6	4	9	4	9	7	38	4	13	0	38	4	16	8	38	5	0	10	117
64	4	12	9	64	4	16	3	64	4	19	9	64	5	3	6	64	5	7	6	118
8				8				8				8				8				119
103				103				103				103				103				120

Age 51.				Age 52.				Age 53.				Age 54.				Age 55.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d		£	s.	d.	
37	4	2	2	37	4	5	5	37	4	8	11	37	4	12	8	37	4	16	8	1
98	4	5	6	98	4	9	5	98	4	13	5	98	4	17	8	80	5	1	6	2
80	4	8	2	80	4	11	3	13	4	14	7	80	4	17	11	98	5	1	11	3
111	4	9	1	13	4	12	4	80	4	16	7	52	5	0	6	52	5	4	4	4
13	4	9	4	111	4	12	10	52	4	16	10	29	5	0	11	29	5	4	8	5
40	4	9	6	79	4	13	3	111	4	16	11	39	5	0	11	39	5	4	8	6
14	4	9	7	52	4	13	5	79	4	17	0	45	5	0	11	45	5	4	8	7
79	4	9	8	40	4	13	6	29	4	17	5	84	5	0	11	84	5	4	8	8
52	4	10	1	14	4	13	7	45	4	17	5	79	5	1	0	95	5	5	1	9
5	4	10	2	5	4	13	10	39	4	17	5	13	5	1	2	94	5	5	1	10
17	4	10	5	29	4	14	1	84	4	17	5	111	5	1	2	100	5	5	1	11
75	4	10	6	39	4	14	1	112	4	17	8	112	5	1	4	79	5	5	3	12
48	4	10	8	112	4	14	1	5	4	17	9	94	5	1	6	112	5	5	4	13
112	4	10	9	45	4	14	2	14	4	17	11	95	5	1	7	115	5	5	6	14
58	4	10	10	84	4	14	2	40	4	18	0	100	5	1	7	111	5	5	10	15
60	4	10	10	17	4	14	3	94	4	18	2	5	5	1	11	13	5	6	1	16
77	4	10	10	48	4	14	6	95	4	18	3	115	5	1	11	5	5	6	4	17
29	4	10	11	77	4	14	7	100	4	18	3	108	5	2	5	34	5	6	4	18
39	4	10	11	75	4	14	8	17	4	18	6	14	5	2	7	53	5	6	4	19
99	4	10	11	99	4	14	10	77	4	18	6	40	5	2	7	82	5	6	4	20
45	4	11	0	60	4	14	11	115	4	18	6	105	5	2	8	88	5	6	4	21
84	4	11	0	58	4	15	0	48	4	18	7	77	5	2	9	108	5	6	4	22
22	4	11	4	94	4	15	0	108	4	18	7	17	5	2	10	105	5	6	11	23
25	4	11	4	95	4	15	0	105	4	18	10	34	5	2	10	77	5	7	3	24
105	4	11	7	100	4	15	0	99	4	18	11	53	5	2	10	92	5	7	6	25
108	4	11	8	108	4	15	0	75	4	19	0	82	5	2	10	14	5	7	8	26
94	4	12	0	105	4	15	1	60	4	19	4	88	5	2	10	17	5	7	8	27
95	4	12	0	115	4	15	3	58	4	19	4	48	5	3	0	40	5	7	8	28
97	4	12	0	25	4	15	4	34	4	19	7	99	5	3	4	48	5	7	9	29
100	4	12	0	22	4	15	8	53	4	19	7	68	5	3	7	68	5	7	11	30
115	4	12	1	81	4	16	0	82	4	19	7	75	5	3	8	99	5	8	1	31
81	4	12	4	97	4	16	0	88	4	19	7	92	5	3	8	90	5	8	6	32
120	4	12	4	68	4	16	1	25	4	19	8	61	5	3	9	6	5	8	8	33
89	4	12	5	30	4	16	5	68	4	19	8	58	5	4	0	61	5	8	8	34
30	4	12	7	34	4	16	5	81	4	19	11	60	5	4	0	65	5	8	8	35
68	4	12	8	53	4	16	5	92	4	19	11	81	5	4	1	73	5	8	8	36
117	4	12	9	82	4	16	5	97	5	0	0	25	5	4	3	85	5	8	8	37
4	4	13	0	88	4	16	5	22	5	0	2	4	5	4	6	4	5	8	9	38
86	4	13	0	92	4	16	5	4	5	0	4	97	5	4	6	75	5	8	9	39
92	4	13	0	4	4	16	6	61	5	0	4	30	5	4	7	42	5	8	11	40
91	4	13	1	120	4	16	6	30	5	0	5	49	5	4	8	49	5	8	11	41
41	4	13	4	101	4	16	7	49	5	0	9	116	5	4	8	63	5	8	11	42
61	4	13	5	89	4	16	8	42	5	0	11	6	5	4	9	30	5	9	1	43
34	4	13	6	61	4	16	10	89	5	1	0	65	5	4	9	58	5	9	1	44
49	4	13	6	117	4	17	0	120	5	1	0	73	5	4	9	60	5	9	1	45
53	4	13	6	49	4	17	0	6	5	1	1	85	5	4	9	67	5	9	1	46
82	4	13	6	86	4	17	1	65	5	1	1	90	5	4	9	81	5	9	1	47
88	4	13	6	42	4	17	2	73	5	1	1	42	5	4	10	25	5	9	2	48
42	4	13	8	41	4	17	6	85	5	1	1	101	5	4	10	97	5	9	6	49
21	4	13	10	63	4	17	6	63	5	1	2	22	5	5	0	109	5	9	6	50
109	4	13	10	109	4	17	6	90	5	1	4	63	5	5	0	91	5	10	2	51
35	4	13	11	6	4	17	7	109	5	1	4	67	5	5	2	22	5	10	4	52
63	4	14	0	65	4	17	7	91	5	1	5	91	5	5	2	70	5	10	4	53
6	4	14	3	73	4	17	7	116	5	1	6	109	5	5	4	89	5	10	4	54
12	4	14	3	85	4	17	7	117	5	1	6	89	5	5	6	35	5	10	6	55
56	4	14	3	91	4	17	8	86	5	1	7	120	5	5	9	86	5	10	6	56
65	4	14	3	35	4	17	8	67	5	1	8	35	5	5	11	15	5	10	10	57
70	4	14	3	56	4	17	11	35	5	1	9	70	5	6	0	20	5	10	10	58
73	4	14	3	90	4	17	11	70	5	1	11	86	5	6	1	120	5	10	10	59
85	4	14	3	70	4	18	0	56	5	1	11	56	5	6	3	28	5	10	11	60

Age 51.				Age 52.				Age 53.				Age 54.				Age 55.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
93	4	14	5	21	4	18	1	41	5	2	0	117	5	6	3	41	5	11	0	61
96	4	14	5	15	4	18	4	28	5	2	4	15	5	6	6	56	5	11	0	62
7	4	14	6	20	4	18	4	15	5	2	5	20	5	6	6	76	5	11	2	63
15	4	14	6	67	4	18	4	20	5	2	5	28	5	6	6	116	5	11	2	64
20	4	14	6	116	4	18	4	44	5	2	6	41	5	6	8	19	5	11	5	65
23	4	14	6	12	4	18	5	33	5	2	7	76	5	6	9	44	5	11	5	66
90	4	14	6	44	4	18	5	21	5	2	8	44	5	6	10	117	5	11	5	67
44	4	14	6	28	4	18	6	76	5	2	9	12	5	7	0	12	5	11	9	68
119	4	14	6	33	4	18	6	119	5	2	9	19	5	7	0	119	5	12	0	69
33	4	14	7	96	4	18	6	12	5	2	10	119	5	7	3	104	5	12	5	70
36	4	14	7	119	4	18	6	96	5	2	10	33	5	7	5	33	5	12	6	71
51	4	14	7	7	4	18	10	19	5	2	11	21	5	7	6	96	5	12	6	72
106	4	14	7	93	4	18	10	10	5	3	5	96	5	7	6	46	5	12	8	73
110	4	14	7	118	4	18	10	7	5	3	6	46	5	8	0	55	5	12	8	74
118	4	14	7	36	4	18	11	23	5	3	6	104	5	8	0	21	5	12	9	75
32	4	14	9	51	4	18	11	36	5	3	6	93	5	8	2	72	5	13	0	76
10	4	14	10	76	4	18	11	46	5	3	6	10	5	8	3	54	5	13	4	77
28	4	14	10	106	4	18	11	51	5	3	6	55	5	8	3	57	5	13	4	78
16	4	15	0	110	4	18	11	55	5	3	6	36	5	8	4	59	5	13	4	79
67	4	15	2	10	4	19	0	93	5	3	6	72	5	8	4	93	5	13	4	80
46	4	15	3	19	4	19	0	106	5	3	6	106	5	8	4	102	5	13	4	81
19	4	15	4	23	4	19	0	110	5	3	6	110	5	8	4	10	5	13	7	82
116	4	15	4	32	4	19	0	118	5	3	7	118	5	8	4	36	5	13	7	83
76	4	15	5	46	4	19	3	104	5	3	8	51	5	8	5	106	5	13	7	84
113	4	15	6	16	4	19	4	32	5	3	9	23	5	8	6	7	5	13	8	85
104	4	15	9	113	4	19	5	113	5	3	9	113	5	8	6	27	5	13	8	86
26	4	16	0	55	4	19	6	16	5	4	0	7	5	8	7	51	3	13	8	87
72	4	16	0	104	4	19	9	72	5	4	0	54	5	8	8	71	5	13	8	88
55	4	16	0	72	4	19	10	54	5	4	4	59	5	8	8	87	5	13	8	89
47	4	16	0	69	5	0	0	59	5	4	4	102	5	8	8	110	5	13	8	90
2	4	16	1	54	5	0	2	102	5	4	4	32	5	8	9	118	5	13	8	91
27	4	16	4	59	5	0	2	27	5	4	8	27	5	9	0	23	5	13	9	92
54	4	16	4	102	5	0	2	47	5	4	8	57	5	9	0	32	5	13	9	93
59	4	16	4	47	5	0	3	71	5	4	9	16	5	9	1	113	5	13	9	94
102	4	16	4	2	5	0	5	57	5	4	10	71	5	9	1	69	5	14	4	95
71	4	16	5	26	5	0	5	2	5	5	0	47	5	9	5	47	5	14	5	96
62	4	16	6	27	5	0	5	26	5	5	0	87	5	9	5	16	5	14	8	97
83	4	16	7	71	5	0	6	69	5	5	3	69	5	9	7	2	5	15	4	98
31	4	16	10	57	5	0	10	87	5	5	6	26	5	9	11	26	5	15	4	99
24	4	16	11	62	5	1	0	24	5	5	7	2	5	10	0	1	5	15	5	100
57	4	17	0	24	5	1	1	31	5	5	7	1	5	10	4	62	5	15	10	101
69	4	17	1	31	5	1	1	62	5	5	8	62	5	10	6	24	5	16	0	102
18	4	17	5	83	5	1	3	1	5	6	1	24	5	10	7	31	5	16	0	103
1	4	17	7	87	5	1	7	83	5	6	4	31	5	10	7	11	5	16	4	104
50	4	17	7	1	5	1	8	18	5	6	4	11	5	11	3	74	5	16	4	105
87	4	17	11	18	5	1	9	74	5	6	6	18	5	11	3	78	5	16	4	106
74	4	18	0	50	5	2	0	78	5	6	6	74	5	11	3	114	5	16	4	107
78	4	18	0	74	5	2	1	114	5	6	6	78	5	11	3	18	5	16	5	108
114	4	18	0	78	5	2	1	11	5	6	7	114	5	11	3	50	5	17	2	109
11	4	18	1	114	5	2	1	50	5	6	9	83	5	11	7	83	5	17	4	110
3	4	18	9	11	5	2	2	3	5	8	7	50	5	11	10	66	5	19	10	111
107	4	18	9	3	5	3	6	107	5	8	7	3	5	14	1	3	5	19	11	112
66	5	1	4	107	5	3	6	66	5	10	1	107	5	14	1	107	5	19	11	113
43	5	1	9	66	5	5	7	43	5	10	8	66	5	14	9	43	6	0	8	114
9	5	4	6	43	5	6	1	9	5	13	4	43	5	15	6	38	6	2	9	115
38	5	5	0	9	5	8	9	38	5	13	4	9	5	18	0	9	6	3	0	116
64	5	11	9	38	5	9	0	64	6	0	6	38	5	18	0	64	6	9	6	117
8				64	5	16	0	8				64	6	5	0	8				118
101				8				101				8				101				119
103				103				103				103				103				120

Age 56.				Age 57.				Age 58.				Age 59.				Age 60.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
37	5	1	1	37	5	5	9	37	5	10	9	37	5	16	1	80	6	1	6	1
80	5	5	1	80	5	8	11	80	5	12	11	80	5	17	1	37	6	1	9	2
52	5	8	5	52	5	12	9	94	5	17	1	94	6	1	7	94	6	6	4	3
45	5	8	7	84	5	12	10	95	5	17	1	95	6	1	7	95	6	6	4	4
84	5	8	7	29	5	12	10	45	5	17	4	45	6	2	2	45	6	7	2	5
29	5	8	8	39	5	12	11	52	5	17	4	84	6	2	2	84	6	7	2	6
39	5	8	8	45	5	12	11	84	5	17	4	52	6	2	3	79	6	7	3	7
94	5	8	10	94	5	12	11	29	5	17	5	115	6	2	3	34	6	7	4	8
95	5	8	10	95	5	12	11	39	5	17	5	29	6	2	4	53	6	7	4	9
115	5	9	4	115	5	13	4	79	5	17	6	39	6	2	4	82	6	7	4	10
79	5	9	6	79	5	13	6	115	5	17	8	79	6	2	6	88	6	7	4	11
112	5	9	7	34	5	14	0	34	5	18	2	82	6	2	7	112	6	7	4	12
34	5	10	1	53	5	14	0	53	5	18	2	34	6	2	8	115	6	7	4	13
53	5	10	1	82	5	14	0	82	5	18	2	53	6	2	8	29	6	7	5	14
82	5	10	1	88	5	14	0	88	5	18	2	88	6	2	8	39	6	7	5	15
88	5	10	1	112	5	14	0	112	5	18	2	112	6	2	8	52	6	7	6	16
111	5	10	10	5	5	15	11	92	6	0	10	90	6	5	8	90	6	10	7	17
5	5	11	0	92	5	16	0	90	6	1	1	105	6	6	2	67	6	11	6	18
13	5	11	4	105	5	16	1	105	6	1	1	92	6	6	5	105	6	11	6	19
105	5	11	5	111	5	16	2	5	6	1	2	6	6	6	7	6	6	11	10	20
92	5	11	7	90	5	16	8	6	6	1	9	65	6	6	7	65	6	11	10	21
1	5	12	3	13	5	17	0	65	6	1	9	67	6	6	7	73	6	11	10	22
108	5	12	3	6	5	17	1	73	6	1	9	73	6	6	7	85	6	11	10	23
68	5	12	5	65	5	17	1	85	6	1	9	85	6	6	7	63	6	12	5	24
90	5	12	6	73	5	17	1	111	6	1	10	5	6	6	10	92	6	12	6	25
77	5	12	7	85	5	17	1	67	6	1	11	63	6	7	2	5	6	12	9	26
17	5	12	8	68	5	17	3	63	6	2	3	42	6	7	6	42	6	13	0	27
6	5	12	9	1	5	17	6	42	6	2	5	111	6	7	7	111	6	13	2	28
48	5	12	9	108	5	17	6	68	6	2	5	68	6	7	11	68	6	13	10	29
65	5	12	9	63	5	17	7	4	6	2	9	4	6	8	3	49	6	14	1	30
73	5	12	9	67	5	17	7	108	6	2	11	49	6	8	5	1	6	14	8	31
85	5	12	9	42	5	17	9	1	6	3	0	1	6	8	9	4	6	15	0	32
63	5	13	1	4	5	18	0	13	6	3	0	108	6	8	9	108	6	15	0	33
99	5	13	2	17	5	18	1	49	6	3	1	13	6	9	1	13	6	15	1	34
14	5	13	3	49	5	18	1	17	6	4	0	48	6	9	9	17	6	15	7	35
40	5	13	3	48	5	18	3	48	6	4	0	17	6	9	10	30	6	15	8	36
42	5	13	3	77	5	18	4	109	6	4	1	30	6	9	10	101	6	15	9	37
4	5	13	4	99	5	18	8	30	6	4	3	109	6	10	0	99	6	15	10	38
49	5	13	4	109	5	18	9	101	6	4	4	99	6	10	2	70	6	16	0	39
67	5	13	4	30	5	18	11	77	6	4	5	70	6	10	3	76	6	16	2	40
30	5	13	10	61	5	19	3	99	6	4	5	77	6	10	7	21	6	16	4	41
61	5	13	10	14	5	19	5	86	6	4	8	86	6	10	10	109	6	16	4	42
109	5	13	11	40	5	19	6	70	6	4	10	21	6	11	0	28	6	16	7	43
101	5	14	0	86	5	19	6	61	6	5	0	76	6	11	1	77	6	16	8	44
25	5	14	4	70	5	19	9	76	6	5	1	61	6	11	2	25	6	17	8	45
81	5	14	6	25	5	19	10	25	6	5	8	28	6	11	5	61	6	17	8	46
60	5	14	7	58	6	0	1	35	6	5	9	35	6	11	7	35	6	17	9	47
58	5	14	8	35	6	0	4	28	6	5	10	25	6	11	8	86	6	17	9	48
70	5	14	11	60	6	0	4	21	6	6	0	58	6	12	3	57	6	18	2	49
86	5	15	0	81	6	0	4	40	6	6	0	89	6	12	4	87	6	18	6	50
35	5	15	2	28	6	0	7	41	6	6	0	57	6	12	6	89	6	18	7	51
89	5	15	5	21	6	0	9	14	6	6	1	41	6	12	8	91	6	18	7	52
91	5	15	5	76	6	0	9	58	6	6	1	81	6	12	9	59	6	18	8	53
15	5	15	6	89	6	0	10	91	6	6	1	91	6	12	9	60	6	18	11	54
20	5	15	6	41	6	1	0	81	6	6	4	60	6	12	10	81	6	18	11	55
120	5	15	6	91	6	1	0	89	6	6	5	44	6	13	0	41	6	19	0	56
28	5	15	7	15	6	1	2	60	6	6	6	87	6	13	0	58	6	19	0	57
41	5	15	8	20	6	1	2	19	6	7	0	56	6	13	1	113	6	19	0	58
76	5	15	10	19	6	1	4	44	6	7	1	14	6	13	2	44	6	19	3	59
21	5	16	0	120	6	1	4	56	6	7	1	19	6	13	2	56	6	19	5	60

Age 56.				Age 57.				Age 58.				Age 59.				Age 60.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
22	5	16	0	56	6	1	5	57	6	7	2	40	6	13	2	104	6	19	5	61
56	5	16	1	44	6	1	7	15	6	7	4	119	6	13	5	119	6	19	6	62
19	5	16	2	22	6	2	2	20	6	7	4	15	6	13	6	15	6	19	10	63
44	5	16	4	104	6	2	3	120	6	7	7	20	6	13	6	20	6	19	10	64
12	5	16	9	119	6	2	3	87	6	7	9	104	6	13	8	71	6	19	10	65
55	5	16	11	57	6	2	4	104	6	7	9	120	6	13	11	19	6	19	11	66
117	5	17	0	55	6	2	8	119	6	7	9	113	6	14	0	120	7	0	0	67
119	5	17	0	87	6	2	10	22	6	8	6	71	6	14	1	72	7	1	1	68
104	5	17	2	46	6	3	0	55	6	8	7	46	6	14	9	27	7	1	2	69
57	5	17	8	117	6	3	0	71	6	8	8	55	6	14	9	46	7	1	3	70
116	5	17	8	72	6	3	2	46	6	8	9	72	6	14	9	54	7	1	3	71
46	5	17	9	12	6	3	3	113	6	8	9	22	6	15	0	102	7	1	3	72
96	5	17	10	113	6	3	5	72	6	8	10	54	6	15	0	55	7	1	5	73
33	5	17	11	54	6	3	6	54	6	9	0	102	6	15	0	22	7	1	6	74
72	5	17	11	71	6	3	6	102	6	9	0	27	6	15	1	14	7	1	8	75
54	5	18	2	102	6	3	6	27	6	9	3	117	6	15	7	40	7	1	9	76
87	5	18	2	33	6	3	7	117	6	9	3	33	6	15	10	96	7	1	10	77
102	5	18	2	96	6	3	7	33	6	9	8	96	6	15	10	69	7	2	0	78
113	5	18	5	27	6	3	9	96	6	9	8	69	6	16	1	117	7	2	0	79
71	5	18	6	116	6	4	2	12	6	9	9	12	6	16	3	33	7	2	5	80
27	5	18	7	69	6	4	6	69	6	10	1	10	6	17	3	12	7	2	9	81
10	5	19	1	10	6	4	8	116	6	10	9	116	6	17	4	116	7	3	11	82
69	5	19	2	7	6	5	0	10	6	10	10	47	6	17	10	10	7	4	0	83
93	5	19	2	93	6	5	4	47	6	11	5	7	6	18	0	32	7	4	6	84
7	5	19	3	110	6	5	4	7	6	11	6	93	6	18	0	93	7	4	6	85
23	5	19	3	36	6	5	5	93	6	11	8	23	6	18	3	23	7	4	8	86
32	5	19	3	47	6	5	5	23	6	11	9	32	6	18	3	47	7	4	8	87
36	5	19	3	51	6	5	5	32	6	11	9	36	6	18	4	118	7	4	8	88
51	5	19	3	106	6	5	5	118	6	11	9	51	6	18	4	7	7	4	9	89
106	5	19	3	118	6	5	5	36	6	11	10	106	6	18	4	9	7	4	9	90
110	5	19	3	23	6	5	6	51	6	11	10	110	6	18	4	36	7	4	9	91
118	5	19	3	32	6	5	6	106	6	11	10	118	6	18	4	51	7	4	9	92
47	5	19	9	16	6	6	1	110	6	11	10	16	6	18	8	106	7	4	9	93
16	6	0	1	2	6	7	2	16	6	12	3	24	7	0	2	110	7	4	9	94
26	6	1	0	26	6	7	3	24	6	13	7	31	7	0	2	16	7	5	6	95
2	6	1	2	24	6	7	5	31	6	13	7	74	7	0	4	2	7	6	11	96
31	6	1	7	31	6	7	5	2	6	13	9	78	7	0	4	26	7	6	11	97
24	6	1	8	78	6	7	8	26	6	13	9	114	7	0	4	24	7	7	1	98
62	6	1	8	114	6	7	8	11	6	13	11	2	7	0	5	31	7	7	1	99
11	6	1	10	11	6	7	9	18	6	13	11	11	7	0	5	78	7	7	1	100
74	6	1	10	18	6	7	9	74	6	13	11	26	7	0	5	114	7	7	1	101
78	6	1	10	74	6	7	9	78	6	13	11	18	7	0	6	11	7	7	2	102
114	6	1	10	62	6	7	10	114	6	13	11	62	7	1	0	74	7	7	2	103
18	6	1	11	50	6	8	7	62	6	14	4	50	7	1	5	18	7	7	6	104
50	6	2	9	83	6	10	4	50	6	14	10	66	7	3	8	62	7	7	9	105
83	6	3	7	43	6	11	0	66	6	17	2	83	7	4	6	50	7	8	4	106
66	6	5	3	66	6	11	0	83	6	17	5	43	7	4	9	66	7	10	9	107
43	6	6	2	3	6	13	2	43	6	18	2	9	7	5	10	83	7	11	7	108
3	6	6	4	107	6	13	2	38	6	19	6	3	7	6	9	43	7	11	9	109
107	6	6	4	9	6	13	10	9	6	19	8	38	7	7	9	3	7	14	11	110
9	6	8	3	38	6	13	10	3	7	0	5	107	7	7	9	107	7	14	11	111
38	6	8	4	64	6	19	6	107	7	0	5	64	7	9	6	64	7	15	0	112
64	6	14	6	8				64	7	4	6	8				38	7	15	6	113
8				59				8				59				8				114
59				75				59				75				59				115
75				97				75				97				75				116
97				98				97				98				97				117
98				100				98				100				98				118
100				101				100				101				100				119
103				103				103				103				103				120

Age 21.				Age 22.				Age 23.				Age 24.				Age 25.				
	£	s.	d.		£	s.	d		£	s.	d.		£	s.	d.		£	s.	d.	
27	1	12	5	5	1	13	4	5	1	14	1	5	1	15	0	5	1	15	10	1
119	1	12	6	27	1	13	4	27	1	14	3	9	1	15	3	9	1	16	0	2
5	1	12	7	119	1	13	4	119	1	14	4	27	1	15	3	83	1	16	1	3
42	1	12	10	42	1	13	8	9	1	14	6	83	1	15	3	27	1	16	3	4
100	1	12	11	9	1	13	9	42	1	14	6	42	1	15	4	42	1	16	3	5
99	1	13	0	100	1	13	9	83	1	14	6	119	1	15	4	119	1	16	6	6
9	1	13	1	83	1	13	10	100	1	14	8	100	1	15	7	100	1	16	7	7
12	1	13	2	99	1	13	11	12	1	24	10	12	1	15	9	47	1	16	8	8
47	1	13	2	12	1	14	0	47	1	14	10	47	1	15	9	12	1	16	9	9
83	1	13	2	47	1	14	0	99	1	14	11	30	1	15	10	30	1	16	9	10
17	1	13	4	30	1	14	2	30	1	15	0	86	1	15	11	86	1	16	10	11
77	1	13	4	86	1	14	2	86	1	15	0	99	1	15	11	99	1	16	11	12
86	1	13	4	97	1	14	3	17	1	15	1	41	1	16	0	19	1	17	0	13
30	1	13	5	17	1	14	3	41	1	15	2	17	1	16	1	40	1	17	0	14
97	1	13	5	77	1	14	3	77	1	15	2	19	1	16	1	64	1	17	0	15
41	1	13	6	41	1	14	4	97	1	15	2	72	1	16	1	72	1	17	0	16
57	1	13	6	57	1	14	4	101	1	15	2	77	1	16	1	101	1	17	0	17
101	1	13	7	101	1	14	5	19	1	25	3	101	1	16	1	17	1	17	1	18
21	1	13	8	19	1	14	6	57	1	15	3	97	1	16	2	77	1	17	1	19
60	1	13	8	72	1	14	6	72	1	15	3	40	1	16	3	10	1	17	2	20
81	1	13	8	21	1	14	7	60	1	15	5	57	1	16	3	41	1	17	2	21
96	1	13	8	58	1	14	7	10	1	15	6	64	1	16	3	97	1	17	2	22
105	1	13	8	60	1	14	7	40	1	15	6	10	1	16	4	57	1	17	3	23
58	1	13	9	76	1	14	7	45	1	15	6	21	1	16	5	45	1	17	4	24
72	1	13	9	81	1	14	7	58	1	15	6	45	1	16	5	84	1	17	4	25
76	1	13	9	96	1	14	7	76	1	15	6	60	1	16	5	60	1	17	5	26
10	1	13	10	105	1	14	7	81	1	15	6	81	1	16	5	21	1	17	6	27
19	1	13	10	10	1	14	8	84	1	15	6	84	1	16	5	58	1	17	6	28
45	1	13	10	45	1	14	8	96	1	15	6	96	1	16	5	76	1	17	6	29
84	1	13	10	84	1	14	8	105	1	15	6	105	1	16	5	81	1	17	6	30
15	1	13	11	40	1	14	9	21	1	15	7	58	1	16	6	96	1	17	6	31
20	1	13	11	15	1	14	10	64	1	25	7	76	1	16	6	105	1	17	6	32
40	1	14	0	20	1	14	10	15	1	15	9	15	1	16	8	52	1	17	7	33
2	1	14	2	64	1	14	11	20	1	15	9	20	1	16	8	35	1	17	8	34
3	1	14	2	3	1	15	1	52	1	15	10	52	1	16	8	59	1	17	8	35
107	1	14	2	52	1	15	1	2	1	16	0	35	1	16	10	15	1	17	9	36
23	1	14	3	107	1	15	1	35	1	16	0	59	1	16	10	18	1	17	9	37
64	1	14	3	35	1	15	2	59	1	16	0	2	1	16	11	20	1	17	9	38
35	1	14	4	59	1	15	2	104	1	16	0	18	1	16	11	104	1	17	10	39
59	1	14	4	104	1	15	2	3	1	16	1	104	1	16	11	44	1	17	11	40
104	1	14	4	2	1	15	3	107	1	16	1	44	1	17	0	94	1	17	11	41
52	1	14	5	23	1	15	3	18	1	16	2	66	1	17	1	2	1	18	1	42
66	1	14	5	66	1	15	3	44	1	16	2	3	1	17	2	66	1	18	1	43
50	1	14	6	18	1	15	4	66	1	16	2	94	1	17	2	82	1	18	2	44
62	1	14	6	44	1	15	4	23	1	16	3	107	1	17	2	89	1	18	2	45
89	1	14	6	62	1	15	4	89	1	16	3	23	1	17	3	3	1	18	3	46
113	1	14	6	50	1	15	5	62	1	16	4	82	1	17	3	23	1	18	3	47
16	1	14	7	82	1	15	5	82	1	16	4	89	1	17	3	107	1	18	3	48
44	1	14	7	89	1	15	5	50	1	16	5	62	1	17	4	62	1	18	4	49
82	1	14	7	113	1	15	5	94	1	16	5	25	1	17	5	67	1	18	4	50
18	1	14	8	16	1	15	6	16	1	16	6	50	1	17	5	25	1	18	5	51
108	1	14	8	25	1	15	7	25	1	16	6	108	1	17	5	108	1	18	5	52
25	1	14	9	108	1	15	7	108	1	16	6	16	1	17	6	50	1	18	6	53
92	1	14	10	94	1	15	8	113	1	16	6	67	1	17	6	74	1	18	6	54
74	1	14	11	74	1	15	9	74	1	16	7	74	1	17	6	85	1	18	6	55
94	1	14	11	92	1	15	9	92	1	16	8	113	1	17	6	16	1	18	7	56
120	1	14	11	26	1	15	10	67	1	16	9	92	1	17	7	92	1	18	7	57
26	1	15	0	120	1	15	10	120	1	16	9	85	1	17	8	113	1	18	8	58
51	1	15	2	67	1	16	0	26	1	16	10	120	1	17	9	120	1	18	10	59
67	1	15	2	51	1	16	1	85	1	17	0	26	1	17	10	91	1	18	11	60

Age 21.				Age 22.				Age 23.				Age 24.				Age 25.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
69	1	15	3	7	1	16	2	7	1	17	1	55	1	18	0	14	1	18	11	61
7	1	15	4	55	1	16	3	51	1	17	1	7	1	18	1	26	1	18	11	62
46	1	15	4	69	1	16	3	55	1	17	1	14	1	18	1	55	1	19	0	63
55	1	15	4	85	1	16	3	14	1	17	3	51	1	18	1	71	1	19	0	64
118	1	15	5	46	1	16	4	69	1	17	3	91	1	18	1	111	1	19	0	65
85	1	15	6	14	1	16	5	91	1	17	3	111	1	18	1	7	1	19	2	66
14	1	15	7	91	1	16	5	111	1	17	3	71	1	18	2	51	1	19	2	67
111	1	15	7	111	1	16	5	1	1	17	5	69	1	18	3	69	1	19	3	68
117	1	15	7	118	1	16	5	46	1	17	5	109	1	18	4	109	1	19	3	69
78	1	15	8	117	1	16	6	71	1	17	5	118	1	18	4	116	1	19	3	70
91	1	15	8	78	1	16	7	109	1	17	5	46	1	18	5	78	1	19	4	71
114	1	15	8	109	1	16	7	117	1	17	5	78	1	18	5	114	1	19	4	72
109	1	15	9	114	1	16	7	118	1	17	5	114	1	18	5	1	1	19	5	73
116	1	15	10	71	1	16	8	78	1	17	6	1	1	18	6	46	1	19	6	74
1	1	15	11	116	1	16	8	114	1	17	6	116	1	18	6	117	1	19	7	75
36	1	15	11	1	1	16	9	116	1	17	7	117	1	18	6	118	1	19	7	76
106	1	15	11	24	1	16	10	24	1	17	9	24	1	18	9	24	1	19	8	77
24	1	16	0	31	1	16	10	31	1	17	9	31	1	18	9	48	1	19	8	78
31	1	16	0	36	1	16	10	11	1	17	10	6	1	18	10	4	1	19	9	79
71	1	16	0	106	1	16	10	36	1	17	10	11	1	18	10	11	1	19	9	80
11	1	16	1	6	1	17	0	106	1	17	10	48	1	18	10	31	1	19	9	81
6	1	16	2	11	1	17	0	6	1	17	11	70	1	18	10	6	1	19	10	82
70	1	16	3	70	1	17	0	70	1	17	11	4	1	18	11	70	1	19	10	83
33	1	16	4	33	1	17	3	48	1	18	0	36	1	18	11	36	2	0	1	84
115	1	16	4	48	1	17	3	4	1	18	1	106	1	18	11	106	2	0	1	85
53	1	16	5	115	1	17	3	33	1	18	2	33	1	19	2	33	2	0	2	86
4	1	16	6	4	1	17	4	115	1	18	3	115	1	19	3	43	2	0	2	87
48	1	16	6	53	1	17	5	53	1	18	4	43	1	19	4	115	2	0	2	88
32	1	16	7	32	1	17	6	61	1	18	5	61	1	19	4	28	2	0	4	89
61	1	16	7	61	1	17	6	32	1	18	6	53	1	19	5	54	2	0	4	90
54	1	16	9	54	1	17	8	54	1	18	6	54	1	19	5	61	2	0	4	91
102	1	16	9	102	1	17	8	102	1	18	6	102	1	19	5	102	2	0	4	92
63	1	16	10	63	1	17	9	43	1	18	7	28	1	19	6	53	2	0	6	93
28	1	17	1	28	1	17	10	28	1	18	8	32	1	19	7	68	2	0	7	94
43	1	17	1	43	1	17	10	63	1	18	8	63	1	19	8	32	2	0	8	95
56	1	17	2	56	1	18	1	65	1	19	0	65	1	19	11	63	2	0	8	96
65	1	17	3	65	1	18	1	56	1	19	1	88	2	0	0	88	2	0	10	97
90	1	17	5	90	1	18	4	90	1	19	3	56	2	0	1	65	2	0	11	98
88	1	17	10	88	1	18	7	88	1	19	4	68	2	0	1	56	2	1	1	99
68	1	18	2	68	1	18	10	68	1	19	6	90	2	0	3	90	2	1	3	100
87	1	18	7	87	1	19	7	87	2	0	8	87	2	1	9	87	2	2	11	101
8				8				8				8				8				102
13				13				13				13				13				103
22				22				22				22				22				104
29				29				29				29				29				105
34				34				34				34				34				106
37				37				37				37				37				107
38				38				38				38				38				108
39				39				39				39				39				109
49				49				49				49				49				110
73				73				73				73				73				111
75				75				75				75				75				112
79				79				79				79				79				113
80				80				80				80				80				114
93				93				93				93				93				115
95				95				95				95				95				116
98				98				98				98				98				117
103				103				103				103				103				118
110				110				110				110				110				119
112				112				112				112				112				120

Age 26.				Age 27.				Age 28.				Age 29.				Age 30.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
5	1	16	8	5	1	17	8	5	1	18	7	5	1	19	6	5	2	0	7	1
9	1	16	10	9	1	17	9	9	1	18	8	9	1	19	7	9	2	0	8	2
42	1	17	2	42	1	18	2	42	1	19	3	42	2	0	4	42	2	1	5	3
83	1	17	2	27	1	18	4	27	1	19	6	64	2	0	5	64	2	1	5	4
27	1	17	4	83	1	18	4	64	1	19	6	27	2	0	7	17	2	1	7	5
119	1	17	6	119	1	18	6	30	1	19	7	30	2	0	7	27	2	1	8	6
30	1	17	8	30	1	18	7	83	1	19	7	83	2	0	8	30	2	1	8	7
47	1	17	8	40	1	18	7	40	1	19	8	17	2	0	9	83	2	1	9	8
100	1	17	8	64	1	18	7	47	1	19	9	19	2	0	10	19	2	1	10	9
12	1	17	9	47	1	18	8	19	1	19	10	40	2	0	10	72	2	1	11	10
40	1	17	9	100	1	18	9	86	1	19	10	47	2	0	10	40	2	2	0	11
64	1	17	9	12	1	18	10	72	1	19	10	72	2	0	11	47	2	2	0	12
86	1	17	9	19	1	18	10	119	1	19	10	100	2	0	11	100	2	2	0	13
19	1	17	11	86	1	18	10	100	1	19	10	12	2	1	0	12	2	2	2	14
72	1	18	0	72	1	18	11	12	1	19	11	86	2	1	0	86	2	2	2	15
99	1	18	0	101	1	19	0	17	2	0	0	119	2	1	0	119	2	2	2	16
101	1	18	0	99	1	19	2	101	2	0	0	101	2	1	2	101	2	2	4	17
17	1	18	1	10	1	19	3	10	2	0	4	10	2	1	5	10	2	2	6	18
10	1	18	2	17	1	19	3	41	2	0	4	35	2	1	5	35	2	2	6	19
77	1	18	2	77	1	19	3	45	2	0	4	52	2	1	5	58	2	2	6	20
41	1	18	3	97	1	19	3	84	2	0	4	59	2	1	5	59	2	2	6	21
57	1	18	3	41	1	19	4	99	2	0	4	99	2	1	5	99	2	2	6	22
97	1	18	3	45	1	19	4	52	2	0	5	18	2	1	6	18	2	2	7	23
45	1	18	4	84	1	19	4	77	2	0	5	41	2	1	6	45	2	2	7	24
84	1	18	4	52	1	19	5	97	2	0	5	45	2	1	6	77	2	2	7	25
52	1	18	6	57	1	19	5	18	2	0	6	77	2	1	6	84	2	2	7	26
60	1	18	6	35	1	19	6	35	2	0	6	84	2	1	6	94	2	2	7	27
81	1	18	6	59	1	19	6	57	2	0	6	97	2	1	6	97	2	2	7	28
96	1	18	6	18	1	19	7	59	2	0	6	57	2	1	8	41	2	2	9	29
105	1	18	6	44	1	19	7	94	2	0	8	94	2	1	8	85	2	2	9	30
21	1	18	7	21	1	19	8	21	2	0	9	44	2	1	10	57	2	2	10	31
35	1	18	7	58	1	19	8	60	2	0	9	85	2	1	10	60	2	2	11	32
58	1	18	7	60	1	19	8	81	2	0	9	108	2	1	10	44	2	2	11	33
59	1	18	7	76	1	19	8	96	2	0	9	21	2	1	11	81	2	2	11	34
76	1	18	7	81	1	19	8	44	2	0	10	58	2	1	11	96	2	2	11	35
18	1	18	8	94	1	19	8	58	2	0	10	60	2	1	11	105	2	2	11	36
94	1	18	9	96	1	19	8	76	2	0	10	76	2	1	11	21	2	3	0	37
104	1	18	9	105	1	19	8	104	2	0	10	81	2	1	11	55	2	3	0	38
15	1	18	10	104	1	19	9	105	2	0	10	96	2	1	11	67	2	3	0	39
20	1	18	10	15	1	19	11	85	2	0	11	104	2	1	11	76	2	3	0	40
44	1	18	10	20	1	19	11	108	2	1	0	105	2	1	11	104	2	3	0	41
2	1	19	1	66	2	0	1	15	2	1	1	67	2	2	1	108	2	3	0	42
66	1	19	1	67	2	0	2	20	2	1	1	15	2	2	3	119	2	3	2	43
82	1	19	2	85	2	0	2	66	2	1	2	20	2	2	3	15	2	3	5	44
89	1	19	2	89	2	0	2	67	2	1	2	66	2	2	4	20	2	3	5	45
67	1	19	3	108	2	0	2	89	2	1	3	89	2	2	4	66	2	3	6	46
85	1	19	3	2	2	0	3	2	2	1	4	2	2	2	6	71	2	3	6	47
108	1	19	3	82	2	0	3	82	2	1	5	25	2	2	7	2	2	3	7	48
3	1	19	5	25	2	0	5	25	2	1	6	71	2	2	7	25	2	3	7	49
25	1	19	5	23	2	0	6	71	2	1	7	74	2	2	8	116	2	3	7	50
62	1	19	5	74	2	0	6	74	2	1	7	82	2	2	8	74	2	3	10	51
107	1	19	5	62	2	0	7	23	2	1	9	23	2	2	9	89	2	3	11	52
23	1	19	6	3	2	0	8	62	2	1	9	116	2	2	9	92	2	3	11	53
74	1	19	6	50	2	0	8	92	2	1	9	62	2	2	10	14	2	4	0	54
50	1	19	7	71	2	2	8	50	2	1	10	92	2	2	10	23	2	4	0	55
92	1	19	7	92	2	0	8	91	2	1	10	14	2	2	11	62	2	4	0	56
120	1	19	7	107	2	0	8	116	2	1	10	91	2	2	11	82	2	4	0	57
16	1	19	8	120	2	0	8	14	2	1	11	111	2	2	11	111	2	4	0	58
91	1	19	10	16	2	0	10	16	2	1	11	16	2	3	0	16	2	4	1	59
113	1	19	10	91	2	0	10	120	2	1	11	120	2	3	0	91	2	4	1	60

Age 26.				Age 27.				Age 28.				Age 29.				Age 30.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
71	1	19	10	14	2	0	11	3	2	2	0	50	2	3	1	120	2	4	1	61
14	1	19	11	111	2	0	11	107	2	2	0	3	2	3	2	3	2	4	3	62
26	1	19	11	116	2	0	11	111	2	2	0	55	2	3	2	107	2	4	3	63
55	1	19	11	55	2	1	1	55	2	2	1	107	2	3	2	50	2	4	4	64
111	1	19	11	113	2	1	1	26	2	2	4	26	2	3	6	55	2	4	5	65
116	2	0	2	26	2	1	2	109	2	2	4	48	2	3	6	26	2	4	7	66
7	2	0	3	109	2	1	3	113	2	2	5	109	2	3	6	48	2	4	7	67
51	2	0	3	7	2	1	5	1	2	2	6	113	2	3	7	109	2	4	8	68
109	2	0	3	51	2	1	5	48	2	2	6	1	2	3	8	4	2	4	9	69
69	2	0	4	78	2	1	5	78	2	2	6	4	2	3	8	113	2	4	9	70
.78	2	0	4	114	2	1	5	114	2	2	6	78	2	3	8	1	2	4	10	71
114	2	0	4	1	2	1	6	4	2	2	7	114	2	3	8	78	2	4	10	72
1	2	0	5	48	2	1	6	7	2	2	8	7	2	3	10	114	2	4	10	73
48	2	0	7	69	2	1	6	51	2	2	8	69	2	3	10	7	2	4	11	74
4	2	0	8	4	2	1	7	69	2	2	8	51	2	3	11	51	2	4	11	75
46	2	0	8	31	2	1	9	107	2	2	8	6	2	4	0	69	2	4	11	76
117	2	0	8	6	2	1	10	6	2	2	11	24	2	4	0	43	2	5	1	77
118	2	0	8	24	2	1	10	24	2	2	11	31	2	4	0	6	2	5	2	78
24	2	0	9	46	2	1	10	31	2	2	11	43	2	4	0	24	2	5	2	79
31	2	0	9	70	2	1	10	70	2	2	11	70	2	4	0	31	2	5	2	80
6	2	0	10	117	2	1	10	11	2	3	0	11	2	4	2	70	2	5	2	81
11	2	0	10	118	2	1	10	43	2	3	0	118	2	4	2	11	2	5	4	82
70	2	0	10	11	2	1	11	46	2	3	0	46	2	4	3	117	2	5	4	83
43	2	1	1	43	2	2	0	118	2	3	0	117	2	4	3	118	2	5	4	84
33	2	1	2	33	2	2	2	117	2	3	1	28	2	4	4	28	2	5	5	85
28	2	1	3	28	2	2	3	28	2	3	3	54	2	4	4	88	2	5	5	86
36	2	1	3	68	2	2	3	33	2	3	3	102	2	4	4	46	2	5	6	87
106	2	1	3	115	2	2	3	68	2	3	3	33	2	4	5	54	2	5	6	88
115	2	1	3	36	2	2	4	54	2	3	4	88	2	4	5	102	2	5	6	89
54	2	1	4	54	2	2	4	115	2	3	4	61	2	4	6	33	3	5	7	90
61	2	1	4	102	2	2	4	61	2	3	5	68	2	4	6	61	2	5	7	91
68	2	1	4	106	2	2	4	88	2	3	5	115	2	4	6	68	2	5	9	92
102	2	1	4	61	2	2	5	36	2	3	6	36	2	4	8	115	2	5	9	93
53	2	1	7	88	2	2	7	106	2	3	6	106	2	4	8	36	2	5	10	94
32	2	1	8	32	2	2	8	32	2	3	9	32	2	4	10	106	2	5	10	95
63	2	1	8	53	2	2	8	63	2	3	9	63	2	4	10	32	2	5	11	96
88	2	1	9	63	2	2	8	53	2	3	10	53	2	4	11	63	2	5	11	97
65	2	1	11	65	2	3	0	65	2	4	1	65	2	5	3	53	2	6	1	98
56	2	2	2	56	2	3	3	56	2	4	4	56	2	5	6	65	2	6	5	99
90	2	2	4	90	2	3	4	90	2	4	5	90	2	5	7	90	2	6	8	100
87	2	4	1	87	2	5	4	87	2	6	8	87	2	7	11	56	2	6	9	101
8				8				8				8				8				102
13				13				13				13				13				103
22				22				22				22				22				104
29				29				29				29				29				105
34				34				34				34				34				106
37				37				37				37				37				107
38				38				38				38				38				108
39				39				39				39				38				109
49				49				49				49				39				110
73				73				73				73				49				111
75				75				75				75				73				112
79				79				79				79				75				113
80				80				80				80				79				114
93				93				93				93				80				115
95				95				95				95				93				116
98				98				98				98				98				117
103				103				103				103				103				118
110				110				110				110				110				119
112				112				112				112				112				120

Age 31.				Age 32.				Age 33.				Age 34.				Age 35.				
	£	s.	d.		£ · s.		d.		£	s.	d.		£	s.	d.		£	s.	d.	
5	2	1	8	5	2	2	9	5	2	4	0	5	2	5	4	5	2	6	8	1
9	2	1	9	9	2	2	10	9	2	4	1	9	2	5	5	9	2	6	9	2
64	2	2	6	17	2	3	4	17	2	4	5	17	2	5	9	17	2	6	10	3
17	2	2	7	64	2	3	8	64	2	4	11	64	2	6	2	64	2	7	6	4
42	2	2	7	42	2	3	9	42	2	5	0	42	2	6	3	30	2	7	8	5
30	2	2	9	30	2	3	11	30	2	5	1	30	2	6	5	42	2	7	8	6
83	2	2	10	83	2	4	0	83	2	5	3	27	2	6	7	27	2	8	0	7
27	2	2	11	27	2	4	1	27	2	5	4	83	2	6	7	85	2	8	0	8
100	2	3	0	100	2	4	2	100	2	5	4	100	2	6	8	100	2	8	0	9
19	2	3	1	72	2	4	3	40	2	5	6	40	2	6	9	83	2	8	1	10
72	2	3	1	19	2	4	4	72	2	5	7	85	2	6	9	40	2	8	2	11
40	2	3	2	47	2	4	5	85	2	5	7	72	2	6	11	72	2	8	3	12
47	2	3	2	12	2	4	6	19	2	5	8	12	2	7	0	19	2	8	5	13
12	2	3	4	119	2	4	6	119	2	5	8	19	2	7	0	10	2	8	6	14
41	2	3	4	40	2	4	8	12	2	5	9	119	2	7	0	12	2	8	6	15
86	2	3	4	85	2	4	8	47	2	5	9	10	2	7	2	35	2	8	6	16
119	2	3	4	86	2	4	8	10	2	5	11	35	2	7	2	52	2	8	6	17
10	2	3	7	10	2	4	9	35	2	5	11	47	2	7	2	59	2	8	6	18
35	2	3	7	35	2	4	9	52	2	5	11	52	2	7	2	94	2	8	6	19
52	2	3	7	52	2	4	9	59	2	5	11	59	2	7	2	119	2	8	6	20
59	2	3	7	59	2	4	9	41	2	6	0	94	2	7	3	47	2	8	8	21
85	2	3	7	77	2	4	9	77	2	6	0	41	2	7	4	77	2	8	8	22
99	2	3	7	94	2	4	10	86	2	6	0	77	2	7	4	97	2	8	9	23
101	2	3	7	97	2	4	10	94	2	6	0	86	2	7	4	67	2	8	10	24
77	2	3	8	99	2	4	10	97	2	6	0	97	2	7	4	86	2	8	10	25
94	2	3	8	101	2	4	11	99	2	6	0	99	2	7	5	99	2	8	10	26
97	2	3	8	18	2	5	0	18	2	6	3	18	2	7	7	41	2	9	0	27
18	2	3	9	45	2	5	0	101	2	6	3	45	2	7	8	45	2	9	0	28
45	2	3	10	84	2	5	0	45	2	6	4	67	2	7	8	84	2	9	0	29
84	2	3	10	21	2	5	2	84	2	6	4	84	2	7	8	18	2	9	1	30
48	2	4	0	48	2	5	2	21	2	6	5	21	2	7	9	21	2	9	1	31
60	2	4	0	60	2	5	2	48	2	6	5	48	2	7	9	60	2	9	1	32
81	2	4	0	81	2	5	2	58	2	6	5	58	2	7	9	81	2	9	1	33
96	2	4	0	96	2	5	2	60	2	6	5	60	2	7	9	96	2	9	1	34
105	2	4	0	104	2	5	2	76	2	6	5	76	2	7	9	101	2	9	1	35
21	2	4	1	41	2	5	3	81	2	6	5	81	2	7	9	105	2	9	1	36
44	2	4	1	44	2	5	3	96	2	6	5	96	2	7	9	116	2	9	1	37
57	2	4	1	57	2	5	3	105	2	6	5	101	2	7	9	48	2	9	2	38
58	2	4	1	58	2	5	3	57	2	6	6	105	2	7	9	58	2	9	2	39
76	2	4	1	67	2	5	3	67	2	6	6	44	2	7	11	76	2	9	2	40
67	2	4	2	76	2	5	3	44	2	6	7	57	2	7	11	57	2	9	3	41
104	2	4	2	104	2	5	5	71	2	6	9	71	2	8	0	71	2	9	3	42
108	2	4	2	116	2	5	7	104	2	6	9	116	2	8	0	44	2	9	4	43
15	2	4	7	71	2	5	8	116	2	6	9	15	2	8	1	14	2	9	6	44
20	2	4	7	15	2	5	9	15	2	6	11	20	2	8	1	15	2	9	6	45
71	2	4	7	20	2	5	9	20	2	6	11	25	2	8	2	20	2	9	6	46
2	2	4	8	25	2	5	9	25	2	6	11	104	2	8	2	25	2	9	6	47
25	2	4	8	108	2	5	9	2	2	7	1	14	2	8	3	111	2	9	6	48
116	2	4	8	2	2	5	10	14	2	7	1	111	2	8	3	104	2	9	8	49
66	2	4	9	14	2	5	11	111	2	7	1	2	2	8	5	2	2	9	11	50
14	2	5	0	111	2	5	11	89	2	7	5	89	2	8	8	89	2	10	0	51
23	2	5	0	66	2	6	1	108	2	7	5	23	2	8	9	23	2	10	3	52
89	2	5	0	89	2	6	2	23	2	7	6	62	2	8	10	62	2	10	3	53
111	2	5	0	23	2	6	3	62	2	7	6	66	2	8	11	66	2	10	5	54
62	2	5	1	62	2	6	3	66	2	7	6	120	2	9	0	92	2	10	6	55
74	2	5	1	74	2	6	4	92	2	7	8	74	2	9	1	120	2	10	6	56
92	2	5	1	92	2	6	4	120	2	7	8	92	2	9	1	74	2	10	7	57
16	2	5	3	16	2	6	5	16	2	7	9	108	2	9	1	16	2	10	8	58
91	2	5	3	120	2	6	5	74	2	7	9	16	2	9	2	91	2	10	8	59
120	2	5	3	91	2	6	6	91	2	7	10	91	2	9	3	49	2	10	9	60

Age 31.				Age 32.				Age 33.				Age 34.				Age 35.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
82	2	5	4	3	2	6	8	3	2	8	0	49	2	9	4	88	2	10	10	61
55	2	5	5	107	2	6	8	107	2	8	0	3	2	9	6	108	2	10	10	62
3	2	5	6	55	2	6	9	49	2	8	1	50	2	9	6	50	2	10	11	63
107	2	5	6	82	2	6	9	26	2	8	2	107	2	9	6	3	2	11	0	64
50	2	5	7	49	2	6	10	50	2	8	2	26	2	9	7	26	2	11	0	65
49	2	5	8	50	2	6	10	55	2	8	2	55	2	9	7	82	2	11	0	66
26	2	5	9	26	2	6	11	82	2	8	2	82	2	9	7	107	2	11	0	67
4	2	5	11	4	2	7	1	4	2	8	4	88	2	9	8	4	2	11	2	68
109	2	5	11	109	2	7	2	7	2	8	6	1	2	9	9	55	2	11	2	69
113	2	5	11	113	2	7	2	51	2	8	6	4	2	9	9	51	2	11	4	70
1	2	6	0	1	2	7	3	88	2	8	6	7	2	9	11	7	2	11	5	71
51	2	6	0	7	2	7	3	113	2	8	6	51	2	9	11	69	2	11	6	72
7	2	6	1	51	2	7	3	109	2	8	7	109	2	9	11	43	2	11	7	73
78	2	6	1	43	2	7	5	1	2	8	9	113	2	10	0	113	2	11	7	74
114	2	6	1	78	2	7	5	43	2	8	9	43	2	10	1	1	2	11	8	75
69	2	6	2	88	2	7	5	69	2	8	9	69	2	10	1	6	2	11	8	76
43	2	6	3	114	2	7	5	78	2	8	10	6	2	10	3	70	2	11	8	77
6	2	6	4	69	2	7	6	114	2	8	10	70	2	10	3	61	2	11	9	78
70	2	6	4	6	2	7	7	6	2	8	11	78	2	10	3	78	2	11	9	79
24	2	6	5	70	2	7	7	70	2	8	11	114	2	10	3	109	2	11	9	80
31	2	6	5	24	2	7	8	118	2	8	11	31	2	10	4	114	2	11	9	81
88	2	6	5	31	2	7	8	24	2	9	0	118	2	10	4	24	2	11	10	82
118	2	6	5	118	2	7	8	31	2	9	0	24	2	10	5	28	2	11	10	83
117	2	6	6	117	2	7	9	117	2	9	0	28	2	10	5	31	2	11	10	84
11	2	6	7	28	2	7	10	28	2	9	1	61	2	10	5	118	2	11	10	85
28	2	6	7	54	2	7	10	54	2	9	2	117	2	10	5	54	2	11	11	86
54	2	6	8	102	2	7	10	61	2	9	2	54	2	10	6	68	2	11	11	87
102	2	6	8	61	2	7	11	102	2	9	2	102	2	10	6	102	2	11	11	88
33	2	6	9	11	2	8	0	68	2	9	3	68	2	10	7	117	2	11	11	89
46	2	6	9	68	2	8	0	11	2	9	4	11	2	10	9	63	2	12	1	90
61	2	6	9	33	2	8	1	33	2	9	5	33	2	10	9	33	2	12	3	91
68	2	6	10	46	2	8	1	46	2	9	5	63	2	10	9	11	2	12	4	92
36	2	7	0	32	2	8	2	63	2	9	6	36	2	10	10	46	2	12	4	93
106	2	7	0	36	2	8	3	36	2	9	7	46	2	10	10	115	2	12	5	94
115	2	7	0	63	2	8	3	106	2	9	7	106	2	10	10	36	2	12	6	95
32	2	7	1	106	2	8	3	115	2	9	7	115	2	11	0	106	2	12	6	96
63	2	7	1	115	2	8	3	32	2	9	8	32	2	11	2	53	2	12	7	97
53	2	7	4	53	2	8	7	53	2	9	10	53	2	11	2	32	2	12	8	98
65	2	7	8	65	2	9	0	65	2	10	3	90	2	11	7	90	2	13	0	99
90	2	7	11	90	2	9	1	90	2	10	4	65	2	11	8	65	2	13	1	100
56	2	8	0	56	2	9	4	56	2	10	8	56	2	12	1	56	2	13	7	101
8				8				8				8				8				102
13				13				13				13				13				103
22				22				22				22				22				104
29				29				29				29				29				105
34				34				34				34				34				106
37				37				37				37				37				107
38				38				38				38				38				108
39				39				39				39				39				109
73				73				73				73				73				110
75				75				75				75				75				111
79				79				79				79				79				112
80				80				80				80				80				113
87				87				87				87				87				114
93				93				93				93				93				115
95				95				95				95				95				116
98				98				98				98				98				117
103				103				103				103				103				118
110				110				110				110				110				119
112				112				112				112				112				120

Age 36.				Age 37.				Age 38.				Age 39.				Age 40.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
5	2	8	2	17	2	9	8	17	2	11	3	17	2	12	10	85	2	14	5	1
9	2	8	3	5	2	9	9	5	2	11	4	5	2	13	1	17	2	14	6	2
17	2	8	4	9	2	9	10	9	2	11	5	9	2	13	2	5	2	14	10	3
64	2	8	11	85	2	10	3	85	2	11	6	30	2	13	7	9	2	15	1	4
30	2	9	1	64	2	10	5	30	2	12	0	64	2	13	8	30	2	15	3	5
85	2	9	1	30	2	10	6	64	2	12	0	42	2	13	10	64	2	15	5	6
42	2	9	2	42	2	10	8	42	2	12	3	85	2	13	10	42	2	15	7	7
27	2	9	5	27	2	10	10	27	2	12	5	27	2	14	0	27	2	15	9	8
100	2	9	5	40	2	11	0	40	2	12	7	100	2	14	2	116	2	15	9	9
40	2	9	6	100	2	11	0	100	2	12	7	40	2	14	4	100	2	15	11	10
83	2	9	7	83	2	11	2	116	2	12	10	94	2	14	5	94	2	16	0	11
72	2	9	9	72	2	11	3	10	2	12	11	116	2	14	5	40	2	16	1	12
19	2	9	10	10	2	11	4	35	2	12	11	10	2	14	6	10	2	16	3	13
10	2	9	11	35	2	11	4	59	2	12	11	35	2	14	6	35	2	16	3	14
35	2	9	11	59	2	11	4	72	2	12	11	59	2	14	6	59	2	16	3	15
52	2	9	11	94	2	11	4	83	2	12	11	41	2	14	7	19	2	16	5	16
59	2	9	11	19	2	11	5	19	2	13	0	19	2	14	8	83	2	16	5	17
94	2	9	11	52	2	11	6	94	2	13	0	72	2	14	8	12	2	16	6	18
12	2	10	0	116	2	11	6	12	2	13	2	83	2	14	8	41	2	16	6	19
119	2	10	0	12	2	11	7	52	2	13	2	12	2	14	9	67	2	16	6	20
47	2	10	2	77	2	11	8	67	2	13	3	67	2	14	9	72	2	16	6	21
77	2	10	2	119	2	11	8	77	2	13	3	52	2	14	11	71	2	16	7	22
97	2	10	2	67	2	11	9	97	2	13	4	71	2	14	11	77	2	16	8	23
67	2	10	3	97	2	11	9	71	2	13	5	77	2	14	11	14	2	16	9	24
116	2	10	3	47	2	11	10	119	2	13	6	97	2	15	0	52	2	16	9	25
99	2	10	4	71	2	11	11	45	2	13	7	14	2	15	2	97	2	16	9	26
86	2	10	5	99	2	11	11	47	2	13	7	111	2	15	2	111	2	16	9	27
45	2	10	6	45	2	12	0	84	2	13	7	45	2	15	3	45	2	17	0	28
84	2	10	6	84	2	12	0	99	2	13	7	84	2	15	3	84	2	17	0	29
18	2	10	7	86	2	12	1	14	2	13	8	99	2	15	4	119	2	17	0	30
48	2	10	7	101	2	12	1	111	2	13	8	119	2	15	4	99	2	17	1	31
60	2	10	7	18	2	12	2	21	2	13	9	47	2	15	5	21	2	17	2	32
71	2	10	7	21	2	12	2	48	2	13	9	48	2	15	5	25	2	17	2	33
81	2	10	7	48	2	12	2	60	2	13	9	60	2	15	5	48	2	17	2	34
96	2	10	7	58	2	12	2	81	2	13	9	81	2	15	5	60	2	17	2	35
101	2	10	7	60	2	12	2	96	2	13	9	96	2	15	5	81	2	17	2	36
105	2	10	7	76	2	12	2	101	2	13	9	101	2	15	5	96	2	17	2	37
21	2	10	8	81	2	12	2	105	2	13	9	105	2	15	5	101	2	17	7	38
58	2	10	8	96	2	12	2	41	2	13	10	21	2	15	6	105	2	17	2	39
76	2	10	8	105	2	12	2	58	2	13	10	25	2	15	6	58	2	17	3	40
44	2	10	10	14	2	12	3	76	2	13	10	58	2	15	6	76	2	17	3	41
57	2	10	10	57	2	12	3	86	2	13	10	76	2	15	6	47	2	17	4	42
111	2	10	10	111	2	12	3	18	2	13	11	57	2	15	8	57	2	17	5	43
14	2	10	11	41	2	12	4	25	2	13	11	86	2	15	8	44	2	17	7	44
25	2	10	11	44	2	12	4	57	2	13	11	18	2	15	9	86	2	17	7	45
15	2	11	0	25	2	12	5	44	2	14	0	44	2	15	9	15	2	17	8	46
20	2	11	0	15	2	12	6	15	2	14	2	89	2	15	9	18	2	17	8	47
41	2	11	0	20	2	12	6	20	2	14	2	15	2	15	10	20	2	17	8	48
104	2	11	2	89	3	12	9	89	2	14	3	20	2	15	10	88	2	17	9	49
2	2	11	4	104	2	12	9	104	2	14	5	2	2	16	3	2	2	18	0	50
89	2	11	4	2	2	12	10	2	2	14	7	88	2	16	3	89	2	18	0	51
120	2	11	6	120	2	13	0	120	2	14	9	104	2	16	5	104	2	18	2	52
23	2	11	9	23	2	13	3	88	2	14	10	120	2	16	6	92	2	18	3	53
62	2	11	9	62	2	13	4	23	2	15	0	92	2	16	8	120	2	18	3	54
66	2	12	0	88	2	13	5	62	2	15	0	23	2	16	9	4	2	18	6	55
92	2	12	0	92	2	13	6	92	2	15	1	62	2	16	9	23	2	18	6	56
91	2	12	1	91	2	13	7	91	2	15	2	49	2	16	10	62	2	18	6	57
16	2	12	2	49	2	13	8	49	2	15	3	91	2	16	11	50	2	18	8	58
74	2	12	2	66	2	13	8	50	2	15	4	4	2	17	0	49	2	18	8	59
88	2	12	2	74	2	13	9	66	2	15	5	50	2	17	0	82	2	18	10	60

Age 36.				Age 37.				Age 38.				Age 39.				Age 40.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
49	2	12	2	16	2	13	10	74	2	15	5	82	2	17	2	91	2	18	10	61
50	2	12	4	50	2	13	10	4	2	15	6	16	2	17	3	16	2	19	0	62
55	2	12	5	4	2	14	0	16	2	15	6	66	2	17	3	74	2	19	1	63
108	2	12	5	55	2	14	0	82	2	15	7	74	2	17	3	61	2	19	3	64
26	2	12	6	82	2	14	0	109	2	15	7	109	2	17	4	66	2	19	3	65
82	2	12	6	109	2	14	0	55	2	15	9	55	2	17	6	109	2	19	3	66
3	2	12	7	108	2	14	1	26	2	15	10	26	2	17	7	26	2	19	4	67
4	2	12	7	26	2	14	2	108	2	15	10	61	2	17	7	55	2	19	5	68
107	2	12	7	51	2	14	3	61	2	16	0	108	2	17	7	108	8	19	6	69
109	2	12	7	3	2	14	4	69	2	16	1	69	2	17	10	6	2	19	7	70
51	2	12	11	107	2	14	4	3	2	16	2	6	2	17	11	69	2	19	7	71
7	2	13	0	69	2	14	6	107	2	16	2	70	2	17	11	70	2	19	7	72
69	2	13	0	7	2	14	7	6	2	16	3	3	2	18	0	51	2	19	9	73
6	2	13	1	61	2	14	7	7	2	16	3	7	2	18	0	7	2	19	10	74
43	2	13	1	6	2	14	8	70	2	16	3	51	2	18	0	3	2	19	11	75
61	2	13	1	43	2	14	8	43	2	16	5	107	2	18	0	107	2	19	11	76
70	2	13	1	70	2	14	8	31	2	16	6	31	2	18	3	24	3	0	0	77
1	2	13	3	1	2	14	10	51	2	16	6	43	2	18	3	63	3	0	0	78
113	2	13	2	24	2	14	11	24	2	16	7	63	2	18	3	31	3	0	1	79
28	2	13	4	28	2	14	11	28	2	16	7	1	2	18	4	28	3	0	2	80
31	2	13	4	31	2	14	11	54	2	16	7	24	2	18	4	1	3	0	3	81
78	2	13	4	113	2	14	11	102	2	16	7	28	2	18	4	43	3	0	3	82
114	2	13	4	54	2	15	0	63	2	16	8	54	2	18	5	54	3	0	3	83
24	2	13	5	78	2	15	0	68	2	16	8	102	2	18	5	102	3	0	3	84
54	2	13	5	102	2	15	0	113	2	16	8	68	2	18	6	118	3	0	4	85
102	2	13	5	114	2	15	0	1	2	16	9	78	2	18	6	68	3	0	5	86
118	2	13	5	118	2	15	0	78	2	16	9	114	2	18	6	78	3	0	5	87
117	2	13	5	63	2	15	1	114	2	16	9	118	2	18	6	114	3	0	5	88
63	2	13	6	68	2	15	1	117	2	16	9	113	2	18	7	117	3	0	5	89
68	2	13	6	117	2	15	1	118	2	16	9	117	2	18	7	33	3	0	6	90
33	2	13	9	33	2	15	3	33	2	16	11	33	2	18	8	113	3	0	6	91
46	2	13	10	46	2	15	6	53	2	17	1	53	2	18	9	115	3	0	6	92
11	2	13	11	53	2	15	6	46	2	17	2	115	2	18	10	53	3	0	7	93
36	2	14	0	11	2	15	7	115	2	17	2	46	2	18	11	46	3	0	9	94
53	2	14	0	115	2	15	7	11	2	17	4	90	2	19	0	90	3	0	9	95
106	2	14	0	36	2	15	8	90	2	17	5	11	2	19	2	36	3	1	0	96
115	2	14	0	106	2	15	8	106	2	17	5	36	2	19	3	106	3	1	0	97
32	2	14	4	90	2	15	10	36	2	17	5	106	2	19	3	11	3	1	1	98
90	2	14	5	32	2	16	1	32	2	17	10	65	2	19	6	65	3	1	3	99
65	2	14	7	65	2	16	2	65	2	17	10	32	2	19	7	32	3	1	4	100
56	2	15	1	56	2	16	8	56	2	18	4	56	3	0	1	56	3	1	11	101
8				8				8				8				8				102
13				13				13				13				13				103
22				22				22				22				22				104
29				29				29				29				29				105
34				34				34				34				34				106
37				37				37				37				37				107
38				38				38				38				38				108
39				39				39				39				39				109
73				73				73				73				73				110
75				75				75				75				75				111
79				79				79				79				79				112
80				80				80				80				80				113
87				87				87				87				87				114
93				93				93				93				93				115
95				95				95				95				95				116
98				98				98				98				98				117
103				103				103				103				103				118
110				110				110				110				110				119
112				112				112				112				112				120

Age 41.				Age 42.				Age 43.				Age 44.				Age 45.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
85	2	15	9	85	2	17	4	85	2	19	0	85	3	0	7	85	3	2	5	1
17	2	16	6	17	2	18	8	116	3	0	7	116	3	2	5	30	3	4	11	2
5	2	16	10	5	2	18	10	30	3	0	9	30	3	2	9	100	3	4	11	3
30	2	17	0	30	2	18	10	17	3	0	10	100	3	2	11	14	3	5	0	4
9	2	17	1	116	2	19	0	42	3	1	0	14	3	3	2	111	3	5	0	5
64	2	17	3	40	2	19	1	100	3	1	0	17	3	3	2	94	3	5	6	6
42	2	17	4	42	2	19	2	5	3	1	1	111	3	3	2	116	3	5	6	7
116	2	17	4	9	2	19	2	64	3	1	4	42	3	3	3	17	3	5	9	8
100	2	17	7	64	2	19	3	9	3	1	5	5	3	3	5	67	3	5	9	9
27	2	17	10	100	2	19	3	14	3	1	5	94	3	3	6	77	3	5	9	10
94	2	17	10	94	2	19	7	94	3	1	6	64	3	3	7	97	3	5	10	11
40	2	17	11	67	2	19	10	111	3	1	6	9	3	3	9	99	3	5	10	12
10	2	18	1	14	2	19	11	67	3	1	9	12	3	3	9	5	3	5	11	13
35	2	18	1	111	2	19	11	12	3	1	11	67	3	3	9	25	3	6	0	14
59	2	18	1	10	3	0	0	77	3	1	11	77	3	3	9	42	3	6	0	15
67	2	18	2	27	3	0	0	10	3	2	0	41	3	3	10	64	3	6	0	16
12	2	18	3	35	3	0	0	35	3	2	0	97	3	3	10	84	3	6	1	17
19	2	18	3	59	3	0	0	41	3	2	0	99	3	3	11	71	3	6	2	18
41	2	18	3	12	3	0	1	59	3	2	0	71	3	4	0	9	3	6	3	19
14	2	18	4	71	3	0	1	71	3	2	0	10	3	4	1	88	3	6	3	20
71	2	18	4	77	3	0	1	97	3	2	0	25	3	4	1	10	3	6	4	21
77	2	18	4	41	3	0	2	40	3	2	1	35	3	4	2	35	3	6	4	22
111	2	18	4	97	3	0	2	99	3	2	1	59	3	4	2	59	3	6	4	23
72	2	18	5	19	3	0	3	19	3	2	3	21	3	4	4	101	3	6	4	24
97	2	18	5	99	3	0	5	25	3	2	3	48	3	4	4	21	3	6	5	25
52	2	18	8	25	3	0	6	21	3	2	5	60	3	4	4	48	3	6	5	26
83	2	18	8	72	3	0	6	27	3	2	5	81	3	4	4	58	3	6	5	27
45	2	18	9	105	3	0	7	48	3	2	5	96	3	4	4	60	3	6	5	28
84	2	18	9	21	3	0	8	60	3	2	5	101	3	4	4	76	3	6	5	29
99	2	18	9	45	3	0	8	96	3	2	5	105	3	4	4	81	3	6	5	30
25	2	18	10	48	3	0	8	81	3	2	5	19	3	4	5	96	3	6	5	31
119	2	18	10	52	3	0	8	101	3	2	5	58	3	4	5	105	3	6	5	32
21	2	18	11	60	3	0	8	105	3	2	5	76	3	4	5	12	3	6	6	33
48	2	18	11	76	3	0	8	58	3	2	6	88	3	4	5	41	3	6	8	34
60	2	18	11	81	3	0	8	76	3	2	6	119	3	4	6	119	3	6	8	35
76	2	18	11	84	3	0	8	119	3	2	6	45	3	4	8	19	3	6	9	36
81	2	18	11	96	3	0	8	45	3	2	8	84	3	4	8	45	3	6	11	37
96	2	18	11	101	3	0	8	84	3	2	8	40	3	4	9	20	3	7	2	38
101	2	18	11	119	3	0	8	88	3	2	8	27	3	4	11	15	3	7	2	39
105	2	18	11	58	3	0	9	72	3	2	9	15	3	5	0	2	3	7	5	40
58	2	19	0	83	3	0	11	52	3	2	10	20	3	5	0	27	3	7	6	41
57	2	19	4	88	3	0	11	15	3	3	2	72	3	5	0	52	3	7	6	42
88	2	19	4	15	3	1	4	20	3	3	2	52	3	5	1	72	3	7	6	43
47	2	19	5	20	3	1	4	2	3	3	4	2	3	5	3	120	3	7	7	44
15	2	19	6	57	3	1	4	83	3	3	4	120	3	5	6	40	3	7	9	45
20	2	19	6	2	3	1	6	57	3	3	5	57	3	5	8	89	3	7	11	46
44	2	19	7	47	3	1	7	120	3	3	7	89	3	5	8	23	3	8	0	47
86	2	19	8	44	3	1	8	89	3	3	8	23	3	5	9	62	3	8	0	48
2	2	19	9	89	3	1	8	23	3	3	9	83	3	5	10	57	3	8	1	49
18	2	19	9	120	3	1	9	4	3	3	11	62	3	5	11	4	3	8	2	50
89	2	19	9	4	3	1	10	44	3	3	11	4	3	6	0	83	3	8	6	51
120	3	0	0	86	3	1	10	47	3	3	11	44	3	6	3	16	3	8	7	52
4	3	0	1	18	3	1	11	62	3	4	0	47	3	6	4	82	3	8	7	53
92	3	0	1	62	3	2	1	18	3	4	3	16	3	6	5	44	3	8	9	54
104	3	0	2	23	3	2	0	86	3	4	3	82	3	6	5	26	3	8	11	55
23	3	0	3	92	3	2	2	82	3	4	4	18	3	6	8	47	3	9	0	56
62	3	0	3	104	3	2	4	16	3	4	5	86	3	6	8	61	3	9	2	57
50	3	0	6	82	3	2	5	92	3	4	5	26	3	6	10	86	3	9	3	58
49	3	0	7	16	3	2	6	104	3	4	7	92	3	6	10	92	3	9	3	59
82	3	0	7	50	3	2	6	49	3	4	8	49	3	6	11	49	3	9	3	60

Age 41.				Age 42.				Age 43.				Age 44.				Age 45.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
16	3	0	9	49	3	2	7	50	3	4	8	50	3	6	11	18	3	9	4	61
91	3	0	9	91	3	2	9	26	3	4	10	61	3	7	0	50	3	9	4	62
61	3	1	0	61	3	2	11	91	3	4	10	104	3	7	1	7	3	9	5	63
74	3	1	0	26	3	3	0	61	3	4	11	91	3	7	2	51	3	9	5	64
109	3	1	1	74	3	3	1	51	3	5	3	7	3	7	3	6	3	9	7	65
26	3	1	2	6	3	3	4	74	3	5	3	51	3	7	3	70	3	9	7	66
66	3	1	3	70	3	3	4	6	3	5	4	6	3	7	5	91	3	9	7	67
55	3	1	4	51	3	3	5	7	3	5	4	70	3	7	5	104	3	9	7	68
108	3	1	5	55	3	3	5	70	3	5	4	74	3	7	7	3	3	9	11	69
6	3	1	6	66	3	3	5	108	3	5	7	3	3	7	9	107	3	9	11	70
69	3	1	6	108	3	3	5	55	3	5	8	107	3	7	9	74	3	10	0	71
70	3	1	6	7	3	3	6	3	3	5	8	108	3	7	10	118	3	10	0	72
51	3	1	7	69	3	3	6	66	3	5	8	69	3	7	11	108	3	10	1	73
7	3	1	8	109	3	3	6	69	3	5	8	118	3	7	11	117	3	10	1	74
3	3	1	9	3	3	3	9	107	3	5	8	55	3	8	0	69	3	10	2	75
107	3	1	9	107	3	3	9	109	3	5	10	117	3	8	0	36	3	10	3	76
63	3	1	11	63	3	3	11	118	3	5	10	66	3	8	1	106	3	10	3	77
24	3	2	0	24	3	4	0	117	3	5	11	36	3	8	3	55	3	10	5	78
31	3	2	0	31	3	4	0	68	3	6	0	68	3	8	3	66	3	10	8	79
28	3	2	1	117	3	4	0	24	3	6	1	106	3	8	3	113	3	10	8	80
68	3	2	1	118	3	4	0	31	3	6	1	31	3	8	4	115	3	10	8	81
54	3	2	2	68	3	4	1	63	3	6	1	109	3	8	4	24	3	10	9	82
102	3	2	2	28	3	4	2	54	3	6	4	24	3	8	5	31	3	10	9	83
118	3	2	2	1	3	4	3	113	3	6	4	63	3	8	5	68	3	10	9	84
1	3	2	3	54	3	4	3	102	3	6	4	113	3	8	6	63	3	10	10	85
117	3	2	3	102	3	4	3	1	3	6	5	115	3	8	6	109	3	10	11	86
43	3	2	4	53	3	4	4	28	3	6	5	54	3	8	7	33	3	11	0	87
33	3	2	5	113	3	4	4	53	3	6	6	102	3	8	7	53	3	11	0	88
53	3	2	5	33	3	4	5	115	3	6	6	1	3	8	9	54	3	11	0	89
78	3	2	5	115	3	4	5	33	3	6	7	28	3	8	9	90	3	11	0	90
113	3	2	5	43	3	4	6	36	3	6	7	33	3	8	9	102	3	11	0	91
114	3	2	5	78	3	4	7	90	3	6	7	53	3	8	9	28	3	11	3	92
115	3	2	5	90	3	4	6	106	3	6	7	90	3	8	9	1	3	11	4	93
90	3	2	6	114	3	4	7	43	3	6	10	78	3	9	2	65	3	11	7	94
46	3	2	9	36	3	4	9	78	3	6	10	114	3	9	2	78	3	11	7	95
36	3	2	10	106	3	4	9	114	3	6	10	43	3	9	3	114	3	11	7	96
106	3	2	10	46	3	4	11	46	3	7	2	65	3	9	4	43	3	11	11	97
11	3	3	1	65	3	5	2	65	3	7	2	46	3	9	7	46	3	12	0	98
65	3	3	2	11	3	5	3	11	3	7	6	11	3	9	10	11	3	12	4	99
32	3	3	6	32	3	5	8	32	3	7	10	32	3	10	1	32	3	12	4	100
56	3	3	10	56	3	5	10	56	3	8	0	56	3	10	3	56	3	12	7	101
8				8				8				8				8				102
13				13				13				13				13				103
22				22				22				22				22				104
29				29				29				29				29				105
34				34				34				34				34				106
37				37				37				37				37				107
38				38				38				38				38				108
39				39				39				39				39				109
73				73				73				73				73				110
75				75				75				75				75				111
79				79				79				79				79				112
80				80				80				80				80				113
87				87				80				80				87				114
93				93				93				93				93				115
95				95				95				95				95				116
98				98				98				98				98				117
103				103				103				103				103				118
110				110				110				110				110				119
112				112				112				112				112				120

Age 46.				Age 47.				Age 48.				Age 49.				Age 50.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
85	3	5	6	85	3	8	9	111	3	11	5	111	3	14	1	85	3	17	0	1
100	3	7	0	111	3	9	1	100	3	11	11	85	3	14	8	111	3	17	0	2
111	3	7	0	100	3	9	4	85	3	12	0	100	3	14	9	100	3	17	11	3
94	3	7	8	94	3	10	1	99	3	12	7	99	3	15	5	99	3	18	6	4
14	3	7	10	99	3	10	1	94	3	12	9	25	3	15	8	67	3	18	8	5
30	3	7	10	30	3	10	3	67	3	12	10	67	3	15	9	25	3	18	9	6
99	3	7	10	65	3	10	4	25	3	12	11	77	3	15	9	77	3	19	0	7
77	3	7	11	77	3	10	4	77	3	12	11	97	3	15	10	97	3	19	1	8
67	3	8	0	25	3	10	5	97	3	13	0	94	3	16	0	71	3	19	3	9
25	3	8	1	97	3	10	5	71	3	13	6	71	3	16	4	94	3	19	4	10
97	3	8	1	14	3	10	10	96	3	13	6	30	3	16	5	116	3	19	6	11
116	3	8	2	48	3	10	11	105	3	13	6	60	3	16	5	10	3	19	7	12
71	3	8	5	60	3	10	11	101	3	13	6	101	3	16	5	21	3	19	8	13
17	3	8	6	71	3	10	11	17	3	13	7	17	3	16	6	35	3	19	8	14
5	3	8	7	96	3	10	11	21	3	13	7	21	2	16	6	48	3	19	8	15
21	3	8	7	105	3	10	11	30	3	13	7	48	3	16	6	59	3	19	8	16
48	3	8	7	101	3	10	11	48	3	13	7	58	3	16	6	60	3	19	8	17
58	3	8	7	17	3	11	0	58	3	13	7	76	3	16	6	96	3	19	8	18
60	3	8	7	21	3	11	0	60	3	13	7	81	3	16	6	101	3	19	8	19
64	3	8	7	58	3	11	0	76	3	13	7	96	3	16	6	105	3	19	8	20
76	3	8	7	76	3	11	0	81	3	13	7	105	3	16	6	17	3	19	9	21
81	3	8	7	81	3	11	0	10	3	13	10	10	3	16	8	30	3	19	9	22
96	3	8	7	35	3	11	2	35	3	13	11	35	3	16	8	58	3	19	9	23
101	3	8	7	59	3	11	2	59	3	13	11	59	3	16	8	76	3	19	9	24
105	3	8	7	116	3	11	2	14	3	14	2	116	3	17	0	81	3	19	9	25
10	3	8	9	10	3	11	3	119	3	14	2	45	3	17	1	45	4	0	0	26
35	3	8	9	5	3	11	4	120	3	14	2	84	3	17	1	84	4	0	0	27
42	3	8	9	119	3	11	4	5	3	14	4	120	3	17	2	120	4	0	5	28
59	3	8	9	64	3	11	5	42	3	14	4	119	3	17	4	4	4	0	9	29
119	3	8	10	42	3	11	6	45	3	14	4	42	3	17	5	82	4	0	9	30
9	3	8	11	120	3	11	7	84	3	14	4	5	3	17	6	2	4	0	10	31
88	3	8	11	45	3	11	8	64	3	14	6	2	3	17	8	119	4	0	10	32
120	3	9	2	84	3	11	8	2	3	14	7	14	3	17	8	5	4	0	11	33
12	3	9	3	9	3	11	9	116	3	14	7	12	3	17	9	19	4	1	0	34
19	3	9	3	19	3	11	10	19	3	14	8	19	3	17	9	42	4	1	1	35
45	3	9	3	88	3	11	10	9	3	14	10	64	3	17	9	64	4	1	2	36
84	3	9	3	2	3	12	1	88	3	14	10	41	3	17	11	9	4	1	4	37
41	3	9	6	12	3	12	1	12	3	14	11	9	3	18	0	14	4	1	4	38
2	3	9	7	41	3	12	2	15	3	15	0	89	3	18	0	23	4	1	6	39
15	3	9	8	15	3	12	3	20	3	15	0	4	3	18	1	89	4	1	7	40
20	3	9	8	20	3	12	3	41	3	15	0	82	3	18	1	88	4	1	7	41
52	3	10	1	23	3	12	6	23	3	15	3	88	3	18	2	15	4	1	8	42
72	3	10	1	62	3	12	6	62	3	15	3	15	3	18	3	20	4	1	8	43
62	3	10	2	89	3	12	8	89	3	15	3	20	3	18	3	12	4	1	9	44
89	3	10	2	52	3	12	10	4	3	15	5	23	3	18	3	62	4	1	10	45
23	3	10	3	72	3	12	10	82	3	15	7	62	3	18	3	6	4	2	0	46
27	3	10	4	4	3	12	11	72	3	15	9	72	3	18	10	41	4	2	0	47
4	3	10	5	82	3	13	2	52	3	15	10	52	3	19	1	86	4	2	0	48
57	3	10	8	27	3	13	4	57	3	16	3	57	3	19	1	70	4	2	0	49
40	3	10	10	57	3	13	4	83	3	16	3	6	3	19	2	72	4	2	1	50
82	3	10	10	83	3	13	5	26	3	16	4	70	3	19	2	57	4	2	3	51
83	3	10	10	16	3	13	7	16	3	16	6	26	3	19	5	61	4	2	4	52
16	3	11	0	26	3	13	8	27	3	16	6	83	3	19	5	44	4	2	5	53
26	3	11	2	40	3	14	0	6	3	16	8	61	3	19	5	52	4	2	6	54
44	3	11	4	61	3	14	0	61	3	16	8	86	3	19	6	26	4	2	9	55
61	3	11	6	44	3	14	1	70	3	16	8	16	3	19	9	83	4	3	0	56
92	3	11	8	6	3	14	2	86	3	16	10	27	3	19	9	92	4	3	3	57
7	3	11	9	7	3	14	3	7	3	16	11	51	3	19	11	7	4	3	4	58
47	3	11	9	51	3	14	2	51	3	16	11	7	4	0	0	16	4	3	4	59
6	3	11	10	70	3	14	2	44	3	17	0	92	4	0	1	27	4	3	4	60

Age 46.				Age 47.				Age 48.				Age 49.				Age 50.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
50	3	11	10	92	3	14	3	40	3	17	1	40	4	0	2	40	4	3	4	61
49	3	11	10	86	3	14	4	92	3	17	1	44	4	0	2	51	4	3	4	62
51	3	11	10	49	3	14	6	49	3	17	3	49	4	0	3	49	4	3	5	63
70	3	11	10	50	3	14	7	113	3	17	4	113	4	0	4	115	4	3	6	64
86	3	11	10	47	3	14	8	50	3	17	7	118	4	0	8	113	4	3	8	65
18	3	12	1	118	3	14	10	47	3	17	8	50	4	0	9	108	4	3	10	66
91	3	12	2	91	3	14	11	117	3	17	8	115	4	0	9	118	4	4	0	67
104	3	12	2	117	3	14	11	118	3	17	8	117	4	0	9	50	4	4	2	68
118	3	12	4	3	3	15	0	91	3	17	10	108	4	0	10	117	4	4	2	69
3	3	12	5	18	3	15	0	3	3	17	11	47	4	1	0	65	4	4	3	70
107	3	12	5	107	3	15	0	107	3	17	11	91	4	1	0	69	4	4	3	71
117	3	12	5	104	3	15	1	108	3	17	11	3	4	1	2	74	4	4	4	72
74	3	12	7	113	3	15	1	115	3	18	0	69	4	1	2	47	4	4	5	73
108	3	12	7	108	3	15	2	18	3	18	1	74	4	1	2	91	4	4	5	74
69	3	12	9	74	3	15	3	74	3	18	1	107	4	1	2	104	4	4	7	75
55	3	12	11	69	3	15	5	104	3	18	1	104	4	1	3	90	4	4	8	76
113	3	12	11	115	3	15	5	69	3	18	2	65	4	1	5	3	4	4	9	77
115	3	13	0	55	3	15	7	36	3	18	6	18	4	1	5	107	4	4	9	78
36	3	13	2	36	3	15	8	55	3	18	6	36	4	1	8	53	4	4	10	79
106	3	13	2	106	3	15	8	106	3	18	6	55	4	1	8	18	4	4	11	80
24	3	13	3	31	3	15	11	65	3	18	9	90	4	1	8	55	4	4	11	81
31	3	13	3	24	3	16	0	90	3	18	9	106	4	1	8	31	4	5	2	82
66	3	13	4	66	3	16	1	24	3	18	10	53	4	1	10	36	4	5	2	83
68	3	13	5	90	3	16	1	31	3	18	10	31	4	1	11	54	4	5	2	84
90	3	13	5	53	3	16	2	53	3	18	11	24	4	2	0	102	4	5	2	85
53	3	13	6	33	3	16	3	54	3	19	1	54	4	2	0	106	4	5	2	86
63	3	13	6	54	3	16	3	66	3	19	1	102	4	2	0	24	4	5	3	87
33	3	13	7	63	3	16	3	102	3	19	1	109	4	2	2	68	4	5	3	88
54	3	13	7	65	3	16	3	33	3	19	2	33	4	2	3	109	4	5	5	89
102	3	13	7	102	3	16	3	63	3	19	2	68	4	2	3	33	4	5	6	90
109	3	13	7	109	3	16	4	109	3	19	2	66	4	2	3	32	4	5	7	91
65	3	13	10	68	3	16	7	68	3	19	4	63	4	2	4	63	4	5	7	92
28	3	13	11	28	3	16	8	28	3	19	7	28	4	2	9	66	4	5	8	93
1	3	14	0	1	3	16	10	1	3	19	11	32	4	2	10	28	4	6	1	94
78	3	14	3	78	3	17	0	78	3	19	11	78	4	3	0	78	4	6	4	95
114	3	14	3	114	3	17	0	114	3	19	11	114	4	3	0	114	4	6	4	96
43	3	14	8	32	3	17	4	32	4	0	1	1	4	3	2	1	4	6	7	97
46	3	14	8	46	3	17	6	46	4	0	6	46	4	3	7	56	4	6	9	98
32	3	14	10	43	3	17	7	56	4	0	7	56	4	3	7	46	4	6	11	99
11	3	15	0	56	3	17	9	11	4	0	9	11	4	3	11	11	4	7	3	100
56	3	15	1	11	3	17	10	43	4	0	9	43	4	4	0	43	4	7	6	101
8				8				8				8				8				102
13				13				13				13				13				103
22				22				22				22				22				104
29				29				29				29				29				105
34				34				34				34				34				106
37				37				37				37				37				107
38				38				38				38				38				108
39				39				39				39				39				109
73				73				73				73				73				110
75				75				75				75				75				111
79				79				79				79				79				112
80				80				80				80				80				113
87				87				87				87				87				114
93				93				93				93				93				115
95				95				95				95				95				116
98				98				98				98				98				117
103				103				103				103				103				118
110				110				110				110				110				119
112				112				112				112				112				120

Age 51.				Age 52.				Age 53.				Age 54.				Age 55.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
85	3	19	6	85	4	2	5	85	4	5	0	85	4	7	8	85	4	10	4	1
111	4	0	3	111	4	3	7	111	4	7	3	116	4	10	4	111	4	15	3	2
100	4	1	5	100	4	5	0	116	4	7	8	111	4	11	1	67	4	16	2	3
67	4	1	10	116	4	5	0	67	4	8	8	67	4	12	3	116	4	16	4	4
99	4	2	0	67	4	5	2	100	4	9	0	45	4	12	11	45	4	16	8	5
25	4	2	1	25	4	5	8	45	4	9	5	84	4	12	11	84	4	16	8	6
116	4	2	5	99	4	5	8	71	4	9	5	100	4	13	2	71	4	17	2	7
71	4	2	6	71	4	5	10	84	4	9	5	71	4	13	3	10	4	17	5	8
77	4	2	6	10	4	6	1	25	4	9	6	10	4	13	5	35	4	17	5	9
97	4	2	7	35	4	6	2	99	4	9	6	35	4	13	5	59	4	17	5	10
94	4	2	8	45	4	6	2	10	4	9	8	59	4	13	5	100	4	17	9	11
10	4	2	9	84	4	6	2	35	4	9	8	25	4	13	7	4	4	17	10	12
35	4	2	9	77	4	6	3	59	4	9	8	99	4	13	8	6	4	17	10	13
57	4	2	9	97	4	6	4	77	4	10	3	4	4	14	1	25	4	18	0	14
45	4	3	0	94	4	6	8	4	4	10	4	94	4	14	2	94	4	18	0	15
84	4	3	0	30	4	6	9	94	4	10	4	6	4	14	3	99	4	18	2	16
17	4	3	3	4	4	6	10	97	4	10	4	77	4	14	6	82	4	18	10	17
21	4	3	3	82	4	6	11	82	4	10	6	82	4	14	6	77	4	19	1	18
48	4	3	3	60	4	7	0	6	4	11	0	97	4	14	7	61	4	19	1	19
60	4	3	3	101	4	7	0	17	4	11	1	61	4	14	8	97	3	19	2	20
81	4	3	3	105	4	7	0	21	4	11	1	30	4	15	0	5	4	19	3	21
96	4	3	3	21	4	7	1	48	4	11	1	21	4	15	4	19	4	19	11	22
105	4	3	3	48	4	7	1	58	4	11	1	48	4	15	4	57	4	19	11	23
30	4	3	4	81	4	7	1	60	4	11	1	60	4	15	4	17	5	0	0	24
58	4	3	4	96	4	7	1	76	4	11	1	96	4	15	4	21	5	0	0	25
76	4	3	4	17	4	7	2	81	4	11	1	101	4	15	4	30	5	0	0	26
4	4	3	8	58	4	7	2	96	4	11	1	105	4	15	4	48	5	0	0	27
82	4	3	8	76	4	7	3	105	4	11	1	5	4	15	5	58	5	0	0	28
120	4	4	0	6	4	7	10	30	4	11	2	17	4	15	5	60	5	0	0	29
19	4	4	5	120	4	7	10	61	4	11	6	58	4	15	5	76	5	0	0	30
2	4	4	6	19	4	7	11	5	4	11	8	76	4	15	5	81	5	0	0	31
5	4	4	6	5	4	8	0	19	4	11	8	81	4	15	5	96	5	0	0	32
119	4	4	8	61	4	8	4	120	4	12	0	19	4	15	9	105	5	0	0	33
6	4	4	10	2	4	8	5	2	4	12	5	57	4	16	2	115	5	0	0	34
64	4	4	10	64	4	8	8	57	4	12	6	120	4	16	3	52	5	0	1	35
9	4	5	0	9	4	8	9	64	4	12	7	52	4	16	5	65	5	0	6	36
14	4	5	0	88	4	8	9	88	4	12	7	115	4	16	5	42	5	0	8	37
42	4	5	0	119	4	8	10	14	4	12	8	64	4	16	8	92	5	0	10	38
88	4	5	1	14	4	8	10	9	4	12	10	88	4	16	8	120	5	0	11	39
23	4	5	3	42	4	8	11	42	4	12	10	42	4	16	9	64	5	1	0	40
89	4	5	3	23	4	9	0	52	4	12	10	2	4	16	10	88	5	1	5	41
61	4	5	4	57	4	9	0	89	4	12	10	89	4	16	10	89	5	1	1	42
57	4	5	7	89	4	9	0	72	4	13	0	65	4	16	11	108	5	1	1	43
72	4	5	7	70	4	9	2	115	4	13	0	92	4	17	0	69	5	1	2	44
15	4	5	8	72	4	9	2	23	4	13	3	9	4	17	1	90	5	1	2	45
20	4	5	8	52	4	9	4	70	4	13	3	14	4	17	1	2	5	1	6	46
62	4	5	8	62	4	9	7	92	4	13	4	72	4	17	2	53	5	1	6	47
12	4	5	9	15	4	9	8	119	4	13	4	108	4	17	3	72	5	1	6	48
41	4	5	9	20	4	9	8	65	4	13	6	23	4	17	6	9	5	1	8	49
70	4	5	9	12	4	9	9	62	4	13	8	69	4	17	6	14	5	1	9	50
86	4	5	9	41	4	9	9	108	4	13	8	70	4	17	6	49	5	2	1	51
52	4	5	11	115	4	9	9	12	4	13	9	90	4	17	7	70	5	2	1	52
113	4	6	2	86	4	9	10	49	4	13	11	12	4	17	9	40	5	2	2	53
26	4	6	5	92	4	9	10	40	4	14	0	49	4	17	10	23	5	2	3	54
92	4	6	6	108	4	10	2	41	4	14	0	53	4	17	11	86	5	2	4	55
115	4	6	7	40	4	10	3	86	4	14	0	40	4	18	0	12	5	2	10	56
40	4	6	8	49	4	10	3	69	4	14	1	41	4	18	0	41	5	2	10	57
49	4	6	9	65	4	10	3	15	4	14	2	86	4	18	0	62	5	2	10	58
44	4	6	11	113	4	10	3	20	4	14	2	119	4	18	0	44	5	3	0	59
7	4	7	0	26	4	10	4	90	4	14	3	62	4	18	1	119	5	3	0	60

Age 51.				Age 52.				Age 53.				Age 54.				Age 55.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
51	4	7	0	44	4	10	7	113	4	14	4	44	4	18	8	63	5	3	4	61
83	4	7	0	69	4	10	9	26	4	14	6	113	4	18	8	113	5	3	5	62
108	4	7	0	7	4	11	0	44	4	14	6	15	4	18	9	15	5	3	6	63
27	4	7	1	27	4	11	0	53	4	14	7	20	4	18	9	20	5	3	6	64
16	4	7	2	51	4	11	0	27	4	15	2	26	4	19	0	68	5	3	8	65
65	4	7	2	90	4	11	0	51	4	15	2	27	4	19	4	26	5	3	9	66
69	4	7	6	16	4	11	1	7	4	15	3	68	4	19	4	27	5	3	10	67
90	4	7	9	83	4	11	2	16	4	15	5	63	4	19	6	7	5	4	2	68
50	4	7	10	53	4	11	3	68	4	15	5	7	4	19	8	74	5	4	2	69
74	4	7	10	74	4	11	6	74	4	15	5	51	4	19	8	54	5	4	3	70
118	4	7	10	55	4	11	9	55	4	15	9	74	4	19	8	102	5	4	3	71
117	4	7	11	118	4	11	9	83	4	15	9	16	5	0	0	51	5	4	6	72
47	4	8	0	68	4	11	10	63	4	15	10	54	5	0	0	104	5	4	6	73
53	4	8	0	50	4	11	10	91	4	15	10	55	5	0	0	109	5	4	6	74
55	4	8	0	91	4	11	10	104	4	15	11	102	5	0	0	91	5	4	8	75
91	4	8	1	104	4	11	10	47	4	16	0	91	5	0	1	55	5	4	9	76
104	4	8	1	117	4	11	11	54	4	16	0	104	5	0	1	16	5	4	11	77
68	4	8	5	47	4	12	0	102	4	16	0	47	5	0	4	46	5	4	11	78
18	4	8	7	54	4	12	2	50	4	16	1	109	5	0	4	47	5	5	0	79
54	4	8	7	102	4	12	2	118	4	16	1	83	5	0	6	50	5	5	6	80
102	4	8	7	32	4	12	4	117	4	16	2	118	5	0	7	66	5	5	6	81
31	4	8	9	63	4	12	4	109	4	16	4	50	5	0	8	118	5	5	6	82
24	4	8	10	18	4	12	6	18	4	16	8	117	5	0	9	28	5	5	8	83
32	4	8	10	109	4	12	6	33	4	16	9	46	5	1	0	83	5	5	8	84
36	4	8	10	31	4	12	7	24	4	16	10	66	5	1	0	117	5	5	8	85
106	4	8	10	24	4	12	8	31	4	16	10	18	5	1	1	18	5	5	10	86
109	4	8	10	33	4	12	10	32	4	16	10	28	5	1	3	33	5	6	2	87
3	4	8	11	36	4	12	10	66	4	16	10	31	5	1	4	1	5	6	3	88
63	4	8	11	106	4	12	10	28	4	17	1	33	5	1	4	24	5	6	4	89
107	4	8	11	66	4	12	11	36	4	17	3	24	5	1	5	31	5	6	4	90
33	4	9	1	3	4	13	2	106	4	17	3	36	5	1	9	78	5	6	8	91
66	4	9	2	107	4	13	2	46	4	17	4	106	5	1	9	114	5	6	8	92
28	4	9	7	28	4	13	3	78	4	17	7	32	5	1	10	32	5	6	10	93
78	4	9	10	78	4	13	7	114	4	17	7	78	5	2	0	36	5	6	10	94
114	4	9	10	114	4	13	7	3	4	17	9	114	5	2	0	106	5	6	10	95
56	4	10	3	46	4	13	9	107	4	17	9	56	5	2	3	56	5	7	0	96
1	4	10	4	56	4	13	11	56	4	17	11	1	5	2	5	11	5	7	9	97
46	4	10	4	1	4	14	3	1	4	18	5	3	5	2	9	3	5	8	0	98
11	4	10	10	11	4	14	7	11	4	18	8	107	5	2	9	107	5	8	0	99
43	4	11	3	40	4	15	3	43	4	19	5	11	5	3	0	43	5	8	7	100
8				43	4	15	3	8				43	5	3	10	8				101
13				8				13				8				13				102
22				13				22				13				22				103
29				22				29				22				29				104
34				29				34				29				34				105
37				34				37				34				37				106
38				37				38				37				38				107
39				38				39				38				39				108
73				39				73				39				73				109
75				73				75				73				75				110
79				75				79				75				79				111
80				79				80				79				80				112
87				80				87				80				87				113
93				87				93				87				93				114
95				93				95				93				95				115
98				95				98				95				98				116
101				98				101				98				101				117
103				103				103				103				103				118
110				110				110				110				110				119
112				112				112				112				112				120

Age 56.				Age 57.				Age 58.				Age 59.				Age 60.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
85	4	17	2	85	5	4	5	45	5	9	4	45	5	14	8	111	5	19	11	1
111	4	19	9	111	5	4	7	84	5	9	4	84	5	14	8	45	6	0	0	2
67	5	0	6	84	5	4	10	111	5	9	8	111	5	14	10	84	6	0	0	3
45	5	0	7	45	5	4	10	67	5	10	0	67	5	15	0	71	6	0	1	4
84	5	0	7	67	5	5	0	71	5	10	2	71	5	15	0	67	6	0	6	5
71	5	1	4	71	5	5	8	4	5	10	6	4	5	15	5	5	6	0	10	6
35	5	1	8	4	5	6	2	35	5	10	11	94	5	16	0	82	6	1	0	7
59	5	1	8	35	5	6	2	59	5	10	11	35	5	16	1	4	6	1	6	8
10	5	1	9	59	5	6	2	94	5	11	1	59	5	16	1	35	6	1	6	9
94	5	1	11	10	5	6	5	10	5	11	2	5	5	16	3	59	6	1	6	10
4	5	2	0	94	5	6	5	5	5	11	9	10	5	16	4	94	6	1	10	11
116	5	2	4	5	5	7	5	85	5	11	9	82	5	16	6	115	6	1	10	12
25	5	2	9	82	5	7	8	82	5	12	1	115	5	16	9	57	6	1	11	13
99	5	2	11	115	5	7	9	115	5	12	2	57	5	16	10	90	6	2	3	14
5	5	3	3	25	5	7	11	57	5	12	3	65	5	17	1	10	6	2	6	15
82	5	3	3	57	5	7	11	52	5	12	8	52	5	17	5	52	6	2	6	16
115	5	3	8	21	5	8	1	65	5	12	8	90	5	17	7	53	6	2	6	17
57	5	3	10	105	5	8	1	90	5	13	2	21	5	17	9	65	6	2	6	18
61	5	3	10	99	5	8	2	19	5	13	3	105	5	17	9	21	6	2	8	19
52	5	4	0	52	5	8	3	25	5	13	4	53	5	17	11	105	6	2	8	20
77	5	4	0	116	5	8	4	105	5	13	5	69	5	18	1	69	6	2	11	21
19	5	4	1	65	5	8	4	21	5	13	5	17	5	18	5	17	6	3	1	22
21	5	4	2	19	5	8	6	69	5	13	6	19	5	18	5	76	6	3	1	23
105	5	4	2	61	5	8	9	53	5	13	7	76	5	18	5	101	6	3	4	24
17	5	4	4	17	5	8	10	99	5	13	7	25	5	18	10	19	6	4	1	25
65	5	4	4	76	5	8	10	17	5	13	8	30	5	19	0	108	6	4	2	26
76	5	4	4	90	5	8	11	76	5	13	8	108	5	19	0	25	6	4	4	27
30	5	4	5	30	5	9	0	101	5	13	8	99	5	19	1	30	6	4	4	28
101	5	4	6	69	5	9	2	30	5	13	11	42	5	19	4	99	6	4	6	29
58	5	4	11	92	5	9	3	92	5	14	0	85	5	19	6	42	6	5	3	30
92	5	4	11	53	5	9	4	61	5	14	0	92	5	19	6	92	6	5	6	31
48	5	5	0	77	5	9	4	108	5	14	1	61	5	19	7	61	6	5	6	32
60	5		0	108	5	9	5	116	5	14	5	64	6	0	4	64	6	5	10	33
81	5	5	0	42	5	9	10	42	5	14	6	89	6	0	6	70	6	5	10	34
90	5	5	0	58	5	9	10	77	5	14	11	70	6	0	6	63	6	5	11	35
96	5	5	0	64	5	10	3	58	5	15	2	116	6	0	6	89	6	6	0	36
120	5	5	0	89	5	10	3	64	5	15	2	77	6	0	7	77	6	6	3	37
69	5	5	1	48	5	10	4	89	5	15	3	58	6	0	8	116	6	6	7	38
108	5	5	1	60	5	10	4	70	5	15	6	63	6	0	11	58	6	6	8	39
42	5	5	2	81	5	10	4	48	5	16	0	49	6	1	7	49	6	7	3	40
53	5	5	4	96	5	10	4	60	5	16	0	81	6	1	8	46	6	7	4	41
64	5	5	6	120	5	10	4	81	5	16	0	48	6	1	9	48	6	7	4	42
89	5	5	7	70	5	10	9	120	5	16	0	60	6	1	9	85	6	7	4	43
72	5	6	1	72	5	11	0	96	5	16	0	72	6	1	9	96	6	7	4	44
70	5	6	4	49	5	11	3	72	5	16	2	96	6	1	9	120	6	7	4	45
2	5	6	6	9	5	11	8	63	5	16	3	120	6	1	9	60	6	7	5	46
9	5	6	6	40	5	11	8	49	5	16	3	40	6	2	2	81	6	7	5	47
49	5	6	6	63	5	11	9	40	5	16	9	46	6	2	8	72	6	7	8	48
14	5	6	9	2	5	12	0	9	5	17	1	9	6	2	11	40	6	8	1	49
40	5	6	9	44	5	12	8	2	5	17	9	12	6	3	6	12	6	8	8	50
23	5	7	3	23	5	12	9	46	5	17	11	41	6	3	6	9	6	9	1	51
86	5	7	4	68	5	13	1	44	5	17	11	44	6	3	6	41	6	9	4	52
63	5	7	5	14	5	13	1	68	5	18	3	2	6	3	7	44	6	9	5	53
44	5	7	8	12	5	13	2	12	5	18	4	68	6	3	9	2	6	9	6	54
12	5	8	0	46	5	13	4	41	5	18	8	54	6	4	2	68	6	9	8	55
41	5	8	0	41	5	13	6	54	5	18	8	102	6	4	2	54	6	10	0	56
62	5	8	0	62	5	13	6	102	5	18	8	23	6	4	6	102	6	10	0	57
68	5	8	2	86	5	13	6	23	5	18	9	27	6	4	7	23	6	10	3	58
119	5	8	4	27	5	13	7	27	5	18	11	109	6	5	0	27	6	10	6	59
27	5	8	7	54	5	13	7	109	5	19	1	62	6	5	2	91	6	11	2	60

Age 56.				Age 57.				Age 58.				Age 59.				Age 60.				
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
113	5	8	7	102	5	13	7	62	5	19	3	104	6	5	2	62	6	11	3	61
54	5	8	9	109	5	13	9	104	5	19	6	91	6	5	4	104	6	11	3	62
102	5	8	9	113	5	14	1	14	5	19	8	119	6	5	8	109	6	11	6	63
15	5	8	10	104	5	14	2	91	5	19	9	74	6	5	9	113	6	11	8	64
20	5	8	10	119	5	14	2	86	6	19	10	14	6	5	10	74	6	11	10	65
26	5	8	11	74	5	14	5	74	5	19	11	113	6	5	10	28	6	12	1	66
109	5	8	11	15	5	14	6	113	6	0	1	28	6	6	2	26	6	12	3	67
46	5	9	0	20	5	14	6	119	6	0	4	26	6	6	4	55	6	12	6	68
7	5	9	1	26	5	14	6	26	6	0	5	47	6	6	4	47	6	12	7	69
104	5	9	2	91	5	14	6	15	6	0	6	55	6	6	5	14	6	12	8	70
74	5	9	2	7	5	14	7	20	6	0	6	66	6	6	5	66	6	12	8	71
91	5	9	2	47	5	15	0	47	6	0	6	15	6	6	6	16	6	12	9	72
55	5	9	8	55	5	15	0	55	6	0	6	20	6	6	6	15	6	13	1	73
51	5	9	9	16	5	15	2	28	6	0	7	16	6	6	8	20	6	13	1	74
47	5	9	9	66	5	15	3	16	6	0	8	86	6	6	11	51	6	13	2	75
16	5	9	10	28	5	15	4	66	6	0	8	7	6	7	0	86	6	13	2	76
66	5	10	2	51	5	15	4	7	6	1	3	51	6	7	3	119	6	13	2	77
28	5	10	4	50	5	15	9	51	6	1	3	50	6	7	3	7	6	13	3	78
50	5	10	6	18	5	16	1	50	6	1	4	18	6	7	9	50	6	13	6	79
118	5	10	8	118	5	16	4	18	6	1	9	118	6	8	4	18	6	14	2	80
18	5	10	10	117	5	16	7	118	6	2	4	32	6	8	5	118	6	14	2	81
117	5	10	11	33	5	16	9	24	6	2	6	33	6	8	5	32	6	14	7	82
33	5	11	4	24	5	16	10	31	6	2	6	24	6	8	6	117	6	14	7	83
24	5	11	6	31	5	16	10	33	6	2	6	31	6	8	6	1	6	14	8	84
31	5	11	6	78	5	17	1	117	6	2	6	117	6	8	7	33	6	14	8	85
78	5	11	8	114	5	17	1	32	6	2	8	78	6	8	8	31	6	14	10	86
114	5	11	8	32	5	17	2	78	6	2	9	114	6	8	8	78	6	14	10	87
32	5	11	11	56	5	17	5	114	6	2	9	1	6	8	9	114	6	14	10	88
56	5	12	1	1	5	17	6	1	6	3	0	56	6	9	1	24	6	14	11	89
36	5	12	2	36	5	17	10	56	6	3	1	36	6	9	11	56	6	15	5	90
106	5	12	2	106	5	17	10	11	6	4	0	106	6	9	11	36	6	15	10	91
1	5	12	3	11	5	18	3	36	6	4	0	11	6	10	0	106	6	15	10	92
11	5	12	10	43	5	18	11	106	6	4	0	43	6	10	7	11	6	16	3	93
43	5	13	7	3	5	19	11	43	6	4	7	3	6	13	0	43	6	17	0	94
107	5	13	9	107	5	19	11	3	6	6	5	107	6	13	0	3	6	19	6	95
3	5	13	9	83	6	0	7	107	6	6	5	83	6	17	3	107	6	19	6	96
83	5	14	6	6				83	6	10	7	6				83	7	4	0	97
6	5	15	5	8				6				8				6				98
8				13				8				13				8				99
13				22				13				22				13				100
22				29				22				29				22				101
29				34				29				34				29				102
34				37				34				37				34				103
37				38				37				38				37				104
38				39				38				39				38				105
39				73				39				73				39				106
73				75				73				75				73				107
75				79				75				79				75				108
79				80				79				80				79				109
80				87				80				87				80				110
87				88				87				88				87				111
88				93				88				93				88				112
93				95				93				95				93				113
95				97				95				97				95				114
97				98				97				98				97				115
98				100				98				100				98				116
100				101				100				101				100				117
103				103				103				103				103				118
110				110				110				110				110				119
112				112				112				112				112				120

A NEW MODE OF CONSTRUCTING NEW TABLES.

Office 16 had a set of Original Tables calculated for its special use. In the course of a few years' business it was considered prudent, for the safety both of the Company and the Assured, to raise its Premiums with one year, viz., by charging, for instance, for Age 30 what it formerly charged for 31, and so on. A new Office was ushered into life, *i. e.*, No. 58, which, without ceremony, adopted the old rates of No. 16. How Office 58 is better able to manage with lower rates then No. 16, is of course a matter of speculation.

The Commercial and General Life Office had equally a new set of Tables of its own, but it being wound up, its Tables have been made use of by Offices Nos. 15 and 20.

The Mercantile, Professional, and General Life Insurance and Guarantee Company. This long-titled establishment was called into existence by one William John Frederick Norfolk.* Seven clerks, himself included, formed the Company, which was incorporated on the 17th of June, 1864, under the Limited Liability Act of 1862. A set of Tables has been prepared for that Company by myself, on the basis of the last English Life Table, 3½ per cent. A more judicious Table could scarcely have been recommended. Had 3 per cent. been used, the Premiums would have exceeded those of all other Offices, as is shown by the Government Tables; but 3½ per cent. I considered as safe, and at the same time as producing very moderate rates. But this Company having been metamorphosed four times in one year, by having made experiments in Titles, Managers, Directors, Actuaries, etc., its present Manager, Captain R. R. Hutchinson, F.R.G.S., and F.A.S.L., considered the rates, as calculated by me, too high for competition, and therefore has adopted a very ingenious plan for re-constructing the same. The Manager, previous to his present exalted function, happened to be agent to the Western Life Office. Having had great faith in the Tables of this latter Company, the Captain thought he could do no better than cut away 5 per cent. of the Western rates, and thereby obtain a Table of a new and superior quality. How was this effected? In a very novel and interesting style. In order to explain the subject the better, I shall produce here my own table, styled "*The English*," "*The Western*," and "*The National Standard*." The first thing the Captain did was to knock off of "*The Western*" rates 5 per cent. from age 25 to 50; while, of "*The English*" rates, he wrenched off 10 per cent. from age 50 to 60. But then the new table exhibited still some imperfection, especially in the younger ages; well, another remedy was resorted to, 2d. were put on the premium of age 20, 3d. on age 21, 4d. on age 22, and so on, always increasing the loading with one extra penny at each age up to age 40, where the loading reached 1s. 7d. But no extras were allowed from age 40 to 50. On what data the Captain could support his own theorem in constructing tables in the manner described remains, naturally, a puzzle.† Perhaps the Captain will be kind enough to state his theory in full, as it might possibly benefit the actuary, and might shorten the labour in framing new tables when required. The Table here described being too late for insertion in the body of the present Tables, will be found at page 39.

Some offices have changed their Tables by either raising or reducing their rates. One of these is office 38, "*The Friend in Need*," which has reduced its rates considerably. The rates embodied in the present Tables are those formerly adopted, but the new rates will be found on the opposite page.

* The well known-promoter of "The Life Association of England," "The International Agency Company," etc., both of which institutions succumbed within last year, after a few months' existence. This Company was first christened "The Southern." Subsequently, in September last, it assumed the very long title of "Mercantile, etc," and now calls itself "The National Standard." Its Chairman is a Mr. T. Bendyshe, styled a barrister.

† Independently of this peculiar mode of constructing his table, the Captain committed two cardinal errors: at age 47 he states his premium to be £3 0s. 1d., in lieu of £4 0s. 1d.! and at age 60, £6 15s. 3d., for £6 15s. 8d.!

Annual Premiums for the Assurance of £100, payable at Death.

Age.	WITH PROFITS.												WITHOUT PROFITS.			Age.
	English.			Western.			National Standard.			Friend in Need.						
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
20	1	18	2	1	17	9	1	16	0	1	18	6	1	12	4	20
21	1	19	2	1	18	7	1	17	0	1	19	9	1	13	6	21
22	2	0	2	1	19	7	1	18	2	2	1	2	1	14	6	22
23	2	1	2	2	0	6	1	19	2	2	2	4	1	15	7	23
24	2	2	2	2	1	6	2	0	2	2	3	6	1	16	7	24
25	2	3	3	2	2	7	2	1	1	2	4	9	1	17	8	25
26	2	4	5	2	3	9	2	2	3	2	6	0	1	18	8	26
27	2	5	7	2	4	10	2	3	5	2	7	2	1	19	8	27
28	2	6	10	2	6	0	2	4	7	2	8	4	2	0	8	28
29	2	8	1	2	7	4	2	5	9	2	9	6	2	1	9	29
30	2	9	5	2	8	6	2	7	0	2	10	8	2	2	10	30
31	2	10	10	2	9	10	2	8	4	2	12	0	2	4	0	31
32	2	12	4	2	11	4	2	9	9	2	13	6	2	5	2	32
33	2	13	11	2	12	10	2	11	3	2	15	0	2	6	6	33
34	2	15	7	2	14	5	2	12	10	2	16	6	2	7	10	34
35	2	17	3	2	15	11	2	14	2	2	18	0	2	9	4	35
36	2	19	1	2	17	8	2	16	7	2	19	8	2	10	8	36
37	3	1	0	2	19	5	2	18	0	3	1	4	2	12	4	37
38	3	3	0	3	1	4	3	0	0	3	3	0	2	14	0	38
39	3	5	1	3	3	5	3	1	10	3	4	8	2	15	8	39
40	3	7	4	3	5	6	3	2	0	3	6	4	2	17	4	40
41	3	9	8	3	7	8	3	4	5	3	7	10	2	19	0	41
42	3	12	1	3	9	11	3	6	8	3	9	10	3	1	2	42
43	3	14	8	3	12	3	3	9	1	3	12	4	3	3	4	43
44	3	17	5	3	14	1	3	11	8	3	15	0	3	5	8	44
45	4	0	3	3	17	2	3	14	3	3	17	8	3	8	4	45
46	4	3	3	3	19	10	3	17	1	4	0	8	3	11	4	46
47	4	6	6	4	2	10	3	0	1	4	4	0	3	14	6	47
48	4	9	10	4	6	3	4	3	10	4	7	8	3	18	0	48
49	4	13	6	4	9	1	4	6	6	4	11	8	4	1	6	49
50	4	17	4	4	12	0	4	7	7	4	16	0	4	5	0	50
51	5	1	4	4	15	4	4	11	3	5	0	4	4	9	0	51
52	5	5	7	4	18	4	4	15	1	5	4	8	4	13	4	52
53	5	10	1	5	1	6	4	19	1	5	9	0	4	17	10	53
54	5	14	9	5	4	8	5	3	3	5	13	4	5	2	4	54
55	5	19	10	5	11	2	5	7	10	5	17	8	5	7	0	55
56	6	5	3	5	17	8	5	12	9	6	2	8	5	11	8	56
57	6	11	0	6	4	2	5	17	11	6	8	0	5	16	6	57
58	6	17	2	6	10	9	6	3	5	6	13	8	6	1	6	58
59	7	3	8	6	17	4	6	9	4	6	19	4	6	6	8	59
60	7	10	9	7	3	11	6	15	3	7	4	0	6	12	0	60

THE END.